MATHEMATICAL

AND

ASTRONOMICAL TABLES,

FOR THE USE OF

STUDENTS IN MATHEMATICS,

PRACTICAL ASTRONOMERS, SURVEYORS, ENGINEERS,
AND NAVIGATORS;

PRECEDED BY

AN INTRODUCTION,

CONTAINING

THE CONSTRUCTION OF LOGARITHMIC AND TRIGONOMETRICAL TABLES,
PLANE AND SPHERICAL TRIGONOMETRY,

THEIR APPLICATION TO NAVIGATION, ASTRONOMY, SURVEYING,
AND GEODETICAL OPERATIONS,

WITH

AN EXPLANATION OF THE TABLES,

ILLUSTRATED BY

NUMEROUS PROBLEMS AND EXAMPLES.

SECOND EDITION,
GREATLY ENLARGED AND IMPROVED.

BY WILLIAM GALBRAITH, M.A.,
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EDINBURGH:
PUBLISHED BY
OLIVER & BOYD, TWEEDDALE COURT;
SIMPKIN & MARSHALL, AND J. W. NORIE & CO.,
LONDON.

1834.

Printed by Oliver & Boyd,
Tweeddale Court, High Street, Edinburgh.

TO

SIR GEORGE CLERK, OF PENNYCUICK,

BART., F.R.S., &c. &c. &c.

SIR,

THE following Work, which you have been pleased to allow me the honour of inscribing to you, is intended to serve the purposes of useful instruction, and to promote the advancement of practical science.

THOUGH sensible that the success of a Work of this nature must depend upon its own merits, yet I have been solicitous to avail myself of your patronage, in the hope that practical men, in search of useful knowledge, may be induced to consult a Book sanctioned by a name intimately connected with many recent scientific improvements.

I have the honour to be,

SIR,

With the utmost respect,

Your most obedient servant,

WILLIAM GALBRAITH.

EDINBURGH, *Nov.* 1830.

PREFACE

TO THE SECOND EDITION.

THE application of the mathematical sciences to practical purposes has of late made great advances in accuracy and precision. The perfection also which astronomical and geodetical operations have reached, and the extreme delicacy to which the construction of instruments has been carried, require corresponding improvements in the methods of computation and reduction. Convenient tables of moderate expense must therefore be of great value to those engaged either in the details of practice, or in the art of instruction.

There are two descriptions of tables chiefly in use; one large and expensive, which can with difficulty be procured by the generality of purchasers; the other so limited and defective as to be unfit for constant reference. It has been my study to avoid both these objections, by making such additions to the smaller tables as will render their application more easy, without greatly increasing their bulk,—by selecting the most useful from larger collections,—by supplying some new tables, and simplifying the practical rules. Several laborious processes have therefore been rendered more commodious and precise, while the requisite accuracy for the nicest purposes has been strictly preserved.

In most of our initiatory works for popular instruction, the processes and examples are unfortunately conducted in such a manner as to be comparatively of little advantage in actual practice, and, consequently, what has been learned in youth, must, in a great degree, be forgotten in manhood, while new methods are then to be acquired.

To remedy this inconvenience, some of the most approved modes of treating the problems frequently required by Astrono-

mers, Navigators, and Engineers, have been selected from the works of authors celebrated for their successful application of the exact sciences to the niceties of modern practice.

I have therefore taken many of the Astronomical Rules and Examples from the works of Maskelyne, Pond, Bessel, and Brinkley; and such as relate to other topics from those of Captains Kater, Hall, Sabine, and Parry. To Lieutenant-General Sir Thomas Makdougal Brisbane, K. C. B., and Captain Basil Hall, R. N., I am under considerable obligations, not only for access to their original papers, but also for their friendly advice relative to the application of these methods to practice. In some instances I have also profited by the remarks of my learned preceptor, Professor Leslie. I am indebted to Mr Ivory for his very accurate Table of Astronomical Refractions, which I have endeavoured to improve by expanding it, and adding proportional parts to the subsidiary tables, thereby facilitating their practical application. To Mr Francis Baily, whose ardour for the advancement of astronomy is well known, is due the method of determining the longitude by meridional transits of the moon, reprinted from his valuable collection of Astronomical Tables and Formulæ. Besides endeavouring to improve many of the ordinary Tables, I have added several which are new, chiefly for the purpose of simplifying some operations and rendering others more accurate.

The first hundred pages of the general tables, after a careful comparison with others of the highest character for accuracy, were stereotyped in the first edition. In expectation of this second edition they have again been examined, and every error which was discovered has been corrected. It may be observed, however, that the first edition of the *stereotype tables* has proved so accurate, that in the Table of Logarithms of Numbers *one error*, and in the Table of Sines, Tangents, and Secants, *three errors*, are all that have been detected. In addition to the former tables, another concise table has been carefully computed, and stereotyped for the present edition, entitled Reduced Logarithmic Versines, or Log. $\text{Sin}^2 \frac{1}{2}$ P, for the purpose of facilitating the computation of time, or an azimuth from the observed altitude. Besides this, several others throughout the work, too numerous to be particularly noticed here, have been introduced, which will be found very

convenient in Practical Mathematics, Astronomy, and Navigation.

The Introduction is divided into three parts, followed by a copious explanation of the general tables.

In the first I have shortly described the nature, and investigated the more simple series for the computation of Logarithms. I have generally, however, given the more important rules in words at length, without investigation, so as to be readily comprehended by persons who have acquired a knowledge of the elementary principles of mathematics. Indeed, the demonstrations can be understood by those only who have obtained a tolerable knowledge of the elements of geometry and algebra, and, since the generality of books containing these comprehend also the usual investigations in trigonometry, it was thought unnecessary to insert them. On this account, I have given the demonstrations of those propositions only which are less commonly inserted in the usual treatises.

On the Barometric Measurements of Altitudes two different methods have been given. They are in a great degree new, and the first by the original subsidiary tables, calculated expressly for this purpose, will be found easy and accurate.

The second part contains Spherical Trigonometry, with a great variety of its most useful applications. As the rules and examples are either new or selected from the best writers on the subject, it is hoped this section will prove interesting to students of Astronomy and Navigation, since it contains a number of the best methods, illustrated by examples of modern practice.

The third part contains a variety of Rules and Formulæ for the use of Surveyors, Engineers, Navigators, and practical Astronomers. Those for geodetical purposes are selected chiefly for their general utility, and comprehend a sufficient number for usual cases, arranged in as natural an order as possible. Other rules, formulæ, and tables, have also been added, in order to render the work a convenient manual in an observatory, and a useful companion to scientific travellers.

Captain Kater having simplified the problem of determining the figure of the earth by means of the pendulum, and brought the experiment within the reach of our more active and intelligent military and naval officers, I have added the necessary rules and formulæ for that purpose, in order to initiate, as far as pos-

sible, our Cadets and Midshipmen in these interesting researches; as such higher objects of pursuit not only invigorate their faculties, but inspire them with a taste for the acquirement of professional eminence.

The Explanation of the Tables will, it is hoped, be found full and explicit, especially towards the beginning. The explanations of such tables as follow others, analogous in structure or arguments, may sometimes be less full, as it is presumed that those preceding them are well understood.

Uniformity and simplicity, in the methods of calculation, have been particularly studied, so that, with equal or greater facility, by means of the ordinary tables thus improved, those problems may be solved which frequently require the aid of a variety of subsidiary tables. This advantage has been obtained by annexing to several of the principal tables, differences and proportional parts, and to tables V. and LXXV. especially, double arguments in both *arcs* and *time*, by which many problems in Practical Astronomy and Navigation are solved with more facility, and with less chance of error, than when those tables are used which have arguments of arcs only. The trouble of referring to tables LXI. and LXII., to change arcs into time, and the converse, is therefore seldom requisite, though their use is constantly required when the common tables are employed. Hence, also, such tables as those in our usual works on Navigation, entitled Half-elapsed Time, Middle Time, Rising, &c., are unnecessary.

For this second edition, I have been favoured with the remarks of several of our most distinguished mathematicians and practical astronomers, who have testified to me their approbation of the first edition, and I have adopted some of their suggestions. The work has also been carefully revised throughout, many useful additions have been made, and such improvements and corrections introduced as, it is hoped, will render it more perfect, both with regard to accuracy and utility, than any other work of a similar description now before the public.

*** The author having again examined his Tables since the preface was written and the stereotype Tables were thrown off, regrets to find that one error has been found in page 16, Log. of 9462—for 5933, read 5983; and another in page 76, Prop. log. of 24′ 59″—for 32504, read 32594.

CONTENTS

OF

INTRODUCTION.

MISCELLANEOUS TABLES IN THE INTRODUCTION.

Page

BAROMETRIC TABLES.

APPENDIX.

CONTENTS

OF

EXPLANATION OF THE TABLES, &c.

ADDITIONS AND CORRECTIONS.

During the time this work has been in the press, some improvements occurred to the author, some simplifications of formulæ were discovered, and a few additions, chiefly to render the Tables more comprehensive, have been partly inserted in the Appendix, or are here subjoined. Notwithstanding all possible care in superintending the press, several errors have escaped undetected,—a few of which only are of importance. They are also given in the following additions and corrections.

Page 5 Delete lastly, line 5 from bottom.

Add Cor. $\log(N+1) = \text{Log}(N-1) + \frac{2M}{N}$ nearly when N exceeds 100, in tables to six places of decimals.

11 Delete inversely as the cosine of twice the arc, line 1.

14 Denominator of last term 3.3.3.5.7 (35.)

21 for 110° 20′, read 118° 20′, in some copies.

Page 24 for Exmaples read Examples.
22 — AB read AC, line 17.
30 — 51° 49′.0 read 52° 41′.5, Dec. May 12th 18° 11′.9 N.
32 — Piussant read Puissant line above Ex. 1.
40 — AB_2 read AB^2, line 31.
41 — 16.883 read 16.683 for 5 miles in Table.
51 — 7946 read 7949, Ex. 33.
53 — 30 read 50, in Table for Face of Hexagon. Add. Fosse of Ravelin is about two-thirds of that of the polygon.
55 — Schuchburgh read Schuckburgh, line 38.
56 — volumes read volume, line 41.
60 Supply difference, line 28.
61 — Heineker read Heineken.
69 — 16 $(t+t')$ read 15 $(t+t')$, line 28.
Add. The following is a very easy formula for computing heights by the barometer. $H = \left\{ 48500 + 60\ (t+t') \right\} \frac{B - b}{B + b} - 3(\tau - \tau')$ in feet.
78 — 180° read 90°, prop. XIX., cor. 1.
83 — S read N. ex. 2. Ans.
86 Supply N after 14° 14′ 24″ ex. 5.
88 — cot T read cos T, line 12.
89 — Noon read Meridian.
95 Supply to the rule. The dip from Table XI. must be subtracted from altitudes observed at sea before computing the refraction.
96 — KPL read KPZ, line 13, and for $\frac{20.}{2.8}$ read $\frac{20.8}{2}$, line 28.
100 — 7^h read 8^h, line 2 from bottom.
103 Supply Ans. to ex. 2, 48° 30′ 26″ W.
104 — same evening read morning, 8th September, 1825.
113 — —0′.9 in col. of diff. read —1′.9, for —2′.8 read —3′.8 at 50° alt.
Ans. to ex. 2, read 18° 15′.3 N.
114 Delete, of, line 7 from foot-note.
Supply N. to Ans. ex. 3.
115 — 52° 20′, ex. 1, read 53° 20′.
116 — 51° 54′ read 52° 54′, line 12, and for 52′ read 53′, line 2 from bottom.
117 — 40^h read 10^h, ex. 2.
118 Add to note. By omitting log. radius, and adding the correction from Mendoza's Table XII., to the Moon's parallax when computing the parallax in altitude.
120 — 28°6 inches read 29.6, line 4.
138 — 28^m read 8^m, line 6.
141 Supply latitude 34° 37′ S., ex. 2.
Add. Should the log. μ from the Berlin Ephemeris, be used in place of the daily variation in declination, the const. log. is 5.063487.
165 — $\pm$ read $\mp$ before the *last term*, line 2 above note.
167 — 23° 53′ 30″ read 23° 52′ 36″.5, middle of page.
177 Delete second part of foot-note, and read, Captain Kater's Floating Horizontal Collimator may be readily converted into a convenient meridian mark.
181 — 28 read 27 solid feet = 1 solid yard.
197 Delete last sentence of ex. 6.
200 — C lat. 55° 40′ 5″ N. long. 2° 27′ 38″ W. not 37′ read 55° 39′ 54″ N. long. 2° 28′ 0″ W.
214 Deal 1.000460. C. Kater.
219 Add to ex. 9, the angle of elevation being 32° 30′
232 — compressed read expressed, line 35.
238 Delete, true, line 11, from foot-note.
It may be added to the note, that, if the expansion of moist air be employed, such as is generally the case, or 0.00225 instead of

0.002083 for dry air, the very small errors of this table would be still diminished.

Page 239 — page 85 read 110, last line in note.

242 — 1′ 33″ read 0′ 33″, 2d diff., and for 38″ read 33″, 3d diff.

245 — 38° read 33°, line 4 from bottom.

245 and 246, Tables XXVIII. and XXIX. may be carried beyond their present limits, if the horary distance be necessarily greater than that to which the table extends, by the following rule: To half the horary distance in Part I. Table XXVIII. find the corresponding number, which, being quadrupled, will give the reduction very nearly for the time required; or, if the reduction to one-third the meridian distance be multiplied by nine, the result will be the reduction as before, though it will generally be inexpedient to carry it so far. The reduction from Part II. must be multiplied by the fourth power of the denominator denoting the aliquot part of the time. This mode is applicable to Table XXIX. in a similar manner.

247 for June 21 read June 16, line 3, under the table.

— Tables XXVII. and XXVIII. read XVII. and XVIII.

248 — 23° 28′ 16″.50 read 23° 28′ 17.″02 M. Ob. 23° 28′ 6″.93.

The equation of time at noon should have been applied to the ex. to Table XXX. whence $3^h\ 46^m\ 50^s - 5^m\ 27^s.4 = 3^h\ 41^m\ 22^s.6$, to which the reduction being applied, gives $3^h\ 41^m\ 58^s.967$, and the time will be $22^h\ 44^m\ 12^s.507$.

250 — r read r', line 2 from bottom.

251 — E read C, line 18 from top.

267 — $(-a)$ read $(s-a)$ in some copies.

269 — δ read $2 \sin \frac{\delta}{2}$.

TABLES.

Page 103. Nov. 1, Table XLIX. for 0″.360 read 0″.370, and supply accents to annual diminution, and these numbers.

106. Dec. 18, for 0.862, read 0.962, and since the numbers are decimals of a year, the figure before the points should always be 0.

108. To reduce the sun's declination to a future time, add twice the daily variation, reckoning minutes seconds, &c. to the given declination if it be increasing, but subtract it if decreasing; the result will be the true declination for a period four years after, four times if eight years after, &c.; and to May 17, for 29° read 19°.

110. Time of high water at London, for $2^h\ 50^m$ read $2^h\ 15$.

Vienna, for log 17° read 16°.

111. Table LXI., for 1″, &c. read 1′ in the middle column.

116. In Table LXIX. increase the days of the month by unity, that is, for Jan. 0 read Jan. 1, for 10 read 11, &c., throughout the year. Also in the leap years increase the date by 21^h, the first after leap year by 3^h, the second by 9^h, and the third by 15^h.

217. For $\frac{s\,R}{\frac{1}{2}gc}$ read $\sqrt{\frac{s\,R}{\frac{1}{2}gc}}$. Formula 3, I.

235. Tables XIII. and XIV. will serve to correct altitudes observed at land with an artificial horizon by increasing the numbers in XIII. under 16 feet by 4′, and diminishing those in XIV. under 16 feetby 4′, and then applying the sum and remainder, according to the title of each table, to the altitude observed, the result will be the true altitude.

It may be remarked, that Professor Bessel of Konigsberg has found, very lately, that the usual corrections, Section V., page 200, &c., employed to reduce pendulum experiments to a vacuum, will require some modification, which future investigations can alone determine.

PREPARING FOR PUBLICATION,

BY THE SAME AUTHOR.

I.

A KEY to this Work; containing Solutions to all the Problems, and Exemplifications of all the Formulæ, to render their Application easy and familiar. For the use of Teachers, and of private Students who may not have the assistance of a Master.

*** To be printed uniformly with the Tables.

II.

A Short but Comprehensive TREATISE on MATHEMATICAL and ASTRONOMICAL INSTRUMENTS; in which the Principles of those most generally useful will be clearly explained, and their Application to Practice fully illustrated. One volume 8vo.

This work will form a second volume to the Author's Mathematical and Astronomical Tables, which may be considered the first, and the two taken together will constitute a useful introduction to the proper management of Mathematical and Astronomical Instruments, and to the most approved methods of obtaining the results generally derived from them by observation and calculation.

INTRODUCTION.

PART I.

OF LOGARITHMIC AND TRIGONOMETRICAL TABLES.

SECTION I.

Of the Properties of Logarithms.

1. LOGARITHMS are a series of numbers, originally invented by Baron Napier, for the purpose of facilitating arithmetical calculations. This end is attained by their enabling us to perform the operations of multiplication by addition, of division by subtraction, of involution by multiplication, and of the extraction of roots by division.*

2. It is evident that any two series of numbers, the one being in arithmetical and the other in geometrical progression, possess these properties: thus, for example, let the

Ar. series be	0	1	2	3	4	5	&c.
Geo. series	1	10	100	1000	10,000	100,000	

Now, if we add any two numbers in the arithmetical series, such as 2 and 3, which are equal to 5, and multiply the corresponding numbers under them, 100 and 1000, we have 100,000, the number immediately under 5, which was obtained by the addition of 2 to 3. Hence, then, it is clear that, if tables of this kind, sufficiently extensive, were formed, by a reference to them, the operation of multiplication could be performed by means of addition.

In like manner, we perform division by subtraction; for, if from 5 we take 3, the remainder is 2, under which we get 100; that is, 100,000, the number under 5, divided by 1000, that under 3, gives 100 as a quotient.

Roots are readily determined in a similar way; thus, 4, in the arithmetical series, divided by 2, gives 2, under which, in the geometrical series, is 100; that is, the second, or square root of 10,000 the number under 4, is 100, the number under 2, and so on.

Napier called the first series the *logarithms* of the corresponding numbers in the second.

3. Since the two series may be assumed at pleasure, we may have as many different systems of logarithms as we choose.

4. The series in art. 2 being adapted to the common denary scale of arithmetic, is, on the whole, the most convenient for general purposes, though other systems have, in particular cases, their peculiar advantages.

On considering these series, it appears that the logarithm of 1 is

* The identity of this process with that performed upon the exponents of quantities in the corresponding operations of algebra, will be obvious to those who have acquired the rudiments of that branch of mathematics.

0, and that of 10 is 1, and hence the logarithms of all numbers between 1 and 10 are greater than 0 and less than 1, that is, they are fractions. In the same manner, between 10 and 100 they are greater than 1 and less than 2, that is, they are 1 with some fraction annexed, and so on. The *whole numbers* or *integers* in the logarithmic series are hence easily obtained, being always a unit less than the number of figures in the integral part of the corresponding natural number. On this account it is customary, in the common printed tables, to put down only the fractional part in the form of a decimal, the computer supplying the whole number or integer under the name of *index*.

5. In order to generalize, let us assume the two following series:

$$r^x,\ r^{x'},\ r^{x''},\ r^{x'''}, \&c. \quad . \quad . \quad (1)$$
$$y,\ y',\ y'',\ y''', \&c. \quad . \quad . \quad (2)$$

in which r is some given number greater or less than unity, and x, x', x'', x''', &c. any variable quantities chosen in such a manner that $r^x=y$, $r^{x'}=y'$, $r^{x''}=y''$, $r^{x'''}=y'''$, &c., then the several exponents, x, x', x'', x''', &c. of the series (1) are called the logarithms of the corresponding terms in the series (2).

Thus if y, y', y'', y''', &c. be a series of numbers such that $r^x=y$, $r^{x'}=y'$, $r^{x''}=y''$, $r^{x'''}=y'''$, &c., then $x=\log.\ y$, $x'=\log.\ y'$, $x''=\log.\ y''$, $x'''=\log.\ y'''$, &c.

6. For the purpose of adapting the series (1) to the series of natural numbers 1, 2, 3, &c. the given number r must be greater than unity, the first index x must be equal to 0, and the several indices x', x'', x''', &c. must continually increase. For, since by the principles of algebra, $r^\circ=1$, whatever r may be, this series will increase from 1 to infinity; and by properly adjusting the values of x', x'', x''', &c. it is evident that the several quantities $r^{x'}$, $r^{x''}$, $r^{x'''}$, &c. may be made to coincide with the numbers 2, 3, 4, &c. For example, let $r=10$; then since $10^\circ=1$, and $10^1=10$, the indices of 10, which would give $10^{x'}$, $10^{x''}$, $10^{x'''}$, &c. equal to the numbers 2, 3, 4, &c., must be fractions between 0 and 1. If we take the number 3 we have $10^{\frac{1}{2}}=3.16$ nearly, from which we infer that a fraction (x') somewhat less than $\frac{1}{2}$ or 0·5, being made the index of (r) 10, would give $10^{x'}=3$. This fraction is found by calculation to be ·47712; hence $10^{\cdot 47712}=3$; therefore, when $r=10$, the logarithm of 3 is ·47712.

In like manner, if we assume the number 5, whose logarithm is to be found in place of that of 3, we have $10^{\frac{2}{3}}=4.64$; whence a fraction, $x^{(n)'}$ somewhat greater than $\frac{2}{3}$, or ·666, being made the index or exponent of 10, would give $10^{x^{(n)'}}=5$. This fraction more accurately computed is found to be .69897, that is, when $r=10$, the logarithm of 5 is .69897.

7. From this it appears, that the value of the logarithm of any given number depends upon the value of the number r, and that by assuming it equal to different numbers, as many different systems of logarithms may be formed as we please.

In every system, however, since $r^\circ=1$, the logarithm of 1 must be 0. This constant quantity r from the powers of which the natural numbers are formed, is called the *radix* or *base* of the system to which it belongs.

8. In the general equation $r^x=y$, (art. 5.), let us make x vary, and observe the correspondent variations of y.

If r is greater than 1, on making $x=0$, we have $y=1$; when $x=1$ then $y=r$ or the logarithm of the base is $=1$; in proportion as x increases from 0 to infinity, y will increase from 1 towards r, and afterwards to infinity, so that if we suppose x to pass through all the intermediate values, in following *the law of continuity*, y will increase also in the same manner, though much more rapidly.

If we put for x, negative values, we shall have $y=r^{-x}$, or $y=\frac{1}{r^x}$. Here we see, in like manner, that the more x increases the more y or $\frac{1}{r^x}$ decreases, so that in proportion as x augments negatively, y takes all possible values less than 1, as far as 0, in which case x becomes infinite. This was the proposition which Napier made to Briggs on their celebrated meeting at Edinburgh, when conversing on the propriety of changing the logarithmic scale.

If r is less than 1 we shall make $r=\frac{1}{b}$, b being greater than 1, and we have $y=\frac{1}{b^x}$ or $y=b^x$, according as x is positive or negative. We fall here upon the same case, with this difference, that x is positive when y is less than 1, and negative when y is greater than 1. This proposal Briggs made to Napier, but immediately abandoned it on Napier suggesting that mentioned above, which was finally adopted.

If $r=1$, we have $y=1$ whatever x may be.

We may then say generally, that provided r is not unity, there can always be found a value for x, which renders r^x equal to any given number y. The constant use that is made of the properties of the equation $y=r^x$ requires the denominations of its parts to be fixed in order to avoid circumlocution. Hence, as before remarked, x is called the logarithm of the number y, the invariable number r is called the base, and, finally, the logarithm of a number, the power to which the base must be raised in order to produce that number.

With regard to the base r it is arbitrary, and when we write $x=\log. y$ to show that x is the logarithm of the number y, or that $y=r^x$, the base r is always understood, because when once chosen it is supposed to remain fixed. If it should be changed the new base ought to be indicated.

9. From these principles are derived several properties.

1°. In every system of logarithms, the logarithm of 1 is 0, and that of the base r is 1.

2°. If the base r is greater than 1, the logarithms of numbers greater than 1 are positive, the others are negative. The contrary takes place if r is less than 1.

3°. The composition of a table of logarithms consists in determining all the values of x when y is made successively equal to 1, 2, 3, &c. in the equation $y=r^x$.

If we suppose $r^{\varrho}=\mu$, on making

$x=0,\ \varrho,\ 2\varrho,\ 3\varrho$, &c. . . . $n\varrho$

We find $y=1,\ \mu,\ \mu^2,\ \mu^3$, &c. . . . μ^n

The logarithms therefore increase in progression by differences, while the numbers increase in progression by the product or quotient, according as μ is an integer or a fraction.

The ratios are the arbitrary numbers ϱ and μ. We may, therefore, regard the systems of values of x and y which satisfy the equation

$y=r^x$, as classed in these two progressions, which coincides with what has already been said in art. (2.)

10. We shall now demonstrate algebraically the various properties of logarithms.

Let N and n be any two numbers belonging to the series (1); and for example, let $N=r^x$ and $n=r^{x'}$, then $N\,n=r^x\times r^{x'}=r^{x+x'}$, but, by art. 5, the logarithm of $r^{x+x'}$ is $x+x'=\log. r^x+\log. r^{x'}=\log. N+\log. n$.

In like manner, if n, n', n'' be any set of numbers in the series (1) it might be shown that the logarithm of $n\times n'\times n''$, &c.$=\log. n+\log. n'+\log. n''$, &c., from which we infer that the logarithm of the product of any number of factors is equal to the sum of their logarithms.

11. Again $\frac{N}{n}=\frac{r^x}{r^{x'}}$; but the logarithm of $r^{x-x'}=x-x'$; therefore, the logarithm of $\frac{N}{n}=x-x'=\log. r^x-\log. r^{x'}=\log. N-\log. n$; hence it appears, that the logarithm of the quotient of any two numbers is equal to the difference of their logarithms; and that the logarithm of a fraction $\left(\frac{N}{n}\right)$ is equal to the logarithm of its numerator minus the logarithm of its denominator.

If N be less than n, then $\log. N-\log. n$ is negative; therefore, the logarithms of all proper fractions are negative.

12. Let $N=r^x$ be raised to the m^{th} power, then $N^m=r^{mx}$; but the logarithm of r^{mx} is $=mx$, hence the logarithm of $N^m=mx=m\log. r^x$ $=m\log. N$; for the same reason, since $\sqrt[m]{N}=N^{\frac{1}{m}}=r^{\frac{x}{m}}$, the logarithm of $\sqrt[m]{N}=\frac{x}{m}=\frac{\log. N}{m}$; from which we infer, that the logarithm of the m^{th} power of any number is found by multiplying its logarithm by m, and that of the m^{th} root of any number, by dividing its logarithm by m.

Section II.

Of the Construction of Tables of Logarithms.

13. Let r^x express generally any term of the series, (1), and let N be the corresponding number, then $r^x=N$. Hence to find the logarithm of N is merely to solve the equation $r^x=N$ where x is the unknown quantity. In order to accomplish this purpose let $r=1+b$ and $N=1+n$, then take the y^{th} power of each side of this equation, and we obtain $(1+b)^{xy}=(1+n)^y$, which, by expansion, gives

$$1+xyb+\frac{xy(xy-1)}{2}b^2+\frac{xy\,(xy-1)\,(xy-2)}{2.3}b^3+\&c.=$$
$$1+yn+\frac{y(y-1)}{2}n^2+\frac{y(y-1)\,(y-2)}{2.3}n^3+\&c.$$

Rejecting 1 from each side of the equation, and dividing by y, it becomes

$$x\left\{b+\frac{xy-1}{2}b^2+\frac{(xy-1)\,(xy-2)}{2.3}b^3+\&c.\right\}=$$
$$n+\frac{y-1}{2}n^2+\frac{(y-1)\,(y-2)}{2.3}n^3+\&c.$$

Now if y become o, we have
$x\,(b-\frac{1}{2}\,b^2+\frac{1}{3}\,b^3-\frac{1}{4}\,b^4+\&c.)=n-\frac{1}{2}\,n^2+\frac{1}{3}\,n^3-\frac{1}{4}\,n^4+\&c.$
hence x, the log. $(1+n)=\dfrac{n-\frac{1}{2}\,n^2+\frac{1}{3}\,n^3-\frac{1}{4}\,n^4+\&c.}{b-\frac{1}{2}\,b^2+\frac{1}{3}\,b^3-\frac{1}{4}\,b^4+\&c.}$
but $n=N-1$ and $b=r-1$, therefore, by substitution, the above expression becomes

$$\frac{(N-1)-\frac{1}{2}\,(N-1)^2+\frac{1}{3}\,(N-1)^3-\frac{1}{4}\,(N-1)^4+\&c.}{(r-1)-\frac{1}{2}\,(r-1)^2+\frac{1}{3}\,(r-1)^3-\frac{1}{4}\,(r-1)^4+\&c.}$$

14. Let $\dfrac{1}{(r-1)-\frac{1}{2}\,(r-1)^2+\frac{1}{3}\,(r-1)^3-\frac{1}{4}\,(r-1)^4+\&c.}=M.$

This quantity M, which evidently depends upon the base r, is called the modulus of the particular system of logarithms to which it belongs. As it is obvious the series $n-\frac{1}{2}n^2+\frac{1}{3}\,n^3-\frac{1}{4}\,n^4+\frac{1}{5}\,n^5-$ &c. will not converge when n is any whole number greater than unity, before proceeding to the calculation of the logarithms of any particular system, it will be proper to show the manner in which the value of x in the last article may be expressed in a converging series. This may be effected by means of the following process, in which M is substituted for the quantity

$$\frac{1}{(r-1)-\frac{1}{2}\,(r-1)^2+\frac{1}{3}\,(r-1)^3-\frac{1}{4}\,(r-1)^4+\&c.}; \text{ thus,}$$

$$\text{Log. }(1+n)=M\,(n-\tfrac{1}{2}\,n^2+\tfrac{1}{3}\,n^3-\tfrac{1}{4}\,n^4+\tfrac{1}{5}\,n^5-\&c.) \quad . \quad . \quad (3)^*$$

In the above for n put $-n$, and then

$$\text{Log. }(1-n)=M\,(-n-\tfrac{1}{2}\,r^2-\tfrac{1}{3}\,n^3-\tfrac{1}{4}\,n^4-\tfrac{1}{5}\,n^5-\&c.) \quad . \quad . \quad (4)$$

Subtract (4) from (3), then log. $(1+n)$ — log. $(1-n)$ =

$$\text{log. }\frac{1+n}{1-n}=2\,M\,(n+\tfrac{1}{3}\,n^3+\tfrac{1}{5}\,n^5+\tfrac{1}{7}\,n^7+\&c.) \quad . \quad . \quad . \quad . \quad (5)$$

Let $N=\dfrac{1+n}{1-n}$, then $n=\dfrac{N-1}{N+1}$, hence

$$\text{Log. }N=2\,M\left\{\left(\frac{N-1}{N+1}\right)+\tfrac{1}{3}\left(\frac{N-1}{N+1}\right)^3+\tfrac{1}{5}\left(\frac{N-1}{N+1}\right)^5+\&c.\right\} \quad . \quad (6)$$

Again let $n=\dfrac{1}{2\,N-1}$, then $\dfrac{1+n}{1-n}=\dfrac{N}{N-1}$, hence by substitution in formula (5)

$$\text{Log. }\frac{N}{N-1}=2\,M\left(\frac{1}{2\,N-1}+\frac{1}{3(2\,N-1)^3}+\frac{1}{5(2\,N-1)^5}+\&c.\right) \text{ or}$$

$$\text{Log. }N-\text{log. }(N-1)=2\,M\left(\frac{1}{2\,N-1}+\frac{1}{3(2\,N-1)^3}+\frac{1}{5(2\,N-1)^5}+\&c.\right); \text{ and log. }N=2\,M\left(\frac{1}{2N-1}+\frac{1}{3(2N-1)^3}+\frac{1}{5(2N-1)^5}+\&c.\right)$$

$$+\text{ log. }(N-1) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Lastly, if $\dfrac{1+n}{1-n}=\dfrac{N+1}{N}$, then $n=\dfrac{1}{2\,N+1}$ and log. $(N+1)$ =

$$2\,M\left(\frac{1}{2\,N+1}+\frac{1}{3(2N+1)^3}+\frac{1}{5(2N+1)^5}+\&c.\right)+\text{log. }N. \quad . \quad (8)$$

Let $\dfrac{1+n}{1-n}=\dfrac{N^m+1}{N^m}$, then $n=\dfrac{1}{2\,N^m+1}$ and

$$\text{Log. }(N^m+1)=2\,M\left(\frac{1}{2N^m+1}+\frac{1}{3(2N^m+1)^3}+\frac{1}{5(2\,N^m+1)^5}+\&c.\right)$$

$$+\text{ log. }N^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

---

* By means of this formula the logarithm of a quantity exceeding unity by a very small fraction may be re[illegible] found. See examples in the explanation of the tables.

Since the log. of $1=0$, this last series which converges very rapidly, will give the logarithms of all the natural numbers with facility in succession. To these theorems might have been added others still more convenient, but they are sufficient for ordinary cases.

15. Before proceeding to compute a table of logarithms, some value must be assigned to M. Since the value of r is arbitrary, let it be so assumed that $\frac{1}{(r-1)-\frac{1}{2}(n-1)^2+\frac{1}{3}(r-1)^3-\&c.}$ or M shall be equal to 1, that adopted by Napier. Taking series (8) we have (art. 6.)

$$\text{Log. } 1 \quad \ldots \quad =0.0000000$$
$$2=2\left(\frac{1}{3}+\frac{1}{3^4}+\frac{1}{5.3^5}+\&c.\text{ to 8 terms}\right) \quad . \quad =0.6931472$$
$$3=2\left(\frac{1}{5}+\frac{1}{3.5^3}+\frac{1}{5^6}+\&c.\right)+\log. 2 \quad . \quad =1.0986123$$
$$4=2\log. 2\text{ (art.) }12) \quad \ldots \quad =1.3862944$$
$$5=2\left(\frac{1}{9}+\frac{1}{3.9^5}+\frac{1}{5.9^5}+\&c.\right)+\log. 4 \quad . \quad =1.6094379$$
$$6=\log. 2+\log. 3\text{ (art. 10)} \quad \ldots \quad =1.7917595$$
$$7=2\left(\frac{1}{13}+\frac{1}{3(13)^3}+\frac{1}{5(13)^5}+\&c.\right)+\log. 6 \quad . \quad =1.9459101$$
$$8=3\log. 2\text{ (art. 12)} \quad \ldots \quad =2.0794415$$
$$9=2\log. 3\text{ (art. 12)} \quad \ldots \quad =2.1972246$$
$$10=\log. 2+\log. 5\text{ (art. 10)} \quad \ldots \quad =2.3025851$$
&c.

In this manner the Napierean logarithms of all the natural numbers may be found. As their accuracy, however, depends upon those immediately preceding, being derived successively from each other, it would be necessary to check the computations in the actual construction of a table of logarithms by some independent formula, such as (6), though this in large numbers would be rather inconvenient from its slow convergency.

16. To find the value of r, the base, in this system recourse must be had to the series (3) art. (14). If log. $(1+n)$ or log. N be put $=l$ and $M=1$, we have $l=n-\frac{1}{2}n^2+\frac{1}{3}n^3-\frac{1}{4}n^4+$, &c.; reverting this series, and $1+n$, or $N=1+l+\frac{1}{2}l^2+\frac{1}{2.3}l^3+\frac{1}{2.3.4.}l^4$, &c. Now let $l=1$, then the number whose logarithm is 1, that is, the base $r=1+1+\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}+$, &c. $=2.7182818$. To prevent confusion, however, we shall always designate the base or radix of this system by R, retaining r for that of the common logarithms. Hence $R=2.718,281,828,46$.

These are also called hyperbolic logarithms from their application to the quadrature of the hyperbola; but this designation is improper, as any system may be similarly employed.

17. When we have the logarithm of a number N for any particular value of r, the base, we can readily obtain the logarithm of the same number in every other system. Since, art. (5), when the base is r we have $r^x=N$, we shall likewise have $R^X=N$ when the base is R, in which x is different from X, therefore, $R^X=r^x$.

Now taking the logarithms relatively to the system whose base is r, then

$$l.\ R^X=l.\ r^x$$

but $l.r^x = x$ by hypothesis, and $l.R^X = X\,l.R$, art. (12), whence $X\,l.R = x$, or $X = \frac{x}{l.R}$. But if R is the base, X will be the logarithm of N in the system having that base, and designating this by L.N. to distinguish it from the other, we shall have $L.\,N = \frac{l.N}{l.R}$ . . . (12.)

consequently we obtain the logarithm of N in the second system, by dividing its logarithm taken in the first system by the logarithm of the base of the second system. Again, from formula (12) we get

$$L.N \times l.\,R = l.\,N \qquad \text{. . . . . (13)}$$

Hence in every system the logarithm of any number is the product of its Napierean logarithm by the logarithm of R, called the modulus. Also since $\frac{l.\,N}{L.N} = l.\,R$, there exists between $l.N$ and L.N a constant ratio represented by $l.R$

Since we have by formula (12) $L.N = \frac{l.N}{l.R}$, as $N = 10$, then art. (15.)

$2.3025851 = \frac{1}{M}$, or $M = \frac{1}{2.3025851} = 0.4342944819$, and $2\,M = 0.8685889638$ . . . . (14.)

18. It is now easy to construct a table of common logarithms whose base $r = 10$, for by formula (13) we have $l.N = l.R \times L.N$, but $l.R = M = 0.4342944819$; consequently $l.N = 0.4342944819 \times L.N$. It therefore is only necessary to substitute this value for M in any of the series formerly given for the computation of the Napierean logarithms to obtain the common; thus, if in series (8) for 2M we substitute its value 0.86858896 we shall have

$$\log.\,(N+1) = 0.86858896\left(\frac{1}{2N+1} + \frac{1}{3(2N+1)^3} + \frac{1}{5(2N+1)^5} + \&c.\right)$$

$+ \log.\,N$, and making N successively 1, 2, 3, &c.

Log. 1 =	. . . .	0.0000000
2 =	$\cdot 86858896\left(\frac{1}{3} + \frac{1}{3^4} + \frac{1}{5.3^5} +, \&c.\right)$.	$= 0.3010300$
3 =	$.86858896\left(\frac{1}{5} + \frac{1}{3.5^3} + \frac{1}{5^6} +, \&c.\right) + \log.\,2$	$= 0.4771213$
4 =	2 log. 2	$= 0.6020600$
5 =	$\cdot 86858896\left(\frac{1}{9} + \frac{1}{3.9^3} + \frac{1}{5.9^5} +, \&c.\right) + \log.\,4$	$= 0.6989700$
6 =	log. 2 + log. 3	$= 0.7781513$
7 =	$\cdot 86858896\left(\frac{1}{13} + \frac{1}{3(13)^3} + \frac{1}{5(13)^5} +, \&c.\right) + \log.6.$	$= 0.8450980$
8 =	3 log. 2	$= 0.9030900$
9 =	2 log. 3	$= 0.9542425$
10 =	. . . .	1.0000000

19. After Lord Napier had computed his first tables of logarithms it occurred to him that it would be proper to change the radix $R = 2.7182818$ to $r = 10$, at the same time making the logarithms of integers positive, and those of fractions negative, (art. 8.), as more conformable to the denary scale notation, and more convenient in practice. It appears that Mr Henry Briggs had also conceived the idea of changing the radix, and had computed logarithms on a plan somewhat less commodious, by making the logarithms of integers negative, and those of fractions positive, which, upon a personal

communication with Lord Napier, he rejected, and finally adopted his lordship's views. He soon afterwards published the first thousand logarithms of this kind, under the title of *Logarithmorum Chilias Prima.*

Section III.

Of the Trigonometrical Lines, called Sines, Tangents, &c.

20. The Egyptians and Chaldeans began to study astronomy at a very early period. As the determination of the relations and distances of the heavenly bodies involve the mensuration of lines and angles, it was necessary to invent some method of ascertaining the value of these quantities, at least in an approximate manner, before any useful results could be obtained. Some of the more elementary propositions in geometry must have been discovered in the most remote antiquity, and the inventive genius of the Greeks filled up the general outline. The properties of geometrical figures thus acquired, would, without doubt, be applied to the mensuration of several magnitudes, and the distances of various points in space. About six hundred years before the Christian era, Thales measured the heights of the pyramids in Egypt by means of their shadows,—a method which depends upon the proportionality of the sides of similar triangles. This simple property forms the basis of modern trigonometry. If, for example, a pole or gnomon be set perpendicular to the horizontal plane, it will, in a clear day, when the sun is not vertical, cast a shadow to a given distance, while any other high object, such as a steeple near it, will do the same. If straight lines be conceived to be drawn from the top of these objects to the extremity of each of their shadows, it is evident that, unless they are very distant, by this means triangles nearly similar will be formed, whose sides are proportional; that is, as the shadow of the gnomon is to its height, so is the shadow of the object to its height. Now, suppose the length of the shadow of the gnomon to be made the radius with which an arc of a circle is described commencing at the bottom of the gnomon, and, as will be afterwards explained, measuring the angle between the horizontal line and the line from the extremity of the shadow to the top of the gnomon, that gnomon will, by the principles of geometry, be a tangent to the circle. Whence the former proportion becomes as the radius is to the tangent of the angle of elevation, so is the length of the shadow of the object to its height. It would thus require the length of the shadow of the pole or gnomon to be measured each time any height was determined. This, however, might be avoided by having the measure of a set of triangles whose sides, to an assumed radius, and a corresponding series of angles, are previously determined by computation. By this means, in such cases, it is only necessary to measure the angle of elevation of the object, at a given point, and its distance from it, and comparing it with one of those computed triangles equiangular to it, to determine, in a manner similar to the former, the height of the object. It is obvious that the same principles may be applied to objects situated in any plane, whether vertical, horizontal, or oblique.

Several series of triangles of the kind now mentioned have been actually computed and arranged in tables under the designation of Trigonometrical Tables.

These were not accomplished at once, but were the improvements of successive ages. Hipparchus, about 150 years before the Christian era, supposed similar triangles to be inscribed in circles, and employed in his computations the chords subtending the arcs measuring them in sexagesimal parts of the radius. Nearly 300 years afterwards, Ptolemy, in his Μεγαλη Συνταξις, recomputed the chords, but in his Analemma employs the *half chords* instead of the *chords* approaching very nearly to the use of *sines*, afterwards introduced by the Arabians.

Some notions of the tangents, secants, and versed sines, were, towards the beginning of the tenth century, entertained by the more learned Arabians. About the commencement of the fifteenth century the sciences began to be cultivated in Europe, where the greatest progress has been made. At that period Müller invented the tangents, and shortly affer Maurolycus produced his table of secants. These were all in natural numbers to a given radius now generally taken at unity, and, therefore, their application was in many cases troublesome. To remove this inconvenience as far as possible, Napier, about 1614, invented his logarithms, which have brought them perhaps to the last degree of perfection.

Hipparchus, who has been followed by most of the moderns, employed the circle to measure angles. He supposed the whole circumference to be divided into 360 equal parts, each called a degree. The degree was divided into 60 equal parts called minutes, and the minute into 60 equal parts called seconds, and the sexagesimal division was continued, though now the fractions of seconds are commonly expressed in decimals, which are more convenient for calculation.*

Whence the semicircle contains 180 degrees and the quadrant 90. As *four* right angles can be constituted about a point, 90 degrees must be the measure of a right angle. For the purposes of abbreviation, a degree is marked with a small circle, a minute with one accent, a second with two accents, &c. Thus, $57° 17' 44''.806$, denotes 57 degrees, 17 minutes, 44 seconds, and .806 the decimal, whose value is 806 thousandths of a second. This, being an arc whose length is equal to the radius, as will be afterwards explained, is also expressed in degrees and in decimal parts of a degree, thus $57°.2957795$, a mode of using it which, in some cases, has its advantages.

The number of these parts, in either case, contained in the arc between the lines constituting the angle, of which arc the angular point is the centre, indicates the measure of that angle accordingly.

DEFINITIONS.

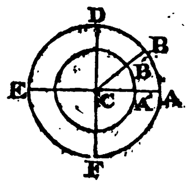
21. If two straight lines intersect one another, in the centre of a circle, the arc of the circumference intercepted between them is called

* The French have lately adopted the centesimal division in some of their works, which, in many cases, is preferable to the sexagesimal. The whole circle is divided into 400 degrees, each degree into 100 minutes, and the centesimal division is continued. Hence the semicircle contains 200 degrees, the quadrant 100, and the ratio of the centesimal to the sexagesimal degree is as 9 to 10.

To convert sexagesimal degrees into centesimal, add $\frac{1}{9}$ of the arc to itself.

The converse is effected by subtracting $\frac{1}{10}$ of the arc from itself.

the measure of the contained angle, whatever be the radius of the circle, since the arcs are proportional to their radii. Thus, the arc AB or A'B', is the measure of the angle ACB, and is expressed in degrees, &c.



22. The *complement* of an arc is its difference from a quadrant, its *supplement*, its difference from a semicircle, and its *explement*, its defect from the whole circumference. Thus if AB be any arc, then BD is the complement, BE the supplement, and BDEFA the explement.

The same thing holds with regard to the angles of which the arcs are the measures, that is, if ACB be any angle, BCD its difference from a right angle is called the complement, BCE the supplement to two right angles, and BCA measured by the arc BDEFA, the explement or difference from four right angles.

23. The *sine* of an arc, or of an angle, of which the arc is the measure, is a perpendicular let fall from one of its extremities upon a radius or diameter passing through the other.

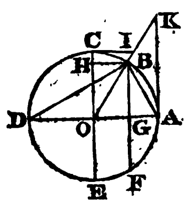
24. The *versed sine* or *versine* of an arc is that part of the diameter intercepted between its sine and the circumference.

25. The *tangent* of an arc is a perpendicular to the extremity of the radius at one end of the arc, and limited by a straight line drawn from the centre, passing through the other.

26. The *secant* of an arc is the straight line drawn from the centre to the extremity of the tangent.

27. It is usual to express the *sine, tangent*, and *secant* of the *complement* of an arc by the abbreviated terms *cosine, cotangent*, and *cosecant*.

28. Let ACDE be a circle of which the diameters AD and CE are at right angles to one another.



Take any arc AB, produce the radius OB, and draw BG, AK perpendicular to AO or AD, and HB, CI perpendicular to CE; then BG is the *sine*, BH or GO the *cosine*, AG the *versine*, CH the *coversine*, DG the *suversine*, and HE the *sucoversine* of the arc AB. Also of that arc AK is the *tangent*, CI the *cotangent*, OK the *secant*, and OI the *cosecant*.

29. Since the diameter which bisects an arc, also bisects the chord of that arc at right angles, therefore the sine of the arc is equal to half the chord of twice the arc. Thus $BG=\frac{1}{2}$ BF=half the chord of the arc BAF, the double of the arc AB.

30. In the right-angled triangle OGB, $BG^2+OG^2=OB^2$, that is, the squares of the sine and cosine are together equal to the square of the radius.

31. The triangle OGB being similar to OAK, OG : GB : : OA : AK, or the cosine of an arc is to the sine as radius is to the tangent.

32. Also the triangles OGB, OAK being similar as before, OG : OB : : OA : OK, the radius is a mean proportional between the cosine and the secant.

33. Since DG : GB : : GB : GA, it follows that the sine is a mean proportional between the versine and the suversine.

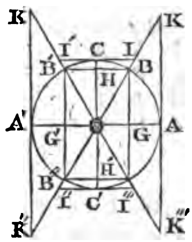
34. Again, AD : AB : : AB : AG, or the chord of an arc is a mean proportional between the diameter and versine.

Cor.—Since $AB^2=AD\,.\,AG$, then because AD is constant, AB^2 varies as AG, or $(\frac{1}{2}\,AB)^2 \propto AG$, that is, the square of the sine varies

directly as the versine of twice the arc, or inversely as the cosine of twice the arc.

35. The triangles OAK and ICO are similar, therefore AK : AO :: OC : CI ; consequently the radius is a mean proportional between the tangent and the cotangent of an arc.

36. In the application of algebra to geometry, where the trigonometrical lines are employed, it is necessary to trace their changes in the several quadrants of the circle, since it is obvious that the same lines treated of above, may be applied to each. In the first quadrant AC, if the sine BG and cosine GO be supposed *positive;* then the sine B′G′ on the same side of the diameter AA′, and in the same direction, still remains *positive;* but the cosine OG′ having changed its position with respect to the centre O, or diameter CC′, becomes *negative.* In the third quadrant, the cosine OG′ and sine G′ B″, having both changed their positions, are both *negative.* In the fourth quadrant, the cosine having resumed its *original position,* OG is now *positive,* while the sine GB‴, remaining as in the third quadrant, is *negative.* The tangents and secants depending upon the sines and cosines, have their signs determined accordingly.*



From article 30, to 35 and inclusive, R being radius, &c. we obtain

1. sin	$= (R^2 - \cos^2)^{\frac{1}{2}}$	7. tan	$= \dfrac{R \times \sin}{\cos}$
2. cos	$= (R^2 - \sin^2)^{\frac{1}{2}}$	8. cot	$= \dfrac{R \times \cos}{\sin}$
3. tan	$= (\sec^2 - R^2)^{\frac{1}{2}}$	9. sec	$= \dfrac{R^2}{\cos}$
4. cot	$= (\mathrm{cosec}^2 - R^2)^{\frac{1}{2}}$	10. cosec	$= \dfrac{R^2}{\sin}$
5. sec	$= (R^2 + \tan^2)^{\frac{1}{2}}$	11. versine	$= \dfrac{\sin^2}{R + \cos}$
6. cosec	$= (R^2 + \cot^2)^{\frac{1}{2}}$	12 covers	$= \dfrac{\cos^2}{R + \sin}$

If radius be supposed unity, then

1. sin	$= (1 - \cos^2)^{\frac{1}{2}}$	7. tan	$= \dfrac{\sin}{\cos}$
2. cos	$= (1 - \sin^2)^{\frac{1}{2}}$	8. cot	$= \dfrac{\cos}{\sin}$
3. tan	$= (\sec^2 - 1)^{\frac{1}{2}}$	9. sec	$= \dfrac{1}{\cos}$
4. cot	$= (\mathrm{consec}^2 - 1)^{\frac{1}{2}}$	10. cosec	$= \dfrac{1}{\sin}$
5. sec	$= (1 + \tan^2)^{\frac{1}{2}}$	11. versine	$= \dfrac{\sin^2}{1 + \cos}$
6. cosec	$= (1 + \cot^2)^{\frac{1}{2}}$	12. covers	$= \dfrac{\cos^2}{1 + \sin}$

* In the above wood-cut, B‴ has been omitted near I‴, which may easily be supplied by the pen.

37. Now, since (7) $\tan.=\frac{\sin}{\cos}$, then it follows from the principles of algebra, that when the signs of the sine and cosine are *like*, the sign of the tangent is *positive*, and when *unlike*, the sign of the tangent is *negative*. In like manner, the signs of the cotangent, secant, and cosecant, may be determined from formulas (8), (9), and (10).

Tables of the Signs of Trigonometrical Lines.

Quadrants.	Sine.	Cosine.	Tangent.	Cotangent.	Secant.	Cosecant.
1 5 9	+	+	+	+	+	+
2 6 10	+	—	—	—	—	+
3 7 11	—	—	+	+	—	—
4 8 12, &c.	—	+	—	—	+	—

Of the Multiples and Powers of Arcs.

38. In most treatises on geometry, such as Leslie's, Legendre's, &c. the elementary propositions containing the principles of trigonometry are also given. It is therefore unnecessary to repeat them here, as it only puts the student to the expense of purchasing the same things in two or three different works. We shall merely give a few of the results most generally useful, referring to those works on geometry and trigonometry where the requisite information may be obtained.*

If a and b are two given arcs of a circle of which the radius is unity, then

$$\sin(a+b)=\sin a\cos b+\sin b\cos a \quad . \quad . \quad (1)$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b \quad . \quad . \quad (2)$$
$$\sin(a-b)=\sin a\cos b-\sin b\cos a \quad . \quad . \quad (3)$$
$$\cos(a-b)=\cos a\cos b+\sin b\sin a \quad . \quad . \quad (4)$$

If we divide these equations, the one by the other in succession, that is, (1) by (2), and (3) by (4), then

$$\tan(a+b)=\frac{\sin a\cos b+\sin b\cos a}{\cos a\cos b-\sin a\sin b} \quad . \quad . \quad (5)$$
$$\tan(a-b)=\frac{\sin a\cos b-\sin b\cos a}{\cos a\cos b+\sin b\sin a} \quad . \quad . \quad (6)$$

Dividing the two terms of the second numbers by $\cos a\cos b$, and substituting $\tan a$ and $\tan b$ for their values in terms of the sine and cosine

$$\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b} \quad . \quad . \quad (7)$$
$$\tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b} \quad . \quad . \quad . \quad (8)$$

expressions which give the tangent of the sum and of the difference of two arcs in terms of the tangents of these arcs.

If we make $a=b$ in the preceding formulæ, they give

$$\sin 2a=2\sin a\cos a, \quad . \quad . \quad . \quad . \quad . \quad (9)$$
$$\cos 2a=\cos^2 a-\sin^2 a \quad . \quad . \quad . \quad . \quad . \quad (10)$$
$$\tan 2a=\frac{2\tan a}{1-\tan^2 a} \quad . \quad . \quad . \quad . \quad (11)$$

* Those we would more particularly recommend are the treatises of Gregory, Woodhouse, Lardner, and Cagnoli. Dr Kelly's Spherics is a very good treatise for teaching the practice of the stereographic projection of spherical triangles.

expressions which give the sine, cosine, and tangent of twice the arc in terms of the sine, cosine, and tangent of the simple arc.

39. Returning to equations (1,) (2), &c. we have by addition and subtraction

$$\sin(a+b)+\sin(a-b) = 2\sin a\cos b \quad . \quad (12)$$
$$\cos(a+b)+\cos(a-b) = 2\cos a\cos b \quad . \quad (13)$$
$$\sin(a+b)-\sin(a-b) = 2\sin b\cos a \quad . \quad (14)$$
$$\cos(a-b)-\cos(a+b) = 2\sin a\sin b \quad . \quad (15)$$

Let $(a+b)=u$, and $(a-b)=v$, then by addition and subtraction $a=\frac{1}{2}(u+v)$, $b=\frac{1}{2}(u-v)$, consequently the preceding formulæ become

$$\sin u+\sin v = 2\sin\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v) \quad . \quad (16)$$
$$\sin u-\sin v = 2\sin\tfrac{1}{2}(u-v)\cos\tfrac{1}{2}(u+v) \quad . \quad (17)$$
$$\cos u+\cos v = 2\cos\tfrac{1}{2}(u+v)\cos\tfrac{1}{2}(u-v) \quad . \quad (18)$$
$$\cos v-\cos u = 2\sin\tfrac{1}{2}(u+v)\sin\tfrac{1}{2}(u-v) \quad . \quad (19)$$

expressions which serve to transform the sum or the difference of the sine or cosine into the product, and thus to unite the two terms into one.

If we divide formula (16) by formula (17) they give

$$\frac{\sin u+\sin v}{\sin u-\sin v}=\frac{\tan\frac{1}{2}(u+v)}{\tan\frac{1}{2}(u-v)} \quad . \quad . \quad (20)$$

If we multiply these equations member by member, observing to substitute $\sin 2a=2\sin. a\cos a$, formula (9), then

$$\sin^2 u-\sin^2 v = \sin(u+v)\cos(u+v) \quad . \quad (21)$$
$$\cos^2 v-\cos^2 u = \sin(u+v)\cos(u+v) \quad . \quad (21)$$

Since $\sin 2a=2\sin a\cos a$, and $\cos 2a=\cos^2 a-\sin^2 a$.

The second of these equations may be put under the two following forms:

$$\cos 2a=1-2\sin^2 a, \text{ and } \cos 2a=2\cos^2 a-1$$

whence $\sin^2 a=\dfrac{1-\cos 2a}{2}$, and $\cos^2 a=\dfrac{1+\cos 2a}{2}$. (22)

These expressions are used when, for the squares of the sine and cosine, the first power of the cosine of the double arc is substituted.

40. Let $2a=u$, then $a=\frac{1}{2}u$ formula (22), these formulæ become

$$\sin^2\tfrac{1}{2}u=\frac{1-\cos u}{2},\ \cos^2\tfrac{1}{2}u=\frac{1+\cos u}{2} \quad . \quad (23)$$

and dividing each corresponding number successively, they give

$$\tan^2\tfrac{1}{2}u=\frac{1-\cos u}{1+\cos u} \quad . \quad . \quad . \quad (24)$$

$$\text{and } \cos u=\frac{1-\tan^2\frac{1}{2}u}{1+\tan^2\frac{1}{2}u} \quad . \quad . \quad . \quad (25)$$

If b in formulæ (1), (2) be made $2a$, $3a$, &c. we may obtain multiple arcs thus:

$$\sin 3a=\sin a\cos 2a+\sin 2a\cos a$$
$$\cos 3a=\cos a\cos 2a-\sin a\sin 2a$$

Substituting for $\sin 2a$ and $\cos 2a$, their values, they become

$$\sin 3a=3\sin a\cos^2 a-\sin^3 a \quad . \quad (26)$$
$$\cos 3a=-3\cos a\sin^2 a+\cos^3 a \quad . \quad (27)$$

These may be put under the form

$$\sin 3a=\cos^3 a(3\tan a-\tan^3 a)$$
$$\cos 3a=\cos^3 a(1-3\tan^2 a)$$

In general n being any integer,

$$\sin n\,a = \cos^n a \left\{ n \tan a - \frac{n\,(n-1).(n-2)}{1.2.3} \tan^3 a + \frac{n.(n-1).(n-2).(n-3).(n-4)}{1.2.3.4.5} \tan^5 a \ldots \&c. \right\} \quad . \quad . \quad (29)$$

$$\cos n a = \cos^n a \left\{ 1 - \frac{n(n-1)}{1.2} \tan^2 a + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4} \tan^4 a, \ \&c. \right\} (29)$$

The coefficients of the different terms are those of the n^{th} power of the binomial, whence these series may be collected under the following form:

$$\sin n\,a = \frac{1}{2\sqrt{-1}} \left\{ \cos a + \sqrt{-1} \sin a \right\}^n - \frac{1}{2\sqrt{-1}} \left\{ \cos a - \sqrt{-1} \sin a \right\}^n (30)$$

$$\cos\ n\ a = \tfrac{1}{2} \left\{ \cos a + \sqrt{-1}\ \sin a \right\}^n + \tfrac{1}{2} \left\{ \cos a - \sqrt{-1}\ \sin a \right\}^n (31)$$

These formulæ, by development, will give the two foregoing series, and are thus easily verified.

41. It may be shown* that if a represent any arc

$$\sin a = a - \frac{a^3}{1.2.3} + \frac{a^5}{1.2.3.4.5} - \frac{a^7}{1.2.3.4.5.6.7} +, \&c. \quad . \quad (32)$$

$$\cos a = 1 - \frac{a^2}{1.2} + \frac{a^4}{1.2.3.4} - \frac{a^6}{1.2.3.4.5.6} +, \&c. \quad . \quad . \quad (33)$$

In these expressions the arc a is supposed to be divided by the radius, which is here taken for the unit of length, and consequently if we wish to restore it we must write $\frac{a}{r}$ in place of a and $\frac{\sin a}{r}$ instead of $\sin a$ in the two members of these equations.

$$\tan a = a + \frac{a^3}{1.3} + \frac{2\,a^5}{1.3.5} + \frac{17\,a^7}{3.3.5.7} + \&c. \quad . \quad . \quad . \quad (34)$$

$$\cot a = \frac{1}{a} - \frac{a}{3} - \frac{a^3}{3.3.5} - \frac{2\,a^5}{3.3.5.7} - \&c. \quad . \quad . \quad . \quad . \quad (35)$$

$$\text{versin}\ a = \frac{a^2}{2} - \frac{a^4}{2.3.4} + \frac{a^6}{2.3.4.5.6} - \&c. \quad . \quad . \quad . \quad . \quad (36)$$

$$a = \sin a + \frac{\sin^3 a}{2.3} + \frac{1.3.\ \sin^5 a}{2.4.5} + \&c. \quad . \quad . \quad (37)$$

$$a = \tan a - \frac{1}{3} \tan^3 a + \frac{1}{5} \tan^5 a - \&c. \quad . \quad (38)$$

These formulæ might be carried much farther than can be introduced into this place. Most of them may be seen by consulting the books already referred to, but, above all, the *Analysis Infinitorum* of Euler.

Tables of Multiples and Powers of Arcs.

1.	2.
1. $\sin\ a = s$, s {being the sine of the arc a}	$\cos\ a = (1-s^2)^{\frac{1}{2}}$
2. $\sin 2\,a = 2s\,(1-\sin.^2)^{\frac{1}{2}}$	$\cos 2\,a = 1 - 2s^2$
3. $\sin 3\,a = 3s - 4s^3$	$\cos 3\,a = (1-4s^2)\,(1-s^2)^{\frac{1}{2}}$
4. $\sin 4\,a = (4s - 8s^3)\,(1-s^2)^{\frac{1}{2}}$	$\cos 4\,a = 1 - 8s^2 + 8s^4$
5. $\sin 5\,a = 16s^5 - 20s^3 + 5s$, &c.	$\cos 5\,a = (1-12s^2+16s^4)(1-s^2)^{\frac{1}{2}}$ &c.)

* Woodhouse's Trigonometry, third edition, page 245,—Gregory, page 42 and 50,—and Cagnoli's copious Treatise.

3.	4.
6. $\tan\ a=t,\ t$ $\left\{\begin{array}{l}\text{being the}\\ \text{tangent } a.\end{array}\right\}$	$\cot\ a=\cot$
7. $\tan 2a=\dfrac{2t}{1-t^2}$	$\cot 2a=\dfrac{\cot^2-1}{2\cot}$
8. $\tan 3a=\dfrac{3t-t^3}{1-3t^2}$	$\cot 3a=\dfrac{\cot^3-3\cot}{3\cot^2-1}$
9. $\tan 4a=\dfrac{4t-4t^3}{1-6t^2+t^4}$	$\cot 4a=\dfrac{\cot^4-6\cot^2+1}{4\cot^3-4\cot}$
10. $\tan 5a=\dfrac{5t-10t^3+t^5}{1-10t^2+5t^4}$, &c.	$\cot 5a=\dfrac{\cot^5-10\cot^3+5\cot}{5\cot^4-10\cot^2+1}$, &c.

5.	6.
11. $\sin\ a=\sin a$	$\cos\ a=\cos a$
12. $2\sin^2 a=1-\cos 2a$	$2\cos^2 a=1+\cos 2a$
13. $4\sin^3 a=3\sin a-\sin 3a$	$4\cos^3 a=3\cos a+\cos 3a$
14. $8\sin^4 a=3-4\cos 2a+\cos 4a$, &c.	$8\cos^4 a=3+4\cos 2a+\cos 4a$.

7. *Differences of Trigonometrical Lines.*

15. $\Delta\sin\ a=+2\sin\frac{1}{2}\Delta a\,.\cos\left(a+\frac{1}{2}\Delta a\right)$
16. $\Delta\cos\ a=-2\sin\frac{1}{2}\Delta a\,.\sin\left(a+\frac{1}{2}\Delta a\right)$
17. $\Delta\tan\ a=+\dfrac{\sin\Delta a}{\cos a\,.\cos(a+\Delta a)}$
18. $\Delta\cot\ a=-\dfrac{\sin\Delta a}{\sin a\,.\sin(a+\Delta a)}$
19. $\Delta\sin^2 a=+\sin\Delta a\,.\sin(2a+\Delta a)$
20. $\Delta\cos^2 a=-\sin\Delta a\,.\sin(2a+\Delta a)$
21. $\Delta\tan^2 a=+\dfrac{\sin\Delta a\,.\sin(2a+\Delta a)}{\cos^2 a\,.\cos^2(a+\Delta a)}$
22. $\Delta\cot^2 a=-\dfrac{\sin\Delta a\,.\sin(2a+\Delta a)}{\sin^2 a\,.\sin^2(a+\Delta a)}$

8. *Differentials of Trigonometrical Lines.*

23. $d\sin\ a=+d\,a\,.\cos a$
24. $d\cos\ a=-d\,a\,.\sin a$
25. $d\tan\ a=+\dfrac{d\,a}{\cos^2 a}$
26. $d\cot\ a=-\dfrac{d\,a}{\sin^2 a}$
27. $d\sin^2 a=+2\,d\,a\,.\sin a\,.\cos a$
28. $d\cos^2 a=-2\,d\,a\,.\sin a\,.\cos a$
29. $d\tan^2 a=+\dfrac{2\,d\,a\,.\tan a}{\cos^2 a}$
30. $d\cot^2 a=-\dfrac{2\,d\,a\,.\cot a}{\sin^2 a}$

42. Having given a short abstract of the more useful formulæ relative to multiples and powers of arcs, we shall now proceed to show the method of constructing the tables of sines, tangents, &c.

When the radius of a circle is unity, the semicircumference is 3.1415926536 nearly. Now there are 180° or 10800′ in a semicircle, consequently, if the former be divided by the latter, the result will

be 0.0002908882, the measure of an arc of one minute, which as the arc is so small, may be considered its sine.

Now, art. 35. 2, $\cos = (1 - \sin^2)^{\frac{1}{2}}$ consequently $\cos' 1' = 0.9999999577$. If these values are substituted in formulæ, (32), and (33), art. 41 the sines and cosines may be obtained through the whole quadrant.

Thus let the arc $a=1'$, and, therefore, $\sin a = 0.0002908882$. Let $a=5^\circ$, then $\frac{5 \times 3.1415926536}{180} = 0.08726646$ the length of a, and

$$a = +0.08726646$$
$$-\frac{a^3}{1.2.3} = -0.00011076$$
$$+\frac{a^5}{1.2.3.4.5} = +0.00000004$$

therefore, $a - \frac{a^3}{1.2.3} + \frac{a^5}{1.2.3.4.5}$, &c.$=0.08715574=$ the natural sine of 5°, the logarithm of which is 8.940296, the log. sine of the same arc. This method is easy when the arc is small, as the series then converges very rapidly; but it is rather laborious when the arc is large, in which case recourse must be had to other methods depending upon the properties of multiple arcs, as may be seen in most of our treatises on trigonometry.

As the sines are computed, the cosines of the same arcs may be found from art. 41, formula (33), or from art. 36, formula (2,) the tangents and cotangents, from formula (7) and (8), and the secants and cosecants from (9) and (10).

Section IV.

Of the Application of Tables of Sines, Tangents, Secants, &c. to Plane Trigonometry.

CASE I.

43. In any plane triangle it is shewn in our usual treatises,* that the sides are proportional to the sines of their opposite angles, or

The sine of any one angle,
Is to the sine of another angle;
As the side opposite to the first,
Is to the side opposite to the second.

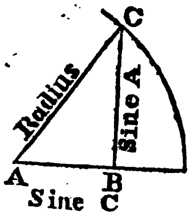
These terms may be taken alternately, inversely, &c.

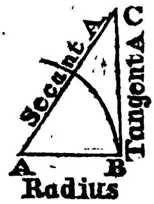
44. When one of the angles is a *right* angle, then the preceding rule may either be applied, or a modification of it derived from the properties which are peculiar to right-angled triangles.

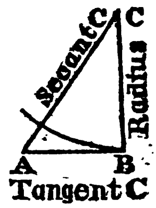
In right-angled triangles, it is usual to call that side subtending the right angle the *hypotenuse*, and the other sides, which contain the right angle, the *legs*, or the one the *base* and the other the *perpendicular*.

Then if one of the sides of any triangle ABC, be assumed equal to the *radius*, the *names* of the other sides must be determined by art. 28, as follows:—

* Leslie's and Legendre's Geometry, Ingram's, Playfair's, Simson's Euclid, &c.







The names of the sides being thus known when three of the parts of a triangle including a side are given, the rest may be found by the following rules:—

I.—*To find a side.*

As the name of the given side,
Is to the name of the required side;
So is the given side,
To the required side.

II.—*To find an angle.*

As the side made radius,
Is to the other given *side*,
So is radius,
To the name of *this side.*

Any side may be made radius to find a *side,* but one of the *given sides* must be made radius to find an *angle.*

In the solution of plane triangles, it must be recollected that all the angles in any triangle are together equal to two right angles, or 180°. Whence if two of the angles are given, the other may be found by subtracting their sum from 180°; when one angle is given the sum of the other two may be found by subtracting it from 180°; and if one be right or 90°, the sum of the other two is also 90°, and the one is the complement of the other.

CASE II.

45. In a plane triangle when the two sides and contained angle are given.

I. As the sum of the given sides
Is to their difference;
So is the tangent of half the sum of the opposite angles
To the tangent of half their difference.

Half the difference added to half the sum of those angles gives the greater, and subtracted from half the sum gives the less.

All the angles being now known, the third side may be found by the rules in Case I.

Or, after having found half the sum and half the difference of the angles, the remaining side may be found without determining the actual angles, in the following manner:*

II. As the sine of half the difference of the opposite angles
Is to the sine of half their sum,
So is the *difference* of the containing sides to the remaining side; or,

III. As the cosine of half the difference of the opposite angles
Is to the cosine of half their sum;
So is the *sum* of the containing sides
To the remaining side.

* Playfair's Euclid, Plane Trigonometry, Prop. IX. and X.

These two methods may be used as a verification to each other, and will be found somewhat more easy in practice than the first method, as several of the quantities may be taken out from the trigonometrical tables at the same time.

Should the sides come out in logarithms from some previous operation, then Gauss' table for finding the logarithm of the sum and difference of two numbers from their logarithms, without first determining the natural numbers themselves, would be some advantage, though it was not thought sufficient to warrant an insertion of it among the tables.

The following method of resolving this problem is convenient, particularly when the *logarithms* of the sides are given.*

IV. From the logarithm of the greater of the two given sides, having its index increased by 10, subtract the logarithm of the less side, the remainder will be the logarithm tangent of an arc, from which, 45° being subtracted, there will be obtained a remainder. To the logarithm tangent of this remainder add the log. tangent of half the sum of the opposite angles, the sum, rejecting 10 in the index, will be the log. tangent of half their difference, from which the angles themselves may be found.

CASE III.

46. In any plane triangle, when the three sides are given,

I. As the base
Is to the sum of the sides;
So is the difference of the sides
To the difference of the segments of the base made by a perpendicular upon it, or upon it produced from the opposite angle.

It may perhaps be convenient to call the longest side the base, in order that the perpendicular may fall within the triangle.

When the three sides of a triangle are given, the difference of the segments of the base may thus be found. Then half the difference added to half the sum, that is, to half the base, will give the greater segment adjacent to the greater side; and half the difference taken from half the sum will give the less. From these the angles may be found by Rule II. § (44).

II. In a plane triangle, as the rectangle under any two sides, is to the rectangle under the excesses of the semiperimeter above those sides; so is the square of the radius to the square of the sine of half their contained angle, as shown in Leslie's Geometry. In practice, this rule, when logarithms are employed, may be stated as follows:

To the arithmetical complements of the logarithms of the two sides containing the required angle, add the logarithms of the differences between those sides and half the sum of the three sides, then half the sum of these four logarithms will be the log. *sine* of half the required angle.

III. To the arithmetical complements of the sides containing the required angle, add the logarithm of half the sum of the three sides, and the logarithm of the difference between this half sum and the side opposite the required angle; half the sum of these four logarithms will be the log. *cosine* of half the required angle.

IV. To the arithmetical complement of the logarithm of half the sum of the three sides, add the arithmetical complement of the difference between half the sum of the three sides and the side opposite the required angle, and the logarithms of the differences between

* See Simson's Euclid, Plane Trigonometry, Prop. IV.

that half sum and the sides containing the required angle; half the sum of those four logarithms will be the log. *tangent* of half the required angle.

It may be remarked, that these three last rules will, in general, be the most commodious in practice, though, in particular cases, each may have its peculiar advantage when great accuracy is required.

When the required angle does not exceed 90°, Rule II. may be used; when it does, Rule III. may be employed; and in either case Rule IV. will give correct solutions. These observations depend upon the variation of the trigonometrical lines in certain parts of the circle, as, for example, near 90°, the sines vary very slowly, so that the true value of an arc cannot be obtained by our ordinary tables, while the tangents always vary by such perceptible quantities as to leave no doubt of the real value of the required arc. These remarks may be easily verified by examining any of our tables extended to six or seven places of decimals.

Of the Construction of Triangles.

47. Previous to the numerical solution of any triangle, it is generally first constructed geometrically. This is accomplished by means of what are termed mathematical instruments, consisting of scales, compasses, &c. contained in a case, at various prices, to suit the convenience of purchasers. Printed descriptions of these, as well as of many others, are to be found in Jones' edition of Adams' Geometrical and Graphical Essays.

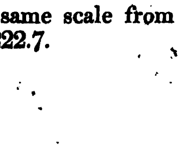
In the construction of plane triangles, the sides are taken from a scale of equal parts, and the angles are laid down by a scale of chords, or more conveniently by a protractor.

Examples.

CASE I.

48. 1. Given the angles and hypotenuse of a right-angled triangle, to find the base and perpendicular.

Let the hypotenuse AC of the right-angled triangle ABC be 288, and the angle A 39° 22′; it is required to find the sides AB and BC.



Construction.—In the indefinite straight line AB take any point A, and by a protractor, or scale of chords, make the angle A equal to 39° 22′; from any convenient scale of equal parts take AC equal to 288, and from C draw CB, perpendicular to AB: then ABC will be the triangle required. In order to simplify and preserve uniformity, the angles may, in general, be denoted by the capital letters A, B, C, and the opposite sides by the small letters a, b, c. The sides a and c being measured by the same scale from which b was taken, will be found to be 182.7 and 222.7.

Calculation.

1. By natural numbers, § (43).

To find a.

As sin B : sin. A : : b : a, or $a=\dfrac{\text{sin. A}\times b}{\text{sin B}}$

$$1 : 0{\cdot}634281 : : 288 : \frac{0{\cdot}634281\times 288}{1}=182.673=a$$

To find c.

And sin B : sin C, or cos A : : b : c

$$1 : 0.773103 : : 288 : \frac{0.773103 \times 288}{1} = 222.654 = c$$

2. By logarithms.

To find a.

As sin B, or radius	10.000000
Is to sin A 39° 22′	9.802282
So is b 288	2.459392
To a 182.673	2.261674

To find c.

As radius	10.000000
Is to cos A 39° 22′	9.888237
So is b 288	2.459392
To c 222.653	2.347629

The solutions may be varied by assuming any of the sides for radius, according to art. (44), and verified by Gunter's scales.

GENERAL RULES.

I. BY GUNTER'S SCALE.

Extend the compasses from the first term to the second, that extent will reach in the *same direction* from the third term to the fourth.

II. BY GUNTER'S SLIDING RULE.

Move the slide till the first term coincide with the second, then, reversing the rule when necessary, opposite the third term will be found the fourth.

Ex.—Extend the compasses from 90°, or radius, to 39° 22′, on the line of sines, that extent will reach from 288 to 182.7 on the line of numbers; or set 90° on the line of sines upon the *slide* to 39° 22′ on the fixed rule, then on reversing the rule, there will be found opposite 288 on the *slide*, 182.7 on the line of numbers.

If *secants* should enter the proportion, read cosines, and turn the compasses the opposite way, or reverse the reading on the slide.

2. Given the angles and one side to find the hypotenuse and the other side.

Let the side AB be 758, and the angle C 39° 26′; to find the angle A, and the sides BC and AC?

Ans.—BC is 921.71, and AC 1193.36, and the angle A 50° 34′.

Construction.—From a scale of equal parts make AB equal to 758, the angle A 50° 34′, the complement of C, and draw BC at right angles to AB; produce AC and BC till they meet in C; then ABC is the triangle required, and a and b measured on the same scale from which c was taken will be found to be about 922 and 1193 respectively.

3. Given the hypotenuse and one side, to find the angles and other side.

Let the hypotenuse AC be 544, and the base AB 464, to find the angles A and C, and the side BC?

Ans.—The angle A is 31° 28′, the angle C is 58° 32′, and BC 284.

Construction.—Make AB equal to 464 from a scale of equal parts, and from B draw BC perpendicular to AB, then from the centre A

at the distance AC, equal to 544, describe an arc intersecting BC in C, join AC, and the triangle is constructed. The angle A being measured by a protractor or scale of chords, will be found to be 31° 28′, consequently C is 58° 32′, and the side BC 284 from the same scale by which the other sides were laid down.

4. Given the base and perpendicular to find the angles and hypotenuse.

Let the base AB be 558, and the perpendicular BC 456; required the angles A and C and the hypotenuse AC?

Ans.—A 39° 15′ 21″, C 50° 44′ 39″, and AC 720.622.

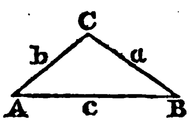
Construction.—Make AB equal to 558, and draw BC perpendicular to AB and equal to 456, join AC, and the triangle is constructed. The angle A will measure 39¼°, and the hypotenuse will be about 721 on the scale of equal parts. The other side may be found by Euclid I. and 47, or Leslie's Geometry, II. 10 and 13.

5. Given the angles and one side of an oblique-angled plane triangle to find the other sides.

In the triangle ABC are given the side AC, 532, the angle A 38° 40′, C 92° 46′, and consequently the angle B 48° 34′, to find the sides AB and BC?

Ans.—AB 708.765, BC 443.345.

Construction.—Draw the indefinite AB, at A make the angle BAC equal to 38° 40′, and from a scale of equal parts make AC 532, at C draw CB, making the angle ACB equal to 92° 46′, it will cut AB in B forming the triangle ABC which was required.



6. Given two sides, and an angle opposite one of them, to find the other angles and the third side.

In the triangle ABC are given the side AB 274, AC 306, and the angle B 78° 13′; required the angles A and C, and the third side BC?

Ans.—The angle C is 61° 14′, the angle A 40° 33′, and the side BC 203.22.

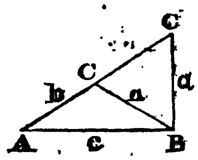
Construction.—Make AB equal to 274, the angle B equal to 78° 13′, and with an extent equal to AC, 306, intersect the line BC in C; ABC is the triangle required.

If in this triangle the side AB be greater than BC, there may be two triangles formed, constituting what is called the ambiguous case, that is, it admits of two solutions, either of which answers the conditions required, unless from some known circumstances one of them must be adopted in preference to the other.

Thus in the oblique-angled triangle ABC there are given AB 318, BC 195, and the angle A 32° 40′.

Ans.—The angle C is 61° 40′ or 118° 20′, the angle B is 85° 40′ or 29°, and the side AC is 360.246 or 175.15.

Construction.—Make AB equal to 318 from any convenient scale of equal parts, the angle A equal to 32° 40′, and with the centre B and distance equal to BC 195 describe an arc cutting AC in C or C′; ABC or ABC′ will be the triangle required.



CASE II.

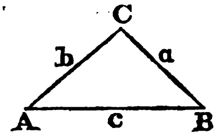
49. Given two sides and the contained angle to find the other angles and the third side.

In the triangle ABC let the side AB be 920 and AC 500, and the

contained angle A 36° 52′; required the angles B and C, and the third side BC?

Ans.—B is 29° 58′ 50″, C 113° 9′ 10″, and BC is 600.31.

Construction.—Make AB equal to 920, at the point A make the angle BAC equal 36° 52′, and AC equal to 500; join BC; ABC is the triangle required.



By Calculation, art. 45, 1.

As AB+AC 1420					3.152288
Is to AB—AC 420					2.623249
So is tan ½ (C+B)	71°	34′	0″		10.477162
To tan ½ (C—B)	41	35	10		9.948123
C	113	9	10		
B	29	58	50		
As sin B	29°	58′	50″		9.698714
Is to sin A	36	52	0		9.778119
So is AB 500					2.698970
To BC 600.31					2.778375

Or by art. 45. II. and III.

As sin ½ (C—B)	41°	35′	10″		9.822001
Is to sin ½ (C+B)	71	34	0		9.977125
So is AB—AC		420			2.623249
To BC		600.31			2.778373
As cos ½ (C—B)	41°	35′	10″		9.873877
Is to cos ½ (C+B)	71	34	0		9.499963
So is AB+AC			1420		3.152288
To BC			600.31		2.778374

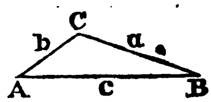
The advantage of these two last methods consists in its being unnecessary to find the values of the angles C and B to determine BC, and that several of the quantities are found among the tables at the same opening of the book, and if computed both ways they are a check upon each other.

CASE III.

50. Given the three sides of a triangle to find the angles.

In the triangle ABC, there are given AB 800, AC 320, and BC 562; to find the angles.

Construction.—Draw the line AB equal to 800 from a scale of equal parts, then from the same scale take an extent equal to AC 320, and with the centre A and distance 320 describe an arc, in like manner, with the centre B and distance BC 562, intersect the former arc in C; ABC is the triangle required.



In the solution of this question, if the angles A and B are first to be determined, then rules II. or IV. § 46, will be found most convenient and accurate; but if C be wanted first, then if great accuracy is required it would be improper to use rule II., but rule III. or IV. should be employed, so as to give the angle with all the requisite accuracy in nice operations.

By Calculation.

RULE II.

AB	800	
AC	320 ar. co.	7.494850
BC	562 ar. co.	7.250264
Sum	1682	
Half	841	
1st diff.	521 log.	2.716838
2d diff.	279 log.	2.445604
Sum		19.907556
Half	64° 1′ 54″.9 sin	9.953778
	2	
C	128 3 49.8	

RULE III.

AB	800	
AC	320 ar. co.	7.494850
BC	562 ar. co.	7.250264
Sum	1682	
Half	841 log.	2.924796
Diff.	41 log.	1.612784
Sum		19.282694
Half	64° 1′ 54″.63 cos	9.641347
	2	
C	128 3 49.26	

RULE IV.

AB	800	
AC	320	
BC	562	
Sum	1682	
Half	841 ar. co.	7.075204
1st diff.	41 ar. co.	8.387216
2d diff.	521 log.	2.716838
3d diff.	279 log.	2.445604
Sum		20.624862
Half	64° 1′ 54″.58 tan.	10.312431
	2	
C	128 3 49.16	

From these solutions it appears that the first and second differ about 0″.54 from each other, while the second and last only differ 0″10.

Had the angle C been nearer 180°, the first and second solutions might perhaps have differed more considerably, while the second and third would have agreed more nearly. Hence it is clear that the proper rules, when great nicety is required, must be chosen according to the nature of the angle.

Exmaples for Exercise.

51. 1. What angle will one foot subtend at the distance of fifty miles? *Ans.*—0″.78.

2. The hypotenuse of a right-angled triangle being 5472 feet, and the acute angle adjacent to the base, 29° 50′ 58″, what are the base and perpendicular?

Ans.—The base 4746.064, and the perpendicular, 2723.538.

3. The hypotenuse of a right-angled triangle is 1963004, and one of the legs 1963000; required the two acute angles?

Ans.—6′ 56.″4, and 89° 53′ 3.″6.

4. If the sides of a plane triangle be in proportion to each other as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, what are the angles?

Ans. 117° 16′ 46″, 36° 20′ 10″, and 26° 23′ 4″.

5. If the base of a plane triangle be 384, and the other two sides 288 and 192, what is the length of the perpendicular upon the base, and the length of the segments of the base made by a line bisecting the vertical angle?

Ans.—Perp. 139.4274, segments 230.4 and 153.6.

6. There are three towns, A, B, C, so situated that the bearing of B and C from A forms an angle double that of A and C from B, and that of A and B from C double that of B and C from A, or the angle opposite *a* is double of the angle opposite *b*, and the angle opposite *b* is double of that opposite *c*, and the circuit round all the three is just one hundred miles; what are their relative distances from each other in succession?

Ans.—19.8073, 35.6861, and 44.5066 miles.

7. In the right-angled triangle right-angled at B, given the base AB 70, and the sum of the hypotenuse and perpendicular AC and BC 200, to find the hypotenuse and perpendicular, and the remaining angles?

Ans.—The angle ACB is 38° 34′ 48″, BAC 51° 25′ 12″, and AC 112.25, and BC 87.75.

8. In an oblique-angled triangle ABC let the side BC be 532, the angle BAC 110° 30′, and the sum of the sides AB, AC 637; required the angles C and B, and the sides AB and AC?

Ans.—The angle C is 45° 5′, B 24° 25′, and the side AB 402.3 and AC 234.7.

9. In the oblique-angled triangle ABC, let the side BC be 250, the angle BAC 96° 50′, also the difference between the sides AB and AC 106; required the angles ACB and ABC, together with the sides AB and AC?

Ans.—ACB is 57° 55′ 40″, ABC 25° 14′ 20″, and AB 213.36, and AC 101.36.

10. Given the base 214, the vertical angle 49° 16′, and the sum of the other two sides 459, to find the sides and remaining angles?

Ans.—The acute angle is 38° 44′ 48″, the obtuse angle is 91° 59′ 12″, the side opposite the acute angle is 176.75, and the side opposite the obtuse angle is 282.245.

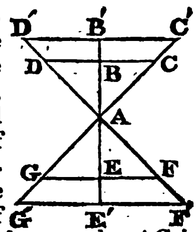
11. Given the base 1514, the verticle angle 75° 24′ 50″, and the perpendicular 972.41; required the remaining sides and angles?

Ans.—The sides are 1298 and 1172, and the angles are 56° 4′ 5″ and 48° 31′ 5″ respectively.

Navigation.

52. The various sailings in navigation are only the applications of trigonometry in particular circumstances.

The course is the angle formed between the meridian and the point on which the ship sails, indicated on the mariner's compass, the distance is the hypotenuse, and the difference of latitude and departure the legs of a right-angled triangle.

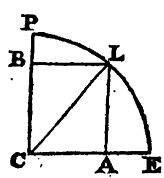


Thus let AB represent a portion of the meridian; then, if a ship sails north-easterly, the line AC is drawn to the right-hand, making an angle BAC equal to the course, shown by the compass, and is generally in points and quarter-points, and AC represents the distance, AB the difference of latitude, and BC the departure. If she sails north-westerly, then BAD is supposed to be the angle of course, AD the distance, AB the difference of latitude, and BD the departure. If she sails south-westerly, AG is the distance, AE the difference of latitude, GE the departure, and GAE the course. Lastly, if the ship sail south-easterly, AF is the distance, AE the difference of latitude, EF the departure, and FAE the course. If, however, AE′ be the meridian difference of latitude, E′F′ is the difference of longitude, E′AF′ is the course, and AF is still the distance. Hence the course and distance between two places can be found, by this method, when their latitudes and longitudes are known. This is commonly called Mercator's sailing.

Parallel, middle latitude, and oblique sailings, may readily be explained on similar principles.

Parallel Sailing.

If the earth be considered as a sphere, from which it does not deviate greatly, the difference of longitude between two places is the angle at the pole, formed by two meridians passing through these places, (Spher. Trig. prop. VI.) and measured by an arc of the equator intercepted between them. But since these meridians approach one another, as their distance from the pole diminishes, the meridian distance corresponding to the same difference of longitude varies according to the latitude.



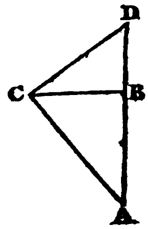
Let PCE be a section of one-fourth of the earth, PE an arc of the meridian from the equator, as E to the pole at P, and L any place on the earth's surface. Draw BL parallel to CE, AL parallel to PC, and join CL. Now CE or CL is to BL as radius is to cosine ECL; therefore, since circles, or any portion of them, are as their radii, any portion of a circle whose radius is CE, is to a similar portion of a circle whose radius is BL, as radius to cosine ECL. The latitude of L, of which BL is the radius of the parallel, is measured by the arc EL, supposing CE the radius of the equator, and PC the polar semiaxis, then the length of an arc on the equator is to the length of the corresponding arc in the latitude L, as radius to the cosine of the latitude. Hence, in any latitude, the radius of the parallel is equal to

the cosine of the latitude, the semidiameter of the equator being considered radius, and since similar arcs are as their radii, it follows, that the distance sailed on any parallel is to the difference of longitude as the cosine of the latitude to the radius. In this manner Table I. was computed.

Middle Latitude Sailing.

This is an easy approximate method of resolving problems in navigation, and is a combination of plane and parallel sailings, in which the difference of longitude is reckoned upon the *middle parallel* between the latitude sailed from and that come to, whence it derives its name.

Let ABC be a figure in plane sailing, in which AB is the difference of latitude, AC the distance, and BC the departure. Connect with this, DBC, a figure in parallel sailing, in which BC is still the departure, DCB an angle equal to the middle latitude, then will DC be the difference of longitude. Resolving these two triangles either separately, or in conjunction, as the case may require, the following rule will be obtained.



Diff. lat. : diff. long. : : cos mid. lat. : tang. course.

This analogy may be taken inversely, &c. according to the *data*.

EXAMPLES.

1. A ship from Bombay in latitude 18° 57′ N. sailed S. W. by S. 224 miles; required the latitude come to, and the departure?

Ans.—The difference of latitude is 186.2, and the departure 124.4

Latitude of Bombay	. .	18° 57′ N.
Diff. of lat. 186 miles=	.	3 6 S.
Latitude come to	. .	15 51 N.

2. A ship from latitude 47° 30′ N. sails S. W. by S. 98 miles; what latitude is she in, and what departure has she made?

Ans.—Difference of latitude 81.48, departure 54.45 miles, and the latitude come to 46° 9′ N

3. A ship from latitude 48° 32′ N. sails between north and west till her departure is 54 miles, and then finds herself in latitude 49° 54′ N.; what course did she steer, and what distance did she run?

Ans.—Course 33° 22′ N. W., and distance 98.18 miles.

4. Required the course and distance from Cape Wrath in Scotland, in latitude 58° 36′ N. longitude 5° 20′ W., to New York in North America, in latitude 40° 28′ N. longitude 74° 2′ W.?

Ans. Course 67° 32′ or W. S. W. nearly, and distance 2847 miles.

5. A ship from latitude 60° 24′ N. and longitude 43° W. sails between south and east till she is in latitude 56° 30′ N., and has made 226 miles of departure; required her course, distance, and longitude?

Ans.—Course S. 44° E. or S. E. nearly, distance 325.4 miles, and the longitude of the ship 35° 47′ W.

6. Required the course and distance between the Isle of May, in latitude 56° 12′ N. longitude 2° 33′ W., and Heligoland, in latitude 54° 12′ N. longitude 7° 53′ E.?

Ans.—Course S. 71° 27′ E. and dist. 377 miles.

7. A ship from Inchkeith, in latitude 56° 3′ N., and longitude 3° 10′ W. sailed E. ½ S. by compass, or, since the variation is about 2¼

points West, E. N. E. true, 3^h 20^m, at the rate of 5.8 knots an hour, what was her situation with regard to the isle of May in latitude 56° 11′ N., and longitude 2° 33′ W.?

Ans.—Latitude in 56° 10′ N. and longitude 2° 47′ W. or about 1 mile south of the isle of May light, and 14′ of longitude west of it.

8. Required the true and magnetic course and distance between Ayr light, in latitude 55° 25′ N. and longitude 4° 26′ W., and the Mull of Cantire, in latitude 55° 18′ N. and longitude 5° 40′ W., the variation of the compass being 2¼ points W.?

Ans.—The true course is S. 80° 47′ W. or W. by S. nearly, the course by compass W. N. W., and the distance 44 miles nearly.

9. Coasting along shore I saw a cape bearing N. E. by N. After standing N. W. 20 miles, the same cape bore E. N. E.; required the distance of the ship at each station?

Ans.—From the first station 33.26, and from the second 35.31 miles.

10. From a ship at sea, I observed a point of land to bear E. by S., and after sailing 12 miles N. E., it bore S. E. by E.; required the distance of the last place of observation from the point of land?

Ans.—26 miles.

11. Sailing N.N.W. at the rate of 6 knots an hour, at 8h. P. M. I discovered two light-houses, the northernmost of which bore N. N. E. and the other E. by N., and at 10h. 30m. the northernmost light bore E. N. E., and the other E. S. E.; the bearing and distance of the lights from each other are required.

Calculation.—In the triangle ACD are given the side AC equal to 15 miles, the angle ADC 3 points, the interval between E by N. and E. S. E. and the angle CAD 4 points, the distance between S. S. E. the opposite point to N. N. W., and E. S. E.; to find CD= 19.09. Again, in the triangle ABC are given AC as before equal to 15 miles, the angle ABC equal to 4 points, the interval between N. N. E. and E. N. E. and the angle ACB also 4 points, the interval between the N. N. W. and N. N. E. points; hence the angle CAB is a right angle; consequently we get BC=21.21.

Lastly, in the triangle BCD are given the sides CB, CD, equal to 21.21 and 19.09 respectively, and the included angle BCD 5 points, the interval between N. N. E. and E. by N.; to find the angles CDB =67° 30′, CBD=56° 15′=5 points, CBF=BCN=2 points, and the distance BD=19.09, and bearing S. E by S.

Most of these problems may be very expeditiously performed by Mercator's chart, of so much utility for laying down a ship's place at sea, and particularly at noon.

When the latitude and longitude are known, the ship's place may be laid down on the chart by laying a ruler first over the latitude on the sides of the chart, and drawing a pencil-line, then over the longitude on the top and bottom, and drawing another line intersecting the former; this point of intersection will be the ship's place. To facilitate the operation of finding the bearing and distance of one place from another, several compasses are drawn on convenient parts of the chart. By laying a ruler over the two places, and extending the points of the compasses from the centre of one of the cards on the chart to the nearest point of the ruler, then sliding both feet of the compasses forward and backward, keeping the one foot close to the ruler, the other will point out the course. The extent between the places being applied to the graduated scale of

latitude on the side of the chart, so that one-half may be above the mean latitude, and the other below it, will give the distance in degrees and minutes, which may be readily reduced to miles. If the distance be too great for the compasses, some aliquot part may be taken, such as a half, a third, &c. whence, by doubling, tripling, &c. the whole distance becomes known.

12. A ship from Aberdeen, in latitude 57° 9′ N. and longitude 2° 8′ W. sailed on the following true courses; required her situation?

Courses.	Dist.	Diff. of Lat. N.	Diff. of Lat. S.	Departure. E.	Departure. W.
S. E.	40		28.3	28.3	
S. S. E.	50		46.2	19.1	
N. E.	20	14.1		14.1	
S. E. *b*. S	60		49.9	33.3	
E. S. E.	200		76.5	184.8	
W. *b*. S.	15		2.9		14.7
N. N. W.	20	18.5			7.7
N. E. *b*. N.	76	63.2		42.2	
E. S. E. ¾ E.	60		14.6	58.2	
S. 71½ E.	378	95.8	218.4	380.0	22.4
			95.8	22.4	
			122.6	357.6	

Diff. of Lat. . . . 2° 3′ S.
Lat. left . . . . 57 9 N.

Lat. in . . 55 6 N.

Lat. left . . 57° 9′ N.
Lat. in . . 55 6 N.

Sum . . 112 15

Half or Mid. Lat. . 56 7½

Now to 56° 7½ as a course, Table VIII., the distance 640′, answering to the departure 357.6, found in a latitude-column, will be the difference of longitude=10° 40′E.

Long. left, . 2 8 W.
Long. made, . 10 40 E.

Long. in . . 8 32 E.

Hence the ship is close on the coast of Jutland opposite Hallam, between the islands Sylt and Romoe.

By computing the courses and run by a ship in this manner daily, from noon to noon, her place will be found each day at twelve, which may be laid down on a chart. Such other matters occurring on board as may be thought worthy of notice are also recorded in a book kept for that purpose, constituting what is called the log-book or journal.

JOURNAL.

The journal is a register of the various transactions which take place on board a ship both in harbour, and more especially at sea. At sea the day begins at noon, 12^h before the civil day, and 24^h before the astronomical day. The first 12^h from noon till midnight are marked P. M., and the next 12^h from midnight to noon, A. M. In ships of the Royal Navy and East India Company's service, the log is hove every hour, but in many trading vessels only once in two hours. To correct the course steered for variation and leeway, the observer is supposed to be placed at the centre of the card of the compass. Now the variation must be allowed to the right hand of the course steered, if it is easterly, but to the left if westerly, and the leeway must be allowed to the right or left, according as the ship is on the larboard or starboard tack.

When the true course is known, that by compass will be obtained by allowing the variation to the right if westerly, but to the left if easterly. When the ship is lying to, the middle point between that to which her head comes up, and that to which it falls off, is taken as the course, and the drift is assumed as the rate per hour in the direction in which the ship is carried. The latitude and longitude obtained by observation is reduced to noon, if necessary, by means of the log-book, showing the run of the ship in the interval. The latitude and longitude should be ascertained by observation daily, if possible. If lunars cannot be conveniently taken, the longitude by chronometer should not be neglected.

Rules for Working a Day's Work.

Having corrected the courses according to the foregoing directions, place these and the distances run in a table, as in example 12, and find the difference of latitute and departure. From the latitude left and that come to, the *middle latitude* may be found, being equal to *half the sum* of the latitudes if they are of the *same name*, and half the difference if of contrary names. Now, in a traverse table, to middle latitude as a course, the distance answering to the departure found in a latitude-column, is the difference of longitude, which being properly applied to the longitude left, according to its name, will be the longitude of the ship by dead reckoning.

The log-board is usually divided into seven columns: the first column on the left hand contains the hours from noon to noon; the second and third contain the knots and tenths of a knot at which the ship is sailing; the fourth contains the courses steered; the fifth the winds; the sixth the leeway; and the seventh the various remarks thought necessary, such as the state of the weather, the management of the sails, the observations for ascertaining the ship's place, the variation of the compass, &c. The log-board is transcribed every day at noon into the log-book, which is ruled and divided in the same manner. The nautical mile is 6076 feet, and since the sand-glass runs out in half a minute, or $\frac{1}{120}$ part of an hour, the knot is $\frac{6076}{120}=50.6$ feet, or say 50 feet, and the tenth part of a knot, in the following journal marked F, or fathom, must be five feet. Hence the number of knots and fathoms run off the reel in half a minute shows the miles and tenths which the ship is sailing per hour.

JOURNAL.

Ship Caledonia from Leith to Hamburgh.

H	K	F	Courses.	Winds.	Lee-way.	Remarks. Monday, May 12, 1828.
1	7	4	N. W.	S. W.		Sailed from Leith Harbour at noon.
2	7	4	E. b. S. ½ S.	. . .		Passed Inchkeith at ½ past 12.
3	7	4	E. S. E.	. . .	.	At 3, P. M. Isle of May Light, bore E. N. E.
4	8	0				At 5, do. it bore . . N. b. W.
5	8	0	S. E.			Distant 11.5 miles by computation.
6	8	0				
7	8	0	. . .	. . .	.	Spoke the Leith and Rotterdam packet, Rhine,
8	8	0				Captain G., bound to Leith, all well.
9	8	0				
10	9	0	. . .	N. N. W.	.	It blew fresh from noon, increasing till about mid-
11	9	0				night, but became gradually more moderate in
12	9	0				the morning.
1	8	6				
2	8	6				
3	8	6	. . .	. . .	.	Passed a ship under French colours.
4	8	6				
5	8	2	. . .	. . .	.	Variation of compass per amplitude, 2½ pts. W.
6	8	2				See Section—Finding the Latitude.
7	8	0				At noon, obs. alt. Sun's lower limb, 50° 49.'0 S.
8	8	0				Cor. for 10 feet, 51° and May . + 12.2
9	8	0				True altitude, . . . 51 1.2
10	7	6				Zenith dist. or comp. to 90° . 38 58.8 N.
11	7	6				Declination, ... 16 19.5 N.
12	7	6				Obs. latitude, ... 55 18.3 N.

Course.	Dist.	Diff. Lat.	Dep.	Latitude by Acc.	Latitude by Obs.	Diff. Long.	Longitude by Acc.	Longitude by Lunar.	Longitude by Chro.	Bearing and Dist. of Heligoland at noon.
S. 71° E.	167	54.6	158	55°17′N.	55°18′N.	280′	2°7′E.		2°8′E.	S. 71° E. Dist. 210 miles.

In sailing down the Firth of Forth, and when about to leave the land, two bearings of the May Light are taken at 3h. and 5h. P.M., from which, and the ship's course and distance run during the interval, her distance at 5h. P.M. from the lighthouse will be found by computation to be 11.5 miles N. by W., which, corrected for variation, gives the ship's bearing and distance in an opposite direction, to be reckoned the first course in the traverse.

Courses.	Dist.	Diff. of Lat. N.	Diff. of Lat. S.	Departure. E.	Departure. W.
S. E. ½ S.	11.5		8.9	7.3	
E. b. S. ½ S.	157.6		45.7	150.7	
S. 71° E.	167		54.6	158.0	

Lat. Isle of May, 56° 11.'5 N.
Diff. of Lat. . . 54.5 S.

Latitude in . 55 17.0 N.

Sum of Lats. . 111 28.5
Half or Mid. Lat. 55 44.2 = 55°.7

Now to mid. lat. 55°.7 as a course, the distance 280, answering to the departure 158, in a lat. col. is the diff. of longitude, (Table VIII.)

Longitude of Isle of May, ... 2° 33′ W.
Diff. of longitude, 280= . . . . 4 40 E.

Longitude in . . . . . . 2 7 E.

Hence the bearing, per compass, of Heligoland Light is S. E. ¼ E. and distance 210 miles.

JOURNAL.

Ship Caledonia from Leith to Hamburgh.

H	K	F	Courses.	Winds.	Leeway.	Remarks. Tuesday, 13th May, 1828.
1	7	4	S.E.b.E ¼E.	E. N. E.	1	Steady fresh breeze.
2	7	4				
3	7	4				
4	8	0				
5	8	0				
6	8	0				
7	8	0	. .	. . .	.	Breeze increasing.
8	8	0				
9	8	0				
10	8	0				
11	9	0				
12	9	0				
1	9	5	. . .	. . .	.	Beginning to blow hard. Took in top gallant sails.
2	9	5				
3	9	5				
4	9	5	. . .	. . .	.	It blows a gale.
5	8	0				
6	8	0	. . .	. . .	.	Gale abating.
7	8	0	. . .	. . .	.	Longitude by Lunars, 6° 32′ E.
8	8	0				——— by Chronometer, 6° 33′ E.
9	8	0				Variation 2¼ points west.
10	8	0				
11	8	0				Latitude by Obs. 54° 20′ N.
12	8	0				

Course.	Dist.	Diff. Lat.	Dep.	Latitude by Acc.	Latitude by Obs.	Diff. Long.	Longitude by Acc.	Longitude by Lun.	Longitude by Chro.	Bearing and Dist. of Heligoland at noon.
E.S.E.¼E.	198	57.5	190	54°21′N	54°20′N	330	7°38′E	7°39′E	7°40′E	S. E. Dist. 12 miles.

At 6 P.M. Wednesday, May 14th, abreast of Newark Light. At 8 anchored before Cuxhaven. At day light next morning, working up the Elbe. At 4 P.M. came to an anchor opposite Hamburgh.

By continuing a series of computations in this manner, the Journal of a ship's voyage may be prolonged to any distance, according to the example here given. It is of great importance to ascertain the ship's place by observation as often as possible. The longitude in long voyages should be ascertained by lunars, at least weekly when circumstances permit, and that by chronometer daily when observations can be obtained. See articles Lunars and Chronometers in a following part of this work.

The common log is a quadrantal piece of wood of five or six inches radius, loaded at the lower circular edge with lead, to make it swim perpendicularly in the water, where it is supposed to remain fixed while the ship leaves it, taking off from the reel a certain number of knots, marked by the log-line passing over the ship's stern. There are other logs, however, such as Massey's patent log, which are much more correct than the common one can be from the nature of its construction. The ship's place deduced from the courses and distances, or by dead reckoning, as it is called, can never be very accurate, and therefore the intelligent navigator will only regard it as necessary to connect his observations, or to guide him during short intervals, when they cannot be got.

Section V.

Application of Plane Trigonometry to the Mensuration of Heights and Distances.

53. One of the most important applications of plane trigonometry is the mensuration of heights and distances. The *data* are some of the sides and angles of a triangle. The sides are measured by rods, lines, tapes, or chains, constructed according to the degree of accuracy required; and the angles are measured by some angular instrument, such as the quadrant, sextant, reflecting circle, repeating circle, or theodolite. The repeating theodolite is perhaps, in general, the most convenient of all for taking the necessary angles, and the chain, properly constructed, the best for measuring the side called the *base*, though, to military engineers, the small pocket circular box-sextant, or semicircle, as improved by Sir Howard Douglas, will be found highly useful, when accompanied by the box-measuring tape. One of Schmalcalder's surveying compasses will also be found very commodious in military and nautical surveying. A complete description* of these instruments would far exceed our limits, and their use is best learnt under the superintendence of a master. In general, it may be remarked, that an allowance must be made for the height of the eye above the horizontal plane; and when the *base* above-mentioned is inclined to the horizon, it must be reduced to it according to the given inclination, though in nice operations, the base is selected so as to be, if not exactly, at least nearly level. Then, from a little attention, by driving in stakes at moderate distances, and levelling their tops, on which deals properly prepared are laid, an exact horizontal line may be obtained. This truly level line is to be most carefully measured, allowance being made for the contraction or expansion of the materials of which the chain is composed according to the state of the thermometer, or by an instrument composed of different metals whose expansions and contractions mutually destroy each other; in nice operations reduced to the level of the sea, and such other precautions as the nature of the case may require must be observed, in order to ensure the greatest possible accuracy; many examples of which may be seen in the Trigonometrical Survey of the British islands under the direction of the Board of Ordnance.† A number of the more useful problems connected with trigonometrical surveying may be seen in the third volume of Hutton's Course of Mathematics by Dr O. Gregory, in Baron Zach's Work on the Attraction of Mountains, in the Base du Systeme de Metrique Decimal, and in Piussant's Geodesie.‡

Example I.

To determine the distance of a tower, inaccessible by reason of an intervening river, I measured, on a horizontal plane, the base

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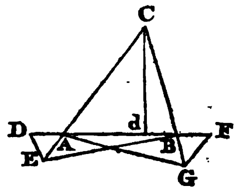
* Those who wish for written descriptions may consult Jones' edition of Adam's Geometrical and Graphical Essays, already mentioned, Biot's Traité d'Astronomie Physique, Delambre's Astronomie, Base du Systeme Metrique, Woodhouse's, Vince's, and Pearson's Treatises of Astronomy.

† There are several methods of approximating to the heights of objects by means of mirrors, shadows staffs, geometrical squares, and Gunter's quadrants; but, as they are seldom used where much accuracy is required, they are omitted here.

‡ A few of the more useful are given in Part III. of this work.

AB, 500 yards, and at each end took the angle included between the other end and the tower, which were 50° 56′ and 75° 10′ respectively; what is the distance of the tower from each end of the base?



In the annexed figure,

AB = 500

CAB = 50° 56′

CBA = 75° 10′, and consequently

Angle C = 180°—(A+B)=53° 54′

Hence, sin C 53° 54′	9.907406	9.907406
Is to AB 500	2.698970	2.698970
So is sin A 50° 56′	9.890093	
To BC 480.46	2.681657	
So is sin B 75° 10′		9.985280
To AC 598.2		2.776844

The perpendicular or nearest distance C d may, if required, be easily found thus:

As radius	10.000000
Is to AC 598.2	2.776844
So is sin A 50° 56′	9.890093
To Cd 464.45	2.666937

Remarks.—These distances might have been determined without an instrument to measure the angles. Thus, suppose that, in the continuation of the base AB, and the lines CA, CB, the four distances, AD, AE, BF, BG, were taken all equal to 100 feet, and DE measured 86, and FG 122 feet, the respective chords, to a radius of 100 feet, of the exterior angles DAE, FBG, which are equal to their vertical interior angles CAB, CBA. Now, since half the chord is the sine of half the angle, we have $\frac{43}{100}$=·43=sin $\frac{1}{2}$ A=25° 28′, and A=50° 56′. In like manner, sin $\frac{1}{2}$ B=·61=37° 35′, and B=75° 10′, which results agree with the former.

Note 1.—The number 100 was chosen for the sake of simplicity; but any other convenient number may be adopted, taking care to divide half the measure of the chord by it.

Note 2.—The same thing may be accomplished when the sides of the triangles bear any proportion to each other, by finding from them the angles DAE, FBG. Also the supplements EAB, ABG of the original angles may be found in the same manner, by joining AG and BE.

Example II.

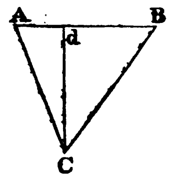
Wanting to know the breadth of a river, I measured 100 yards in a straight line by the side of it; and at each end of this line I found the angles subtended by the other end, and a tree close by the opposite side, to be 53° and 79° 12′; what is its perpendicular breadth?

Ans.—105.89.

Example III.

In order to find the distance between two trees A and B, which could not be directly measured on account of a pool of water which occupied the intermediate space, I measured the distance of each from a third object C, which were 588 and 672 yards respectively,

and then at C took the angle ACB between the two trees 55° 40′; required their distance?



	180° 0′	
Angle C	55 40	
A+B	124 20	
½ (A+B)	62 10	
As BC+AC	1260	3.100371
Is to BC—AC	84	1.924279
So is tan ½ (A+B)	62° 10′ 0″	10.277379
To tan ½ (A—B)	7 11 53	9.101287
Angle A	69 21 53	
Angle B	54 58 7	
As sin A	69° 21′ 53″	9.971203
Is to BC	672	2.827369
So is sin C	55 40 0	9.916859
To AB	592.96	2.773025*

Example IV.

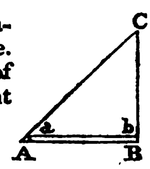
In the Trigonometrical Survey of Britain, Colonel Mudge found, from computations depending on former operations, that the logarithm of the number expressing the distance between Cheviot and Cross Fell in feet was 5.4654017, and between Cheviot and Wisp Hill 5.2672278, and the angle contained by these, corrected for spherical excess, was 53° 30′ 10″; required the other angles, and the distance between Wisp Hill and Cross Fell, without first finding the value of the *given* sides in natural numbers?

Ans.—The angle at Wisp Hill is 87° 14′ 4″.
Cross Fell 39 15 46

The distance of Wisp Hill from Cross Fell 235018.6 feet.

Example V.

In order to determine the height of a tower, I measured, in a direct line, AB 366 feet on a horizontal plane. I then took the angle Cab 37° 30′, the height Aa of my instrument being 5 feet; required BC the height of the tower.

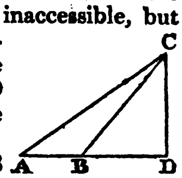


Ans. bC =280.84.
Add Aa 5.00.

Height BC=285.84.

Example VI.

Walking along the side of a river, I observed an obelisk on the opposite side, which, on account of the river, was inaccessible, but whose height I wanted to ascertain. For this purpose I took at B the angle CBD 50° 39′, at A the angle CAB 33° 30′, which was distant from B 360 feet; required the height of the obelisk, and the distance of the station B from its base?



Solution.—Because the angle CBD=CAB+ACB

* In some of the examples the computations in proportion are performed by comparing the *sines* of the angles with the *sides*,—a method sometimes more easy to beginners.

CBD—CAB=ACB=50° 39′—33° 30′=17° 9′, hence sin C : AB :: sin A : BC; and in the right-angled triangle DBC are now given BC and the angle CBD, to find DC and BD, 521 and 427.2 feet respectively.

EXAMPLE VII.

A solution of this problem, more easy and commodious in practice, may be obtained thus:—

Let CD represent any object whose height is to be determined; at the points A and B observe the angles of elevation, and measure the distance AB, the points A,B,C, and D being in the same plane. See preceding figure.

For in the triangles ABC, CBD,
sin ACB : AB :: sin A : BC,
and R : BC :: sin CBD : CD, from which we have sin ACB : AB × BC :: sin A × sin CBD : BC × CD, or sin ACB × BC × CD=sin A × sin CBD × AB × BC; radius being unity.

Hence $CD=\frac{\sin A \times \sin CBD \times AB}{\sin ACB,}$; or, making the terms homogeneous, and substituting cosec for $\frac{1}{\sin}$,

$R^2 \times CD=\sin A \times \sin CBD \times \text{cosec } ACB \times AB$.

That is, to the sines of the observed angles of elevation, add the cosecant of the difference of these angles, and the logarithm of the measured distance; the sum, rejecting 30 from the index, will be the height of the object.

Let the angles of elevation be 55° 54′, and 33° 20′ respectively, and the distance between the stations 100 feet; required the height of the object?

Angles of elevation	55° 54′ sine	9.918062
	33 20 sine	9.739975
Difference . .	22 34 cosec.	10.415942
Distance . .	100 feet	2.000000
Height . .	118.5	2.073979
Height of the eye	5.5	
Height of object	124.0 feet	

EXAMPLE VIII.

In order to determine the distance of two inaccessible objects lying in a direct line from the bottom of a tower 90 feet high, on the top of which I took the angles of depression of the two objects, that of the most remote being 24° 48′, and that of the nearest 58° 36′; required their distance from the tower, and from each other?

Ans.=54.936, 194.778, and 139.842 feet.

EXAMPLE IX.

Wanting to know the distance between two boats lying at anchor in a straight line from a light-house, which is 110 feet high, on the top of which I took the angle of depression of the farthest, and found it to be 18° 26′, and that of the nearest 56° 44′; what was their distance?

Ans.—257.866 feet.

Example X.

From the top of a hill I observed two mile-stones on a horizontal road, which ran straight from its bottom, and took their respective angles of depression below the horizontal plane passing through the place of my eye; that of the nearer mile-stone was 36° 12′, and that of the more distant 15° 26′; required the height of the hill?

Ans.—780.17 yards.

Example XI.

In order to find the height of an obelisk standing on the top of a regularly sloping hill, I measured from its bottom, a distance of 40 feet, and then found the angle formed by the inclined plane, and a line from the top of the obelisk to centre of the instrument, to be 41°; and, after measuring downward in the same direction 60 feet farther, the angle formed as before was only 23° 45′. What was the height of the obelisk and the angle of the inclined plane with the horizon?

Ans.—Height 57.623 feet. Inclination 21° 54½.

Example XII.

Wishing to know the height of an obelisk standing on the top of a regularly sloping hill, to the bottom of which I could not approach on account of a ditch around it, at the outside of which I took the angle formed by the inclined plane, and a line from the centre of the instrument to the top of the obelisk, and found it 41°; but after measuring downward in the same sloping direction 54 feet farther, I found the angle formed in like manner to be 23° 45′. What was the height of the obelisk itself, and that of its top above the last place of observation, supposing the angle formed by the inclined plane and the horizon to be 21° 54′½?

Ans.—51.86 feet the height of the obelisk, the bottom is 33.58 feet, and the top is 85.44 above the last place of observation.

Example XIII.

Being on a horizontal plane, and wanting to know the height of a tower on the top of an inaccessible hill, I took the angle of elevation of the top of the hill 40°, and of the top of the tower 51°; then measuring in a direct line 100 feet farther from the hill, I took in the same vertical plane the angle of elevation of the tower 33° 45. Required the height of the tower?

Ans.—46.666 feet.

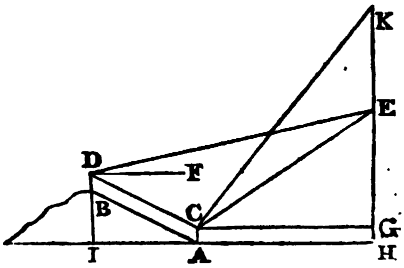
Example XIV.

In order to know the height of a castle standing on a hill, I took the angle of elevation of the top of the castle above the horizontal plane 58°, and of the top of the hill 33°; but could not, as in last example, measure a sufficient distance directly from the castle. I therefore measured in an oblique direction 52 yards, making with the castle an angle of 72° 10′, at the farther end of which the angle, in the same manner, was 64° 30′. What was the height of the castle?

Ans.—34.464 yards.

Example XV.

Wanting to ascertain the height of a tower standing upon a hill, the height of the hill, and the horizontal distance from the nearest place of observation, on account of the nature of the ground, I proceeded as follows:—



At A I took the angle GCK 3° 38′, and GCE 2° 44′ then having set up a staff AC equal in height to the centre of the theodolite, I measured 1810 feet up the sloping ground AB in a direct line with the tower, keeping the points K, E, C, B in the same vertical plane. At B I took the angle FDC = BAI = 1° 54′ and EDF=1° 32′. Required the height of the tower, the height of the hill, and the horizontal distance from the first place of observation?

1. In the triangle DCE are given the side DC=1810 feet, the angle ECD 175° 22′, EDC 3° 26′, and DEC 1° 12′; to find CE=5175.89 feet.*

2. In the triangle CKE, the angle K=86° 22′, CEK=92° 44′, KCE=0° 54′ and CE=5175.89; hence EK=81.463 feet.

3. In the triangle CGE, the angle GCE=2° 44′, and CE=5175.89; hence CG=AH=5170 feet; and GE=246.826.

4. In the triangle ABI, AB=1810, the angle BAI=1° 54′; hence AI=1809 feet, and BI=60.012 feet.

If EK, the height of the tower, were only wanted, it may be found thus:

Sin DEC : DC : : sin CDE : CE=DC sin CDE cosec DEC,
sin K : CE (=DC. sin CDE. cosec DEC) : : sin KCE : KE, and
R^4KE=DC. sin CDE. sin KCE. sec GCK. cosec DEC.

By logarithms.

sin CDE 3 26′	.	8.777333
sin KCE 0° 54′	.	8.196102
sec GCK 3° 38′	.	10.000874
cosec DEC 1° 12′	.	11.678973
log DC 1810	. .	3.257679
EK 81.463	.	1.910961

Example XVI.

At the top of a castle which stood on a hill near the seashore the angle of depression of a ship's hull at anchor was 4° 52′; at the bottom of the castle the angle of depression was 4° 2′. Required the horizontal distance of the vessel, and the height of the hill on which the castle stands above the level of the sea, the castle itself being 64 feet high?

Ans.—4373.75, and 308.4 feet respectively.

Example XVII.

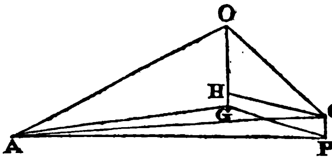
From a window in the lower part of a house, nearly on a level with the bottom of a steeple, I took the angle of elevation of the top of the steeple 40°; and from another window 18 feet directly above

* In calculations where the same number is used which has been found from previous computation, its log. should be reserved from the first to be used in the next, &c.

the former, the same angle of elevation was 37° 30′. Required the height and distance of the steeple?

Ans.—210.44, and 250.79 feet respectively.

Example XVIII.



Suppose A and C to be two stations on sloping ground, O an object on the top of a hill, and the angles OCA, OAC, measured with a sextant, to be 79° 29′ and 63° 11′ respectively; also let the angle of elevation of AO above the horizontal plane be 6° 36′, and that of CO 5° 22′; what are the horizontal distances and height of the object, AC being 410 yards?

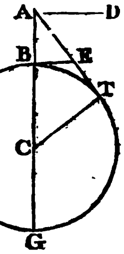
In the triangle AOC are given all the angles, and the side AC; to find AO and CO. Again, in the triangle AGO right-angled at G, are given the angle OAG and the side AO; to find AG=660.3 and OG=76.4. Lastly, in the triangle COH, right-angled at H, are known CO and the angle OCH; to find CH 600.7, and OH 56.4, and OG—OH=76.4—56.4=20 yards nearly=HG=CP, the difference of the heights of the stations, supposing AP to be horizontal. Now in the right-angled triangle APC are given AC and CP, to find $AP=\{(AC+CP)\ (AC-CP)\}^{\frac{1}{2}}=\sqrt{430\times 390}=\sqrt{167700}=409.5$ yards. Hence the sides of the horizontal triangle APG are given, to find the angles, which may be determined by Case III. Plane Trigonometry, to be AGP=37° 31′ 29″, GAP=63° 19′ and GPA=79° 9′ 31″.

The present may serve as an example of reducing hypotenusal lines to their horizontal measure, and of determining the height of an object above each place of observation in most common cases.*

Example XIX.

The height of the mountain called the Peak of Teneriffe was found, barometrically, by the methods afterwards described, to be 12,356 feet, or 2.34 English miles, and the angle of depression of the horizon, from the mean of a great number of observations, 1° 58′ 12″; it is required to determine the diameter of the earth, supposing it to be a perfect sphere?

Ans.—7913.6 miles.



Let C be the centre of the earth, the circle BTG a vertical section passing through the centre, AB the height of the Peak, AT the tangential line drawn from its top to the visible horizon, and AD a line perpendicular to a plumb-line hanging freely: also, let BE, a tangent to the earth's surface at B, meet the other tangent AT in E. Then, in the triangle ABE, right-angled at B, there are given BAE the complement of DAT, the angle of depression=88° 1′ 48″, and AB=2.34, hence R : AB : : tan A : BE : : sec A : AE. But since the triangles CBE, CTE, are right-angled at B and T, have the side CB=CT, and CE common, they are (Leslie's Geom. I, 22, or Hutton's Geom.

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* It may be observed, that the theodolite gives the true horizontal angles, therefore when it is used this method is unnecessary.

theo. 34, cor. 2) equal, and therefore BE=ET; hence, AE+BE =AE+ET=AT. In the triangle ATC, right-angled at T, we have R : AT : : tan. A : TC, the radius of the earth. The operation thus performed occupies but small compass, which may still be farther shortened. For since tan A + sec A = tan (A + ½comp A) we shall, by incorporating the proportions from which AE, BE, and CT are deduced, have

$$R^2\ CT = AB \tan (A + \tfrac{1}{2}\text{comp } A) \tan A;$$

or, log CT = log AB + log tan (A + ½comp A) + log tan A—20, in the index.

The logarithmic computation is as follows:—

Depression	1° 58′ 12″	
Half	59 6	
Comp. depress.	88 1 48 tan	11.463485
Sum	89 0 54 tan	11.764644
Height of Peak	2.34 miles, log	0.369216
Earth's semid	3956.8	3.597345
	2	
Diameter	7913.6	
Distance	136.1	2.133860

If AT were required, we have only to take radius (10) from the sum of the two last lines, and the remainder, 2.133860, is the log of 136.1, the distance sought.*

Note 1.—This method of determining the earth's radius, though elegant in theory, is useless in practice, at least where any thing more than an approximation is wanted, by the great irregularity of the horizontal refractions.

Note 2.—When the diameter of the earth is known, and height of the object given, the distance of the visible horizon may be easily found; for, Euc. III. 36. AB . AG=AT².

By logarithms.

AB	2.34 log	0.369216
BG	7913.6	
AB+BG=AG	7915.94 log	3.898503
		4.267719
As before	136.1 miles, log	2.133859†

Note 3.—The depression of the horizon, or the dip, as it is called at sea, is the angle DAT contained between the true and visible horizon. For if an observer, whose eye is situated at A on the deck of a vessel, takes the altitude of a celestial object with Hadley's quadrant or sextant, by bringing that object to the surface of the water at T, instead of the true horizon AD, the altitude is evidently too great by the angle DAT=TCA. This may be calculated by the usual formulæ of trigonometry for that purpose; but as it will, at any probable altitude, be a small quantity, those which give the co-

* See Dr O. Gregory's Trigonometry.

† See also the method by Leslie in his Geometry.

sine or secant of its value are not sufficiently correct; for which reason we shall give the following method:—

$(BG+AB)\times AB=AT^2$, (Euc. III. 36.), hence $BG\times AB+AB^2=AT^2$, or $2BC\times AB+AB^2=AT^2$, and AB^2 being, at any probable elevation, but a small quantity in comparison of AC, it may be safely neglected; therefore $\sqrt{(2BC\times AB)}=AT$. But CT $(=BC)$: R :: AT $\left[\sqrt{(2BC\times AB)}\right]$: tan C=tan DAT$=\frac{R\sqrt{(2BC.AB)}}{BC}=\sqrt{\frac{2R^2.AB}{BC}}$

Now since $\frac{2R^2}{BC}$ is a constant quantity, and BC being taken in general at 3956 miles=20887680 feet, hence the log of $\frac{2R^2}{BC}$ is 12.98114, and tan DAT$=\frac{1}{2}(12.98114+\log AB)$. Since, in the present case, the arc may be substituted for its tangent, the radius, therefore, becomes 57° 17′ 44″.8=206264″.8; and we have log DAT in seconds $=\frac{1}{2}(3.60999+\log AB$ in feet).

The dip is affected by terrestrial refraction, which is very variable, and by different authors it is estimated at different quantities. Dr Maskelyne estimated it at one-tenth of the whole; M. Delambre, one-eleventh, and Col. Mudge, one twelfth. See Dr Hutton's Course, vol. III. page 138. Taking it at $\frac{1}{12}$, the const. log is 3.53441, or using prop. logs. it is 0.49908.

Ex.—Required the dip, the height of the eye being 40 feet, and estimating the terrestrial refraction at $\frac{1}{12}$.

Constant log		3.60999	or, 3.53441	or, 0.49908
Height of eye 40 feet		1.60206	1.60206	PL. 2.43136
		5.21205	5.13647	2.93044
	403″.6 log	2.60602	2.56823	1.46522
Refrac. sub. $\frac{1}{12}$	33 .6		log. 370″=6′ 10″	PL. 6′ 10″
Dip*	370=6′ 10′.			

Note 4.—Since $AB\times BG+AB_2=AT^2$, therefore

$AB(BG+AB)=AT^2$, and $AB=\frac{AT^2}{BG+AB}$ . . . . (1.)

Now if AB is the unknown quantity, and being small in comparison of BG, it may be found approximately by making, first, $AB'=\frac{AT^2}{BG}$ nearly, substituting this value of AB′ for AB in formula (1.), and

$AB=\frac{AT^2}{BG+AB'}$ . . . . (2.)

which will be sufficiently correct for most purposes. If not, the operation may be repeated till it is so.†

This is useful in determining the height of an object considerably distant.

* The dip in minutes is equal to the square root of the height in feet nearly. If this result be diminished by one hundreth of itself, the remainder will be the dip correctly.

† A repetition will scarcely ever be necessary in any case likely to occur in practice.

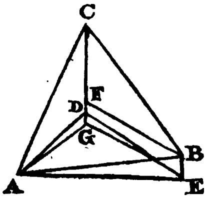
Now, the mean diameter of the earth is about 7912 miles, or 41775360 feet=GB, of which the logarithm is 7.620920, and its arithmetical complement is 2.379080; therefore to twice the log. of AT, in feet add the constant log. 2.379080, the sum, rejecting tens in the index, will give AB′, which will be sufficiently correct if AT does not exceed 50,000 feet, or about 10 miles. If more distant, the operation may be repeated. This correction must always be added to heights determined geometrically, as the usual instruments give their elevation only above the tangent AT.

TABLE OF THE DEVIATION OF THE TANGENT FROM THE CIRCLE OF CURVATURE OF THE EARTH.

Distance in Feet.	Deviation in Feet.	Dist. in Miles.	Deviation in Feet.
100	0.0002	1	0.667
200	0.0010	2	2.669
400	0.0038	3	6.006
600	0.0086	4	10.677
800	0.0153	5	16.883
1000	0.0239	6	24.024
2000	0.0958	7	32.700
3000	0.2154	8	42.710
4000	0.3830	9	54.055
5000	0.5985	10	66.734

EXAMPLE XX.

Required the height of the top of Arthur's Seat above the surface of the water in the drain running parallel to the Duke's Walk, eastward towards Mushat's Cairn, from the following set of observations?



Let C be the summit of the hill, AE a horizontal line passing through the eastern point of observation A. AB the measured base=768.7 feet. The angle BAE the inclination of the base to the horizontal line AE= 0° 30′ 30″.

The angle BAC=84 35 30
ABC=79 50 30
CAD=13 53 30
CBF=13 36 30

The height of the telescope of the theodolite was 4.6 feet above the ground, and the ground 2.5 feet above the surface of the water in the drain.

As radius . .	10.000000	As radius . .	10.000000
Is to sin BAE 0° 30′ 30″	7.948020	Is to cos BAE 0° 30′ 30″	9.999983
So is AB 768.7 feet,	2.885757	So is AB 768.7 feet,	2.885757
To BE 6.82 feet,	0.833777	To AE 768.67 feet, .	2.885740

BAC=84° 35′ 30″
ABC=79 50 30

164 26 0
180

ACB=15 34 0

As sine ACB 15° 34′ 0″ a. c. or cosec	0.571283
Is to sin ABC 79 50 30 . .	9.993138
So is AB′ 768.67	2.885740
To AD 2819.42 feet, . . .	3.450161
As sine ACB 15° 34 0 a. c. or cosec	0.571283
Is to sin BAC 84 35 30 . .	9.998062
So is AE 768.67	2.885740
To BF 2851.59 feet, ...	3.455085

Refraction$=\frac{2830}{1200}=-2''.36$, (page 40,) which being so small, may in this case be omitted.

As radius . .	10.000000	As radius . .	10.000000
Is to tan CAD 13°53′30″	9.393260	Is to tan CBF 13°36′30″	9.383958
So is AD . .	3.450161	So is BF . .	3.455085
To DC 697.30 feet,	2.843421	To FC 690.31 feet,	2.839043

Now since AD and ED are tangents to great circles on the surface of the earth passing through the points A and E, and are in this case nearly equal, the deviation DG, common to both, must be computed by the rule above.

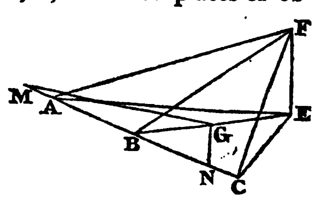
Constant log . .	2.379
Log AD	3.450
Log BF . . .	3.455
Log DG 0.192 foot, .	9.284

Hence DC= .	697.30 feet.	FC= . . .	690.31 feet.
DG= .	0.19	. . . . .	0.19
Height Theodolite Tel.	4.60	. . . . .	4.60
Height above water,	2.50	. . . . . .	2.50
Height of hill above A	704.59	Height of hill above B	697.60
	704.42	Height of B above A or BE=	6.82
Mean of both, . .	704.50	Height of hill above A, when traced through the western station.	704.42

EXAMPLE XXI.

Given the angles of elevation of any distant object, taken at three places in a horizontal straight line, which does not pass through the point directly below the object; and the respective distances between the stations: to find the height of the object, and its distance from either station.

Let AEC be the horizontal plane; FE the perpendicular height of the object F above that plane; A, B, C, the three places of observation; FAE, FBE, FCE, the respective angles of elevation, and AB, BC, the given distances. Then, since the triangles AEF, BEF, CEF, are all right-angled at E, the distances AE, BE, CE, will manifestly be as the cotangents of the angles of elevation at A, B, and C; and we must determine the point E, so that these lines may have that ratio.



Construction.

To effect this geometrically, we must take BM, on AC produced, equal to BC, BN equal to AB; and make

MG : BM (=BC) :: cot A : cot B, and
BN (=AB) : NG :: cot B : cot C

With the lines MN, MG, NG, construct the triangle MNG; and join BG. Draw AE so that the angle EAB may be equal to MGB; this line will meet BG produced in E, the point in the horizontal plane falling perpendicularly under F.

Demonstration.

By the similar triangles AEB, GMB, we have

AE : BE :: MG : MB :: cot A : cot B, and
BE : BA (=BN) :: BM : BG

Therefore the triangles BEC, BGN are similar; consequently BE : EC :: BN : NG :: cot B : cot C. Whence it is obvious that AE, BE, CE, are respectively as cot A, cot B, cot C.

Calculation.

In the triangle MGN are given all the sides, to find the angle GMN, equal to the angle AEB. Then, in the triangle MGB, are given two sides, and the contained angle; to find the angle MGB, equal to the angle EAB. Hence, in the triangle AEB are known the side AB, and all the angles; to find AE and BE. And then EF=AE . tan A=BE . tan B.

Analytically.

Let AB=r, BC=s; also let the cotangents of the angles FAE, FBE, FCE, be denoted by the letters a, b, c, respectively.

Then putting EF=x, we have, to radius 1; $1 : a :: x : ax$=AE, $1 : b :: x : bx$=BE, $1 : c :: x : cx$=CE : and on AC from E, letting fall the perpendicular ED, we have (Euc. II. 12) $a^2 x^2 = b^2 x^2 + r^2 + 2r \cdot \mathrm{BD}$; hence $\mathrm{BD} = \frac{a^2 x^2 - b^2 x^2 - r^2}{2r}$. In like manner $\mathrm{CD} = \frac{b^2 x^2 - c^2 x^2 - s^2}{2s} = \mathrm{BD} - \mathrm{BC} = \mathrm{BD} - s$: whence $\mathrm{BD} = \frac{b^2 x^2 - c^2 x^2 + s^2}{2s}$. Therefore $\frac{b^2 x^2 - c^2 x^2 + s^2}{2s} = \frac{a^2 x^2 - b^2 x^2 - r^2}{2r}$. Hence $x^2 = \frac{r^2 s + r s^2}{s(a^2 - b^2) - r(b^2 - c^2)}$, and $x = \sqrt{\frac{r s(r+s)}{s(a^2 - b^2) - r(b^2 - c^2)}}$.

Otherwise thus:

If AB and CB be conceived to be bisected in M′ and N′, and ED a perpendicular upon AC, which are however omitted to avoid complexity in the figure; then, (Leslie's Geometry, II. 21.) $\mathrm{AE}^2 - \mathrm{BE}^2 = \mathrm{AB} \times 2\mathrm{M'D}$, and $\mathrm{CE}^2 - \mathrm{BE}^2 = \mathrm{BC} \times 2\mathrm{N'D}$; therefore, $\mathrm{AE}^2 \times \mathrm{BC} - \mathrm{BE}^2 \times \mathrm{BC} = \mathrm{AB} \times \mathrm{BC} \times 2\mathrm{M'D}$, and $\mathrm{CE}^2 \times \mathrm{AB} - \mathrm{BE}^2 \times \mathrm{AB} = \mathrm{AB} \times \mathrm{BC} \times 2\mathrm{N'D}$. Adding equals to equals, and $\mathrm{AE}^2 \times \mathrm{BC} + \mathrm{CE}^2 \times \mathrm{AB} - \mathrm{AC} \times \mathrm{BE}^2 = \mathrm{AB} \times \mathrm{BC} \times \mathrm{AC}$; consequently $\mathrm{AE}^2 \times \mathrm{BC} + \mathrm{CE}^2 \times \mathrm{AB} = \mathrm{AC} \times \mathrm{BE}^2 + \mathrm{AC} \times \mathrm{AB} \times \mathrm{BC}$.

If AB=BC, then $\mathrm{AE}^2 + \mathrm{CE}^2 = 2\mathrm{AB}^2 + 2\mathrm{BE}^2$, the line EB being drawn from the vertex E of the triangle ACE, to any point B in the base. Put AB=D, BC=d, EF=x, and then expressing algebraically the foregoing theorem.

The equation thence resulting is,

$$d x^2 \cot^2 A + D x^2 \cot^2 C = (D+d) x^2 \cot^2 B + (D+d) D d.$$

Hence, transposing all the unknown terms to one side of the equation, dividing by the sum of the coefficients, and extracting the square root, we shall have $x=\sqrt{\frac{(D+d)Dd}{d\cot^2 A+D\cot^2 C-(D+d)\cot^2 B}}$. Thus EF becoming known, the distances AE, BE, CE, are found by multiplying cotangents of A, B, and C, respectively, by EF.

Cor.—When $D=d$, or $D+d=2D=2d$, the expression becomes $x=d\div\sqrt{(\frac{1}{2}\cot^2 A+\frac{1}{2}\cot^2 C-\cot^2 B)}$, which is pretty well suited to logarithmic computation. The rule may, in that case, be thus expressed.—Double the logarithm cotangents of the angles of elevation of the extreme stations, find the natural numbers answering thereto, and take half their sum; from which subtract the natural number answering to twice the logarithm cotangent of the middle angle of elevation: then half the log. of this remainder subtracted from the log. of the measured distance between the first and second, or the second and third station, will be the log. of the height of the object.

1. Let AB=60 feet, BC 72 feet; angle FCE=50° 23′, the angle FBE=40° 33′, and the angle FAE=30° 48′; required the distances AE, BE, CE, and EF, the height of the object?

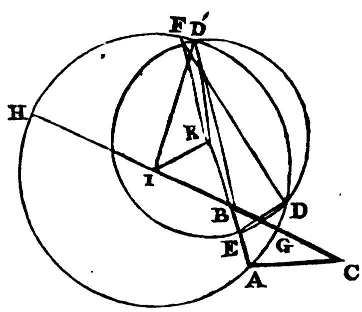
Ans.—AE=159.09 feet, BE=110.84 feet, CE=78.51 feet, and EF=94.84 feet.

2. Let the three angles of elevation be 36° 50′, 21° 24′, and 14°, and the two equal measured distances 84 feet; required the height of the object? *Ans.*—53.964 feet.

Example XXII.

Given the angles of elevation at which an object is seen from three given points in a horizontal plane; to find its position and altitude.

Let A, B, and C be the three points of observation, and D the bottom of the perpendicular from the given object to the horizontal plane. It is evident that the horizontal distances AD, BD, and CD are proportional to the cotangents of the vertical angles at the stations A, B, C; let these cotangents be respectively denoted by L, M, and N.



Divide AB internally and externally at the points E and F in the ratio of L to M; and the lines DE and DF joining in the vertex D of the triangle EDF, must bisect internally and externally (Leslie's Geom. VI. 10.) the angle, whence EDF is a right angle, and contained in a semicircle; wherefore on EF describe a semicircle. In the same manner divide CB internally and externally at G and H in the ratio of M to N, and on GH describe a semicircle. The point D common to both semicircles must occur in their intersection.*

From this construction the trigonometrical calculation is readily

* See Leslie's Geometry, fourth edition, page 275. The same thing may be performed by Prop. XIII. Book III. of Leslie's Geometrical Analysis.

deduced. For $L+M : M :: AB : BE$ and $L-M : M :: AB : BF$; whence $KE=\frac{BE+BF}{2}=\frac{EF}{2}$, or the radius KE is found. In like manner $N+M : M :: CB : BG$, and $N-M : M :: CB : BH$, consequently $DI=\frac{BG+BH}{2}$. In the triangle IBK, the sides BI and BK, with their included angle, $=$ ABC, are given; and, therefore, the angle BKI and the base IK are found. Again, all the sides of the triangle IDK being given, the angle IKD is found. Hence, in the triangle BDK the whole angle BKD and its containing sides are given; and, therefore, the base BL, or the horizontal distance from the station B, and consequently its altitude, is determined.

It is obvious, that the opposite semicircles will likewise, by their intersection, give, on the other side, a second position D′ for that point. In practice, however, this ambiguity could be easily removed. It may be remarked too, that the point D may fall either within or without the triangle.

If the object be seen at the same elevation from all the three points, the arcs of the circles will evidently become tangents, which bisect at right angles the sides of the triangle ABC. The projection D of the object on the horizontal plane will then be the centre of the circle circumscribing that triangle; and, therefore, the radius or distance AD may be found by Prop. 18, Book VI. Leslie's Geometry, as shown in the notes, page 347.

If the three points of observation should lie in the same straight line, the centres of the determining circles will occur in that line or its extension; and hence the process of calculation will be greatly abridged, and will coincide with the foregoing proposition.

Example.—Let the angle of elevation of the object at A, be 50° 45′, that at B 58° 15′, and that at C 46° 45′; also the sides AB 24 yards, AC 38, and BC 50. Required its height?

Hence $L = \cot 50° 45'$, $M = \cot 58° 15'$, and $N = \cot 46° 45'$. From the given sides the angle $ACB = 27° 35' 10''$, $ABC = 47° 9' 22''$, and BAC 105° 15′ 28″. Also $L = 0.8170343$, $M = 0.6188188$, $N = 0.9407061$; therefore, $BE = 10.343$, and $BF = 74.928$, whence $KE = 42.6355$, and $BK = 32.2925$. In like manner, $BG = 19.846$, $BH = 96.123$, hence $DI = 57.9845$, and $IB = 38.1385$. From these the angle $IKB=77° 11' 24''$, and KIB 55° 39′ 14″; and the side $IK = 28.677$. Now from the three sides ID, IK, and KD, the angle $IKD = 107° 10' 26''$. To this, by applying the angle IKB by addition and subtraction, we obtain the angle $BKD' = 184° 21' 50''$, and $BKD = AKD = 29° 59' 2''$.

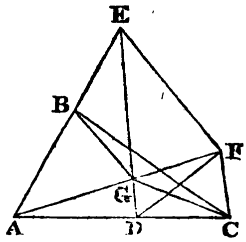
From the sides BK and KD, and the contained angle BKD, are found the angle $KBD = 102° 16' 39''$, and $KBD = 47° 44' 19''$, from which $BD=21.8065$, and the height of the object 35.24 yards.

Should the point D′ be the foot of the perpendicular, the angle $KBD' = 2° 29'$ and $KD'B = 1° 52' 50''$, and $BD' = 74.876$; whence the height above D′ will be 121 yards.

Example XXIII.

Given the angles of elevation of the object from three points in the same plane forming a triangle, of which the sides are known, to find the position of the object referred perpendicularly to that plane and its altitude above it.

Construction.—The perpendicular from the object to the plane may fall either within or without the triangle. In both cases, let A, B, and C be the points of observation, and α, β, and γ, the angles of elevation at these points respectively. Join A, B, and C, and on AB produced, if necessary, make AE equal to AC, and AD to AB, join ED, and upon it construct the triangle EDF, so that cotangent α : cotangent β : : AE : EF, and cotangent α : cotangent γ : : AD : DF. Join AF, and from B draw BG, making the angle ABG equal to the angle AFE, and join CG. The point G in which the straight lines BG and AF intersect each other will be the point at which a perpendicular let fall from the object would meet the plane, thus ascertaining the position of the object, from which, and the given angles, its altitude may be found.



Demonstration.—It is obvious that the straight lines drawn from each of the points of observation to the point at which a perpendicular let fall from the object meets the plane, ought to be in proportion to the cotangents of the angles of elevation at these points respectively. The proposition therefore resolves itself into this. To find a point in a plane from which straight lines drawn to three given points in the same plane, shall have to each other a given ratio which follows from the construction just given.

Solution.—In the triangles ABG, AFE, the angles at B and F are equal by construction, and the angle BAG is common to both; these two triangles are therefore similar. And AG : BG : : AE : EF : : cot α : cot β. Hence $EF = \frac{AC \times \cot\beta}{\cot\alpha}$. Again AG : AE : : AB : AF, or AG : AC : : AD : AF; and as the angle at A is common to the two triangles AGC, and ADF; these triangles are similar, consequently AG : CG : : AD : FD : : cot α : cot γ, whence $FD = \frac{AB \times \cot\gamma}{\cot\alpha}$.

The triangles ADE, ABC having the sides AD, AE of the one equal to the sides AB, AC of the other, and the angle at A, common to both, are equal, and the side ED is equal to the side BC. Therefore in the triangle ADE, the three sides are given, and those of the triangle FDE are already found; whence the angles AED and FED, and consequently the angle AEF may be obtained; and from the angle AEF, with the sides AE and EF, the angle AFE or ABG, which is equal to it, may be determined. Then in the triangle ABG, having the two angles at A and B, and the side AB the distance, BG may be found, consequently, with it and the angle β, the height of the object becomes known.

Example.—Let the side AB be 80 feet, BC=119, and AC=140, also the angle at A or α=50°, that at B or β=66°, at C or γ=55°; required the height of the object?

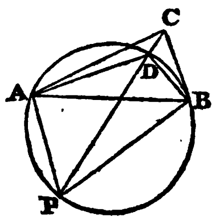
From these EF=96.329, DF=66.758; the angle AED=34° 48′, EDA=87° 6′ 23″, EAD=58° 5′ 37″, GEF=34° 6′ 57″, AFE, or ABG=70° 37′ 8″, FAE or AGB=40° 28′ 16″, BG=55.673; and the height 96.392 feet.

Example XXIV.

From a convenient station P, there could be seen three objects A, B, and C, whose distance from each other were AB=8 miles, AC=6 miles, BC=4 miles; I took the horizontal angles APC=33° 45′, BPC = 22° 30′. It is hence required to determine the respective distances of my station from each object. Here it will be necessary, as illustrative and preparatory to the computation, to describe the manner of

Construction.

Draw the given triangle ABC from any convenient scale. From the point A draw a line AD to make with AB an angle DAB equal to 22° 30′, and from B a line BD to make an angle DBA equal to 33° 45′. Let a circle be described to pass through their intersection D, and through the points A and B. Through C and D draw a straight line to meet the circle again in P, which is the point required. For drawing PA, PB, the angle APD is evidently equal to ABD, since it stands on the same arc AD; and, for a like reason, BPD = BAD. So that P is the point where the angles have the assigned value.



Computation.

In the triangle ABC, all the sides are given; to find the angles. In the triangle ABD, all the angles are known, and the side AB; to find both or one of the other sides AD. Take BAD from BAC, the remainder, DAC is the angle included between two known sides AD, AC; from which the angles ADC and ACD may be found. The angle CAP = 180° — (APC + ACD). Also, BCP = BCA — ACD; and PBC = ABC + PBA = ABC + sup ADC. Hence the three required distances are found by these proportions.

As sin APC : AC : : PAC : PC, and : : sin PCA : PA; and, lastly, as sin BPC : BC : : sin BCP : BP. The operation at length is as under:

By Rule II., Case III., we have

$\text{Sin } \frac{1}{2} \text{ BAC} = \sqrt{\frac{1\times 3}{8\times 6}} = \sqrt{\frac{1}{16}} = \frac{1}{4} = \cdot 25 = \text{sin } 14° 28' 39''$, and
BAC = 28° 57′ 18″.

$\text{Sin } \frac{1}{2} \text{ ABC} = \sqrt{\frac{1\times 5}{8\times 4}} = \frac{1}{8}\sqrt{10} = \cdot 3952847 = \text{sin } 23° 17' 1''\frac{1}{2}$, and
ABC = 46° 34′ 3″.

$\text{Sin } \frac{1}{2} \text{ ACB} = \sqrt{\frac{3\times 5}{6\times 4}} = \sqrt{\frac{5}{8}} = \frac{1}{4}\sqrt{10} = \cdot 7905694 = \text{sin } 52° 14' 19''\frac{1}{2}$, and
ACB = 104° 28′ 39″.

DAB	=22° 30′	CAB	=28° 57′ 18″		180° 0′ 0″
DBA	33 45	DAB	=22 30 0	DAC =	6 27 18
Sum	56 15	DAC =	6 27 18	ADC+ACD =	173 32 42
	180 0			½(ADC+ACD) =	86 46 21
ADB	123 45				

As sin ADB 123° 45′ ar co 0.0801536
Is to AB 8 miles 0.9030900
So is sin ABD 33° 45′ 9.7447390

To AD log 0.7279826
AC 6 miles, log+10 . . . . 10.7781513

Arc . 48° 18′ 7″ tan 10.0501687
Subtract 45 0 0 . . . .

Remainder 3 18 7 tan . . . 8.7611283
$\frac{1}{2}$(ADC+ACD)=86 46 21 tan . . . 11.2487967

$\frac{1}{2}$(ADB—ACD) 45 39 17 tan . . . 10.0099250

ACD = 41 7 4
ACD 41° 7′ 4″ sin 9.8179678
APC 33 45 0 ar co sin. 0.2552610 . . 0.2552610

Sum 74 52 4
180 0 0

PAC 105 7 56 sin 9.9846740
AC 6 miles log 0.7781513 . . 0.7781513

PA 7.10199 miles 0.8513801 . . . . . .

PC 10.42525 miles 1.0180863
ACB=104° 28′ 39″ . . . 180° 0′ 0″
ACD= 41 7 4 BCP+BPC= . 85 51 35

BCP— 63 21 35 PBC= . 94 8 25
As sin BPC 22° 30′ 0″ ar co 0.4171603
Is to BC 4 miles . . . 0.6020600
So is sin BCP 63° 21′ 35″ . 9.9512594

To PB 9.34285 miles . . 0.9704797

The computation of problems of this kind, however, may be a little shortened by means of the following

*General Investigation.**

Put AC$=a$, BC$=b$, APC=P, BPC=P′, ACB=C, and let there be taken for unknown quantities PAC$=x$, PBC$=y$. The triangles PAC and PBC give

Sin APC : sin CAP : : AC : CP, and
Sin BPC : sin CPB : : BC : CP; that is,

$$\text{Sin P} : \sin x : : a : \frac{a \sin x}{\sin \text{P}} = \text{CP, and}$$

$$\text{Sin P}' : \sin y : : b : \frac{b \sin y}{\sin \text{P}'} = \text{CP.}$$

Hence, $\frac{a \sin x}{\sin \text{P}} = \frac{b \sin y}{\sin \text{P}'}$; which may be reduced to $a \sin \text{P}' \sin x - b \sin \text{P} \sin y = 0$.

* See Lacroix Trigonometrie, and Gregory's Trigonometry.

In the quadrilateral ACBP, we have CBP=360°—APC—BPC—ACB—CAP, or $y=360-P-P'-C-x$.

Make $360°-P-P'-C=R$, then we shall have $y=R-x$; and consequently, $a \sin P' \sin x - b \sin P (\sin R \cos x - \cos R. \sin x)=0$.

Dividing by $\sin x$, there results, $a \sin P' - b \sin P (\sin R \frac{\cos x}{\sin x} - \cos. R)=0$.

Whence we have $\frac{\cos x}{\sin x} = \cot x = \frac{a \sin P' + b \sin P \cos R}{b \sin P \sin R}$

This expression being separated into two parts, we have

$$\cot x = \frac{a \sin P'}{b \sin P \sin R} + \frac{\cos R}{\sin R}; \text{ or,}$$

$$\cot x = \frac{\cos R}{\sin R}\left(\frac{a \sin P'}{b \sin P \cos R} + 1\right); \text{ or,}$$

$$\cot x = \cot. R\left(\frac{a \sin P'}{b \sin P \cos R} + 1\right); \text{ or, lastly,}$$

$$\cot x = \frac{a}{b} \sin P' \operatorname{cosec} P \sec R \cot R + \cot R.$$

Hence, x being thus determined, we get y from the equation $y=R-x$; and CP from either of the expressions given above.

We shall now apply the foregoing formula to the solution of the question last proposed.

Example XXV.

Here $a=6$ P= 33° 45′ 0″ PAC=x
$b=4$ P′= 22 30 0 PBC=y
ACB=104 28 39 found by computation

160 43 39
360 0 0

R=199 16 21

$\cot x = \frac{a}{b} \sin P' \operatorname{cosec} P \sec R \cot R + \cot R$; or,

$\cot x = \cot R\left(\frac{a \sin P'}{b \sin P \cos R} + 1\right)$ and using logarithms

we have	$a'=$		3 log	0.4771212
	$b'=$		2 ar co	9.6989700
	P′=	22° 30′ 0″	sin	9.5828397
	P=	33° 45′ 0″	ar co S.	0.2552610
R whose cos is neg		199 16 21	ar co C	0.0250452
	—	1.09458	log	0.0392371
	+	1.00000		
	—	0.09458	log	8.9757993
cot R	+	199° 16′ 21″		10.4563594
cot x	—	105 8 10		9.4321587
As sin	33° 45′ 0″ ar co			0.2552610
Is to sin x 105 8 10				9.9846660
So is 6				0.7781513
To PC		10.4251		1.0180783

Whence the rest may be found.

In using these formulæ, great attention must be paid to the signs of the quantities. See table page 12.

Example XXVI.

Suppose the objects A, B, C, are seen from D, and have their distances AB 7½ miles, BC 12 miles, and AC 8 miles, the angle BDA 25°, and CDA 19°; it is required to determine the distances DA, DB, DC?

Ans.—DA 10.0286, DC 16.7857, DB 14.9095 miles.

Example XXVII.

Suppose the objects A, B, C, are seen from D, and have their distances AB 8 miles, BC 12, and AC 7½; the angle BDC being 17° 47′ 19″. Required the distances DA, DC, and DB?

Ans.—DB 12, DC 22.85, and DA 20 miles.

Example XXVIII.

If AB be 8, AC 7.2, and BC 12 miles, and the angle ADB 107° 56′ 13″. Required the distances DA, DC, and DB?

Ans.—DB 5, DA 4.892, and DC 7 miles.

Example XXIX.

Let the objects A, B, C, be in a straight line; and their distances AC 3.626, AB 12, and BC 8.374, the angle ADC being 19°, and BDC 25°. Required the distances DA, DC, and DB?

Ans.—DA 9.4711, DC 10.861, and DB 16.8485.

Example XXX.

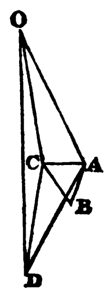
Let the objects A, B, C, as seen from D, be within the triangle; and let the distance AB be 6 miles, BC 12, and AC 9, the angle BDC being 123° 45′, and ADC 132° 22′. Required the distances DA, DC, and DB?

Ans.—DA 1.372, DB 5.523, DC 8.018.

Example XXXI.

To determine the height of a hill from the following observations taken by Captain Sabine at Spitzbergen.

As the point D on the top of the hill could not be seen from B, one of the extremities of the measured base AB, it was necessary to form the triangle ABC so as to have at least two points, A and C, from which the point D could be distinctly observed.



Mean horizontal angles at station A, 9 feet above the half-tide. The base AB was carefully measured by an improved chain on the ice, and was by that means perfectly level, and was found to be 2376 feet.

$$\left.\begin{array}{l} CAB = \ \ 63^\circ\ \ 9'\ 35'' \\ OAB = 128\ \ 27\ \ 15 \\ OAD = 124\ \ \ 8\ \ 34 \\ OAC = \ \ 65\ \ 17\ \ 40 \end{array}\right\} = \text{horizontal angles}$$

The mean zenith distance of the apex of a polished cone of copper, the point observed on the top of the hill D, 44 inches, or 3.67 feet above the summit, was 76° 53′ 48″.

From the station C, where the circle was 9 feet above the half-tide.

$$\left.\begin{array}{l} ACB = 59^\circ\ 9'\ 52''.5 \\ ACD = 102\ 10\ 33 \\ ACO = 96\ 40\ 28 \end{array}\right\} = \text{horizontal angles.}$$

The mean zenith distance of the apex of the cone was 75° 7′ 47″.

From the observatory O, where the circle was 31.5 feet above half-tide, the zenith distance of the cone was 82° 51′ 17″.5.

Hence, AC =2338.35 feet, AO= 7503.2 feet.
AD=7029.40 feet, OD=12842.1 feet.

Now, supposing the correction for refraction to be $\frac{1}{12}$ of the intercepted arc, and the length of the second 100 feet nearly, then $\frac{1}{1200}$ of the horizontal distance will be the correction for refraction in seconds.

Computing the height of the hill D from the stations C, A, and O, and correcting for curvature, there will result,

The height from C, . .	1633.93 feet	
Correction for curvature, (page 41,)	+ 0.91	
Height of centre of circle, .	+ 9.00	
True height, ...	1643.84...	1643.84
Height from A, ...	1636.07	
Correction for curvature, .	+ 1.18	
Height of circle, ...	+ 9.00	
True height, ...	1646.25...	1646.25
Height from O, ...	1609.20	
Correction for curvature, .	+ 3.95	
Height of circle, ...	31.50	
True height, ...	1644.65...	1644.65
Mean of the whole, . . .		1644.91

Example XXXII.

Having occasion to travel through the counties of Kent and Surrey, I perceived the fort built by Lady James, on Shooter's hill, which bore from me N. N. E.; and after going 20 miles in a W. N. W. direction, I perceived the fort again, which now bore N. E. by E. Required my distance from it at each station?

Ans.—29.93 miles, and 36 miles.

Example XXXIII.

At the Observatory on the top of the Calton-hill, 350 feet above the sea at Leith, the angle of depression of the horizon marked by the sea down the frith of Forth was 18′ 12″ by observation. Now, supposing the effect of refraction to be one-twelfth part of the whole, this must be increased by one-eleventh of itself, or the true depression would be 19′ 51″.28. Required the earth's diameter?

Ans.—7946 miles.

Example XXXIV.

Suppose the height of Melville's Monument in St Andrew's Square, Edinburgh, to be 136 feet, and that of the figure placed upon the top of it is 16 feet high, at what distance from the monument may the statue be viewed under an angle of 3°, and what is the greatest angle under which it can be seen?

Ans.—It will be seen, under an angle of 3°, at the distance of 203.93, or 101.37 feet, and the greatest angle under which it can be seen from a point in the horizontal plane, is 3° 11′ 5″.

Example XXXV.

It is required to find the distances from the Edystone lighthouse to Plymouth, Start Point, and the Lizard, respectively, from the following data:

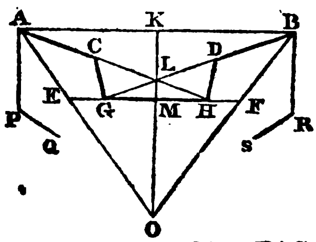
$$\text{The distances from}\left\{\begin{array}{ll}\text{Plymouth to Lizard} & 60\\ \text{Lizard to Start Point} & 70\\ \text{Start Point to Plymouth} & 20\end{array}\right\}\text{miles.}$$

$$\left.\begin{array}{l}\text{Plymouth}\\ \text{Lizard}\\ \text{Start Point}\end{array}\right\}\text{bears from Edystone}\left\{\begin{array}{l}\text{North}\\ \text{W. S. W.}\\ \text{E. by N.}\end{array}\right.$$

$$\textit{Ans.}\text{—From Edystone to}\left\{\begin{array}{ll}\text{Lizard} & 53.04\\ \text{Plymouth} & 14.33\\ \text{Start} & 17.36\end{array}\right\}\text{miles.}$$

Example XXXVI.

The side AB of a pentagon being 180 toises, the face of the bastion AC 50, the normal or perpendicular KL 30; it is required to find, by trigonometrical calculation, all the other lines and angles of the fortification, supposing the line of defence AH to be equal to a line drawn from A to D.



Solution.—Here $\frac{AB}{2}=\frac{180}{2}=90=AK$.

Hence, in the right-angled triangle AKL, AK (90) : R : : KL (30) : tan LAK=18° 26′. Because AB is the side of a regular pentagon, we have $\frac{360^\circ}{5}=$ $72^\circ = AOB$, and $\frac{72^\circ}{2}=36^\circ=AOK$, whence $90^\circ-36^\circ=54^\circ=EAK$, and $54^\circ-18^\circ\,26'=35^\circ\,34'=EAC$, which being doubled, is 71° 8′, the salient angle PAC or DBR. Join BC, then will ABC be a triangle in which are given AB, AC, and their contained angle BAC; to find ABC=6° 48′. Now sin ABC (6° 48′) : AC (50) : : sin BAC (18° 26′) : BC = 133.52, equal to the line of defence AH or BG. In the triangle BCG, ABG—ABC=18° 26′—6° 48′ = 11° 38′ = CBG. Because BC = BG, we have $\frac{180^\circ-11^\circ\,38'}{2}=\frac{168^\circ\,22'}{2}=84^\circ\,11'=CGB$.

Again, because AB and EF are parallel, and AH, BG equal; we have the angles BAH, ABG, AHE, and BGF all equal, that is, each equal to 18° 26′.

In the triangle CGH, we have the angle CGB+BGH=84° 11′ +18° 26′ = 102° 37′ = CGH; 180°—(CGH+CHG) = 180°—(102° 37′+18° 26′) = 58° 57′ = the angle HCG; and the side CH = AH —AC = 133.52—50 = 83.52 = CH. Then sin CGH (102° 37′) : CH (83.52) : : sin CHG (18° 26′) : the flank CG or DH=27.062 : : sin HCG 58° 57′ : the curtain GH=73.323.

TABLE OF THE MEASURES OF THE PRINCIPAL LINES AND ANGLES IN REGULAR FORTRESSES, FROM FOUR TO TWELVE SIDES INCLUSIVE.

Names of Sides and Angles.	Name of Polygons.								
	Square	Pentag	Hexag	Hepta.	Octag.	Nonag.	Decag.	Undec.	Dodec.
Exterior side, in toises -	180.	180.	180.	180.	180.	180.	180.	180.	180.
Radius of exterior side -	127.3	153.1	180.0	207.4	235.2	263.1	291.2	319.4	347.7
Interior side - -	115.5	123.9	130.6	136.2	140.0	142.9	144.3	146.3	148.1
Radius of interior side -	81.7	105.4	130.6	157.0	183.0	208.9	253.4	259.7	286.1
Capital - - -	45.6	47.7	49.3	50.5	52.2	54.2	57.8	59.7	61.7
Normal - - -	22.5	27.0	30.0	32.0	34.0	36.0	39.0	41.0	43.0
Curtin - - -	78.0	77.1	76.4	75.9	75.3	74.7	73.7	71.4	69.3
Flank - - -	20.3	24.5	27.3	29.2	31.1	33.0	35.8	37.0	38.1
Face - - - -	50.0	50.0	30.0	50.0	50.0	50.0	50.0	51.0	52.0
Line of defence - -	133.0	134.2	135.1	135.8	136.4	137.4	138.2	138.2	138.4
Demigorge - - -	18.7	23.4	27.1	30.2	32.4	34.1	35.3	37.4	39.4
Angle of the Centre - -	90° 0′	72° 0′	60° 0′	51°26′	45° 0′	40° 0′	36° 0′	32°44′	30° 0′
Angle of the Polygon -	90 0	108 0	120 0	128 34	135 0	140 0	144 0	147 16	150 0
Angle of the Curtin - -	97 1	98 21	99 13	99 47	100 21	100 54	101 43	102 15	102 46
Angle of the Shoulder -	111 3	115 3	117 39	119 21	121 3	122 42	125 9	126 45	128 18
Angle of Bast., or Flank. Angle	61 56	74 36	83 8	89 26	93 36	96 24	97 8	98 16	98 56
Diminished Angle - -	14 2	16 42	18 26	19 34	20 42	21 48	23 26	24 30	25 32
Exterior Flanking Angle -	151 56	146 36	143 8	140 52	138 36	136 24	133 8	131 0	128 56
Breadth of Foss. in Toises -	15	16	17	18	19	20	21	22	23

BAROMETRIC MEASUREMENT OF ALTITUDES.

Having given a pretty full view of the method of measuring the heights of objects geometrically, we shall here subjoin that of determining them by the barometer, thermometer, and hygrometer.

That the observations may be carefully and properly made, the persons who undertake them should be provided with two portable barometers of the best construction, filled with mercury of the same specific gravity, on which, by means of a vernier properly adapted to the scale, the height of the mercurial columns may be read off to the 1000th part of an inch; each barometer being fitted up with an attached thermometer, set in the wooden frame in the same manner as the barometer-tube is. The ball of each thermometer would be best if nearly of the same diameter as the barometer-tube. Besides these, they must also be provided with two other thermometers detached from the barometers. Of these barometers, one, with its attached and detached thermometers, is to be placed in the shade at the top of the eminence, while the other remains below. Let them continue in their places at least a sufficient time for the detached thermometer to acquire the temperature of the air, that is to say, till the contained fluid is stationary. Then the observer on the eminence must note down the height of the mercurial column in the barometer, as well as the temperatures exhibited by the attached and detached thermometers; and, at the *same time*, the other observer must make like observations upon the instruments below. If, in this manner, three or four sets of observations be taken at each station, after short intervals of time, and the mean of the results furnished by these sets respectively be taken, the probability of error in

the true altitude deduced by the following rules will be much diminished. When our first method of computation is adopted, two of Daniell's hygrometers must be employed to determine the dew-points at each station. If the observations be repeated on several successive days, the position of the instruments ought to be changed at each station alternately, at the same time comparing each pair of instruments to determine their index-error, should there be any. It is also advisable to make the observations in serene weather, between 11 and 12 o'clock. For it has been found that the computed heights are too *small*, when the observations have been made near sunrise or sunset, or when the wind blows fresh from the south; and that, on the contrary, the computed results are too *great* when the observations are made about three o'clock in a hot summer day, or during a brisk wind from the north or east.*

The barometers employed for this purpose are constructed in various plans, according to the judgment of the artist. In some the diameter of the cistern at the surface of the mercury is considerably larger than that of the tube, so that there is little or no capillary action at the cistern, while it may be somewhat considerable in the tube, and this must be corrected by Table I. In others, commonly called syphon-barometers, the diameters of the tube and cistern at the surface of the mercury are equal, and consequently the effect of capillarity is destroyed, since it is equal in both branches. By means of a screw the surface of the mercury in the cistern is always brought to the same height, namely, that from which the attached scale was graduated, by raising or lowering it till a small segment of light is observed between the surface and the frame of the barometer. This is perhaps one of the most convenient in practice.

Barometers with iron cisterns have been frequently constructed of late, and seem capable of great precision. In this case it is necessary to allow for capacity, as it is called, arising from the relative changes of the surfaces of the mercury in the tube and cistern. This is ascertained by measuring the diameters of each, and by knowing the height of each at the time of graduation, the actual height of the surface in the tube above that in the cistern at the time of observation may be found, to which, allowance for capillarity and temperature being applied, the true height at the standard, or freezing-point, will be obtained. This method is pursued in the example from Captain Sabine.

In the Connaissance des Tems for 1829, there is a memoir by Laplace, on the means of destroying the effects of capillarity in barometers, accompanied with a new table of the depression of mercury in glass by Bouvard, and another for estimating the distance of a point of ivory (the method frequently adopted in France for determining the upper surface in the basin) from the sides of the cistern, so as to destroy the capillarity depending on the interior diameter of the tube of the barometer, which seems capable of considerable accuracy.

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* One person may perform the whole operation with one set of instruments, by making the observations two or three times alternately at the top and bottom, and taking a mean of the results at each station. Indeed if two sets of instruments, and two different observers, be employed, the observers should alternately change places and instruments, so as to destroy the errors of reading and capillarity.

The ease with which heights can be measured by the barometer and its accompaniments has ever since its application to this subject been its chief recommendation, and great exertions have been made to perfect the rules and formulæ, to obtain in all cases, in every variety of circumstances, accurate results, without complete success.

As we ascend from the surface of the earth, the density of the air must neccssarily diminish, for each stratum is compressed by the weight of those strata above it; therefore the upper strata are less compressed, and consequently less dense than those directly below them. Now, if the temperature remained the same throughout the atmosphere, it is readily shown, that when the successive heights increase in arithmetical progression, the densities of the corresponding strata decrease in geometrical progression. But the densities are as the compressing forces measured by the columns of mercury in the barometer; therefore the heights from the surface of the earth being taken in arithmetical progression, the columns of mercury in the barometer at those heights will decrease in geometrical progression. Whence logarithms having relatively to their natural numbers the same property, it is obvious that they might be employed with advantage in such computations. For B being the height of a column of mercury at the lower station, and b that at the upper, then the height H of the upper station above the lower will be as the differences of these logarithms, or as log B—log b.

It was now only necessary to find the relation between the exact height in any known measure determined geometrically, and the difference between the logarithms of the barometric columns at these stations, to determine the coefficient requisite to compare the difference of elevation of one place above another in the same measure.

Hence if C be this coefficient, then

$$H = C\,(\log.\ B - \log.\ b.) = C \log.\ \frac{B}{b} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It was soon found, that when the common logarithms were employed, and the temperature at 32° Fahrenheit, or the freezing-point, the coefficient was nearly 10,000 English fathoms. A little experience shewed that this coefficient varied with the temperature, and involved the expansion of the air for successive accessions of heat, amounting to about 0.00244, in a mean state of the atmosphere for one degree of Fahrenheit, according to the experiments of Schuchburgh and Roy. But by the accurate experiments of Gay Lussac, Clement and Desormes, Dulong and Petit, *dry air* expands in volumes from 1 to 1.375, from a change of temperature from the freezing to the boiling points of water. Supposing the increase to be uniform, as it is known to be nearly, this would give 0.002084 only for each degree of Fahrenheit. Laplace lately assumed, that *moist air*, in its usual state, increases by a change of temperature of 180° Fahrenheit from 1 to 1.4, and consequently the variation for 1 degree of Fahrenheit is 0.002222. Both these, however, are less than what is allowed in barometric measures, as determined by the observers above-named. Since there is such a diversity of opinions, then it appears that the exact quantity in every given case is unknown, and all that can be done, in such circumstances, is to give two or more of these rates of expansions, and, by a consideration of the circumstances of the case, to select that most applicable.

The method of determining mean quantities, and employing them

in a general formula, is certainly not to be recommended if the exact quantities under given circumstances can be obtained. It would, therefore, be desirable to reconsider this subject, in order, if possible, to adapt the barometric formula to the exact state of the atmosphere at the time of observation. Though the difficulty of this subject be so great, that the mean quantities have been acquiesced in by such men as Playfair, Biot, and Laplace, yet, from the gradual progress of physical science, it would be unphilosophical to despair of final success. The instruments invented by Leslie, Daniell, &c. have thrown great light upon the nature and constitution of the atmosphere, and it is to be hoped that still farther advances in that department of science will be made. Indeed, it may be presumed, that considerable approximations have been made of late to determine truly the requisite corrections for the effects of aqueous vapour, —one of the most important elements now affecting the accuracy of barometric measurements. By the hygrometer, and the tables of the elastic force of aqueous vapour, by Dalton and Ure, its effects can be pretty accurately estimated. It is known that the humidity of the air is reduced one-half by an ascent of about 4500 feet. Now, as the density of the air is reduced also one-half by an ascent of 18,000 feet, it follows, that the law of the arithmetical and geometrical progressions does not strictly apply in the same manner to a combination of dry air and vapour mixed together, though they may be conformable to it when taken separately.

In permanently-elastic fluids, according to Dalton, when the temperature increases in arithmetical progression, the elasticity and volume increase in geometrical progression, and the increase of temperature from the freezing to the boiling points, according to the experiments of Dulong and Petit, increases the bulk and elasticity of such fluids from 1 to 1.375. If, therefore, the volume at the temperature of freezing be called 1; if x be any number of degrees of Fahrenheit above that temperature, and f the elastic force, then

$$f=(1.375)^{\frac{x}{180}}=\frac{x}{180}\log.(1.375)=0.0007835\,x \quad . \quad . \quad . \quad (2.)$$

From this formula, a table of the logarithms of the expansion or bulk of permanently-elastic fluids, such as dry air, may be readily computed. This Mr Dalton at one time believed would be more conformable to the nature of such fluids, than that derived from a supposed equable increase, and consequently it would be more accurate than that commonly employed in barometric measurements, or $1+0.00244\left(\frac{t+t'}{2}-32^\circ\right)=1+\left\{0.0012(t+t)-0.0768\right\}=0.9232$ $+0.0012\,(t+t')$ nearly, when the air is in its mean state, and consequently also than $1+0.002084\left(\frac{t+t'}{2}-32^\circ\right)$ when the expansion of dry air is assumed as the proper rate. Dalton's theory is perhaps not accurately true in all circumstances, though, in the opinion of some, it is more nearly correct than the usual theory of an equable increase. Dr Ure thinks (Chemical Dictionary, art. Caloric,) that though bodies in general increase in bulk more rapidly in the higher parts of the scale than in the lower, yet it is not so great as Dalton supposes. This principle, however, at one time adopted by Dalton, has been objected to by some chemists, such as Dulong and Petit, who have affirmed, that an equable increase, according to their ex-

periments, ought to be preferred. It is admitted, however, that, at high temperatures, most fluids and liquids, such as even mercury, expand more rapidly than at the lower, which naturally occur in the atmosphere. From these and other considerations, I am inclined to believe that Dalton's hypothesis, perhaps somewhat modified, gives the increase of the volume of air rather the more accurately. Dalton, indeed, himself admits, that the determinations of Dulong and Petit are good approximations, though he seems still, and perhaps justly, to think that his view of the subject is more or less founded in truth. When such authorities differ, all that can be done with propriety is to give a table of the expansion of air according to both these laws, and, by comparing results deduced by each with the accurate geometrical method, experience will determine which in barometric measures ought to be preferred.

But these laws of arithmetical and geometrical progressions are applicable to dry air only disengaged from vapour; consequently it is necessary to restore the same laws to a mixture of dry air and aqueous vapour similar to the actual constitution of the atmosphere.

Let f denote the elastic force of aqueous vapour at the lower station, where the barometric pressure is B, and f' that at the upper, where the pressure corrected for temperature is b. Now, since the weight of a given volume of vapour under the same pressure is $\frac{5}{8}$ of that of dry air, the density of the moist atmosphere would be the same as that of dry air, which would support a pressure indicated by $B-f+\frac{5}{8}f=B-\frac{3}{8}f$ . . . . (3.)

As the modulus of aqueous vapour is about one-fourth of that of dry air, as has already been observed, this expression must receive a correspondent modification before the usual law can be applied. Hence $\frac{1}{4}\times\frac{3}{8}\times f=\frac{1}{10}f$ nearly, for the exact relation is not known with certainty; therefore the corrected expression to which the law of the arithmetical and geometrical progressions may be applied is $B-\frac{1}{10}f$ . . . . (4.)

But Dalton and Gay Lussac found, when a perfectly dry gas is admitted to water, its volume v augments, and becomes $\frac{vp}{p-f}$, in which p is the barometric pressure, and f the elastic force of aqueous vapour. Whence the bulk increased by humidity is to its original bulk as B to $B-f$, or as $1+\frac{f}{B-f}$ to 1; consequently a column of dry air mixed with a quantity of aqueous vapour, whose elastic force is f, will be increased in length in the ratio of 1 to $1+\frac{f}{B-f}$, . (5.)

and since the density of the air is inversely as its bulk, a coefficient of this form must be applied to the barometric formula, on account of the dilatation of the atmospheric column by moisture. The most accurate mode of applying this coefficient will be to take the mean barometric pressure and elastic force at the lower and upper stations. Let f denote the elastic force at the lower station, and f' that at the upper; also B the barometric pressure as the lower, and b that at the upper; then substituting $\frac{f+f'}{2}$ for f in formula (5), and $\frac{B+b}{2}$, for B we get $1+\frac{f+f'}{B+b-(f+f')}$ . . . . . . (6.)

Introducing (2), (4), and (6) into formula (1), there will result

$$H = C \times \left\{ (1.375)^{\frac{\frac{t+t'}{2} - 32^\circ}{180}} \right\} \left(1 + \frac{f+f'}{B+b-(f+f')} \right) \log \frac{B - \frac{1}{10}f}{b - \frac{1}{10}f'} \quad . \quad (7.)$$

by making $\frac{\frac{t+t'}{2} - 32^\circ}{180} = \frac{x}{180}$ in formula, (2) in which t is the temperature of the air by the detached thermometer at the lower station, and t' that at the upper, and consequently $\frac{t+t'}{2} - 32^\circ$ is the excess of the mean temperature above the freezing-point, derived from the temperature of the aerial columns by the thermometers, at the two extremities, and the rate of the expansion of the air, according to Dalton's hypothesis.

If an equable expansion be that assumed, then it becomes

$$H = C \left\{ 1 + 0.002084 \left(\frac{t+t'}{2} - 32^\circ \right) \right\} \left(1 + \frac{f+f'}{B+b-(f+f')} \right)$$
$$\log \frac{B - \frac{1}{10}f}{b - \frac{1}{10}f'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

agreeable to the deductions of Dulong and Petit.

The question now to be determined is the value of the coefficient C under given circumstances. If the densities of air and mercury are compared at any given latitude, such as 45°, and found to have a given relation under a pressure of 30 inches on account of the change of gravity, they will have a different relation in another latitude under the same pressure. This change is proportional to $1 - 0.00268 \cos 2 L$, in which L is the latitude, and 0.00268 the excess of the polar pendulum above the equatorial, divided by twice the length of the latter. The logarithms of this expression at any latitude may be put into a table for the sake of simplifying the calculation. But the gravitating force varies also with the height above the sea. Let r represent the mean radius of the earth, or 20887680 feet, which is sufficiently accurate for this purpose, a the altitude of the lower barometer above the sea in feet, then the gravitating force is diminished proportionally to $\left(\frac{r+a}{r}\right)^2 = 1 + \frac{2a}{r} = 1 + \frac{a}{\frac{1}{2}r}$ nearly. Also it has been shown by Biot in the third volume of his Astronomie Physique, that the correction for the difference of the gravitating forces at the stations of the two barometers is equal to

$C' \left\{ \log \frac{B}{b} + 2 \log \left(1 + \frac{H'}{r} \right) \right\} \left(1 + \frac{H'}{r} \right)$, H' being the height of the higher barometer above the lower, and C' the approximate coefficient.

This expression is rather inconvenient in application, and therefore it has been found necessary to change it. Since $\frac{H'}{r}$ is always a small quantity, the log of $1 + \frac{H'}{r}$ is readily obtained by employing the modulus. We may suppose in that case the $\log 1 + \frac{H'}{r} = \frac{MH'}{r}$, M being the modulus=0.43429445, and afterwards multiplying by the factor $1 + \frac{H'}{r}$, we get $C' \log \frac{B}{b} + \frac{C'}{r} \left\{ \log \frac{B}{b} + 2M \right\} H'$, consequently

$$H' = \frac{C' \log \frac{B}{b}}{1 - \frac{C'}{r}\left\{\log \frac{B}{b} + 2M\right\}}$$

The denominator of this fraction does not differ much from unity, for the coefficient of the second member $\frac{C'}{r} = \frac{60155}{20887680} = \frac{1}{347}$ very nearly, and the other factor $\frac{B}{b} + 0.868589$ never much exceeds 0.868589, since in all ordinary cases the log $\frac{B}{b}$ is a small fraction. For supposing B=30 inches, and b=25, then $\frac{B}{b} = \frac{30}{25} = \frac{6}{5}$, the log of which is 0.079181 for an elevation of about 5000 feet. Hence 0.868589+0.079181=0.948770, a number less than unity.

Whence $\frac{C'}{r}\left\{\log \frac{B}{b} + 2M\right\} = \frac{60155 \times 0.94877}{20887680} = 0.002733$, and log $60155 \times 1.002733 = 4.779272 + 0.001186 = 4.780458$. But $\frac{2\,H}{r} = \frac{H}{\frac{1}{2}r}$, and supposing H=1000 feet, then $\frac{H}{\frac{1}{2}r} = 0.00009575$, therefore the logarithm to be added to that of C′, at the level of the sea, for every 1000 feet of elevation of the lower barometer, is 0.0000416, and an addition of 0.001186 for a difference of 5 inches in the height of the mercury in the two barometers, or 0.000237, for a difference of one inch.

On these data, which are perhaps pretty near the truth, a table of logarithmic barometric coefficients may be constructed.

Supposing the value of C to be found in this manner, and introducing the expansion of mercury for heat, the effect of the variation of gravity for latitude, the effect of aqueous vapour, &c. into formula (7) or (8), and the formula for the correct height will be obtained. In order to simplify, let E denote $(1.375)^{\frac{\frac{t+t'}{2} - 32^\circ}{180}}$ in formula (7), or $1 + 0.002084\left(\frac{t+t'}{2} - 32\right)$ in formula (8) according as the one hypothesis or the other is adopted, then

$$H = C \times E\left\{(1 + 0.00268 \cos 2L)\left(1 + \frac{f+f'}{B + b\{1 + 0.0001(\tau - \tau')\}}\right) \times \log\left(\frac{B - \frac{1}{10}f}{b\{1 + 0.0001(\tau - \tau') - \frac{1}{10}f'\}}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

In which H is the height of the one place above the other, t the temperature of the air by the detached thermometer, at the lower station, t' that at the upper, L the latitude of the place of observation, τ the temperature of the mercury by the attached thermometer at the lower station, and τ' that at the upper, f the elastic force of aqueous vapour, determined by the hygrometer at the lower station, and f' that at the upper.

In this formula f and f' are supposed to be taken directly from a table containing the elastic force of aqueous vapour for a temperature corresponding to the dew-point, which is sufficient in most or-

dinary cases when the temperature is not very high, and the barometric pressure does not differ considerably from 30 inches; otherwise let f'' and f''' be the true elasticities, then

$$f''=\frac{B}{30}\times\left(1+\frac{t-d}{448+d}\right)\times f \quad . \quad . \quad . \quad . \quad (A)$$

$$f'''=\frac{b}{30}\times\left(1+\frac{t'-d'}{448+d'}\right)\times f' \quad . \quad . \quad . \quad (B)$$

Whence the true elasticities become known.

Method I.

By the foregoing formulæ and auxiliary tables derived from them.

Examples for Exercise.

1. Required the height of the Peak of Snowden above Caernarvon Quay, from the following set of observations, the geometrical height being 3555.4 feet?

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
At Caernarvon Quay,	29.984 .	56°.50 .	55°.25 .	50°
At Snowden Peak,	26.271 .	42 .75 .	43 .00 .	41

Hence $\tau = 56°.50$ $\quad t = 55°.25 \quad d = 50 \quad f = 0.3653$

$\tau' = 42 .75 \quad t' = 43 .00 \quad d' = 41 \quad f' = 0.2292$

$\tau - \tau' = 13 .75 \quad t + t' = 98 .25 \quad f + f' = 0.5945$

$\tau - \tau' = 13.\ 75$ log E. 0.000597 (Table III.)

$b = 26.271$ log 1.419476

$b' = 26.307$ log 1.420073

$B = 29.984$

$B + b' = 56.291$

$B - b' = 3.677$ Logarithmic coefficient, (Table VII.) 4.780145

$t + t' = 98°.25$ Logarithm E', (Table V.) Dalton, 0.013158

Lat. $= 53° 4'$ Log. of effect of gravity, (Table VI.) 9.999679

$B - \frac{1}{10} f = 29.948$ Log. 1.476370

$b' - \frac{1}{10} f' = 26.284$ 1.419691

0.056679 logarithm . . 8.753422

$E'' = 1 + \frac{f+f'}{B+b'-(f+f')} = 1.01067$ logarithm . 0.004609

B.H = 3556.4 logarithm 3.551013

G.H = 3555.4

Diff. = + 1.0 foot

2. Required the height of Mount Vesuvius from the following observations of the Right Honourable the Earl of Minto?

At Portici, near Naples, 17th of March, 1822, 3 feet above the sea.

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
Mean of several sets,	30.242 .	73.5 .	64° .	50°

At the old Palo, 3 feet below the summit of Vesuvius.

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
Mean of several sets,	26.188 .	60°.0 .	54.8 .	44°

Bar. height . = 3957.6 feet
1st cor at Portici, = + 3.0
2d cor at summit, = + 3.0

True height, . = 3963.6 feet

3. Required the height of Mont Salève, from the following set of observations, the mean geometrical height from two series of triangles being 2831.8 feet?

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
At lower station,	28.3925	72°.0	73°.5	66°
At upper,	25.7075	76 .0	64 .5	59

Ans.—2828.2 feet, or about 3.6 feet less than the geometrical. It is proper to observe, however, that the geometrical heights were 2835.07 feet by one series, and 2828.45 feet by the other, nearly the same as the barometric.

4. Let the height of Pico Ruivo, in the island of Madeira, be determined from the following observations by Dr Heineker, with one of Newman's iron cistern barometers and Daniell's hygrometer, which were taken at intermediate stations. The height of the cistern was 4 feet above the sea at the lowest station, and the cistern at the highest 20 feet lower than the extreme summit of the Peak.

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
I. St George's beach,	30.230	75°	75°	67°
Mr Welsh's house,	29.171	69	68	65

	Bar.	At. Ther.	Det. Ther.	Dew-Point.
II. Mr Welsh's house,	29.102	67°	65°	57°
Pico Ruivo,	24.290	60	60	56

Latitude about 33° N.

1. Height of Mr Welsh's house by set I. or B.H = 1000.3 feet
2. Height of Peak above Mr Welsh's house, B.H′ = 5059.4
3. First correction, or . . . . . h = 4.0
4. Second correction, or . . . . . h' = 20.0

True height, ... = 6083.7 feet

Mr Bowdich makes the height of the Peak 6164 feet; but Dr Heineker, who made the foregoing observations, found, barometrically, that the height of Mr Veitch's turret, where Mr Bowdich made his observations, was only 97 feet above the level of the sea, instead of 154 feet, the quantity adopted by him; consequently his height ought to be diminished by 57 feet, and 6164—57=6107=Bowdich's height correctly.

Bowdich's height,	= 6107 feet		6107
Heineker's,	= 6084		6084
Difference,	23 feet	Mean,	6095.5

5. At Spitzbergen, in latitude 79° 50′ N. Captain Sabine, by a mean of twenty-six sets of observations, found the height of the barometer, whose cistern was 21 feet above the level of half-tide, to be 29.60669 inches, the temperature of the mercury by the attached thermometer 39°.992 Fahrenheit, that of the air by the detached thermometer 36°.385, and the dew-point 35°.93. At the same time, Lieutenant Foster found, on the top of a hill, the height of the barometer, whose cistern was near the ground, and 44 inches, or 3.67 feet below a copper cone, the point observed geometrically to be compared with the barometric measurement, to be 27.95623 inches, the temperature of the mercury by the attached thermometer 38°.27, that of the air by the detached thermometer 36°.635, and the dew-point 35°.19.

Captain Sabine used two barometers with corresponding thermometers and hygrometers. The one was made by Newman, under

the superintendence of Mr Daniell, the other by Jones; they were both graduated when the pressure of the atmosphere was 30.4 inches. The capacity of the tube of Jones's to that of its cistern was as 1 to 11, or $\frac{1}{11}$, that is, 11 inches in the tube would fill one inch in the cistern, as ascertained by the maker from experiment or calculation. The capacity of the tube of Newman's to that of its cistern was as 1 to 54, or $\frac{1}{54}$. The diameter of Jones's was 0.15 inch; that of Newman's 0.31 inch. Now, to correct the observed height for capacity, as it is called,* the difference between the height of the barometer, when the instrument was graduated, and its actual height at the time of observation, multiplied by the fraction expressed by the ratio of the capacity of the tube to that of the cistern, is to be added to the height by observation, if that height is greater than the height of the barometer at the time of graduation, but subtracted if it is less, the temperature in both cases being the same, or being in general reduced to the freezing-point. Captain Sabine, however, found, from some accident which happened to Jones's barometer, that its neutral point required to be estimated from 30.271, instead of 30.4, with an index-error of 0.1938 to reconcile it with Newman's.

B . .	$= 29.60669$	$b =$	27.95623
Reduction to 32°	$= -0.02369$		-0.01753
Capacity, .	$= -0.06036$		-0.04526
Capillarity, .	$= +0.02732$		$+0.08628$
Index-error,	$= +0.19380$		
		$b' =$	27.97972
B′ . .	$= 29.74376$		

Here $f'' = \frac{29.7438}{30} \times \left\{1 + \frac{36.385-35.93}{448+35.93}\right\} \times 0.21564 = 0.21401$

and $f''' = \frac{27.9797}{30} \times \left\{1 + \frac{36.635-35.19}{448+35.19}\right\} \times 0.20972 = 0.19620$

$B' = 29.74376$		b' . .	$= 27.97972$
$\frac{1}{10}f'' = -0.02140$		$-\frac{1}{10}f'''$ . .	$= -0.01962$
$B'' = 29.72236$	. . .	b'' . .	$= 27.96010$

$B' - b' = \Delta\beta = 29.744 - 27.980 = 1.762$

Hence C . . . . . . $= 4.779694$
$t + t' = 73°.02$, gives E′ . . . . $= 0.003465$
Lat. $= 79° 50'$ . . . . . . $= 9.998908$
$B'' = 29.72236$ log 1.473083
$b'' = 27.96010$ log 1.446539

Diff. 0.026544 log . . $= 8.423966$

$E'' = 1 + \frac{f'' + f'''}{B' + b' - (f'' + f''')} = 1.00715$ log . $= 0.003094$

H $= 1618.55$ . . . log 3.209127
Cor. $= +24.67$
B.H $= 1643.22$
G.H $= 1644.91$
Diff. $= -1.69$ foot.

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* This correction must be always applied to those barometers which have not a contrivance for adjusting the cistern, or bringing the surface of the mercury in it to the same height as it was when graduated. This latter method appears to be the safest, since, unless the tube and cistern be of uniform diameters, their relations may be altered.

BAROMETRIC TABLES.—Table I.

Capillarity,

Or Depression of Mercury in Glass Tubes, to be added to the observed Height of the Mercury in the Barometer. By the Formula of Mr Ivory.

Diam. of Tube.	Capillarity	Diff.	Diam. of Tube.	Capillarity	Diff.	Diam. of Tube.	Capillarity	Diff.	Diam. of Tube.	Capillarity	Diff.
In.	Inch of Mercury.		In.	Inch of Mercury.		In.	Inch of Mercury.		In.	Inch of Mercury.	
0.10	0.1404	146	0.25	0.0409	27	0.40	0.0154	9	0.55	0.0060	4
0.11	0.1258	122	0.26	0.0382	24	0.41	0.0145	9	0.56	0.0056	4
0.12	0.1136	103	0.27	0.0358	24	0.42	0.0136	9	0.57	0.0052	3
0.13	0.1033	90	0.28	0.0334	21	0.43	0.0127	8	0.58	0.0049	3
0.14	0.0943	78	0.29	0.0313	20	0.44	0.0119	7	0.59	0.0046	3
0.15	0.0865	70	0.30	0.0293	18	0.45	0.0112	7	0.60	0.0043	2
0.16	0.0795	62	0.31	0.0275	18	0.46	0.0105	6	0.61	0.0041	3
0.17	0.0733	55	0.32	0.0257	16	0.47	0.0099	6	0.62	0.0038	2
0.18	0.0678	50	0.33	0.0241	15	0.48	0.0093	6	0.63	0.0036	2
0.19	0.0628	45	0.34	0.0226	14	0.49	0.0087	5	0.64	0.0034	3
0.20	0.0583	41	0.35	0.0212	13	0.50	0.0082	5	0.65	0.0031	2
0.21	0.0542	38	0.36	0.0199	13	0.51	0.0077	5	0.66	0.0029	2
0.22	0.0504	34	0.37	0.0186	11	0.52	0.0072	4	0.67	0.0027	2
0.23	0.0470	32	0.38	0.0175	11	0.53	0.0068	4	0.68	0.0025	1
0.24	0.0438	29	0.39	0.0164	10	0.54	0.0064	4	0.69	0.0024	

Table II.

Reduction of the English Barometer to 32° Fah. subtractive.

Temp	Part I.—For Mercury only. Height of the Barometer in Inches.				Part II.—For Mercury and Brass. Height of the Barometer in Inches.			
	28 In.	29 In.	30 In.	31 In.	28 In.	29 In.	30 In.	31 In.
32°	0.0000	0.0000	0.0000	0.0000	0.0088	0.0091	0.0094	0.0097
34	0.0056	0.0058	0.0060	0.0062	0.0138	0.0143	0.0148	0.0152
36	0.0112	0.0116	0.0120	0.0124	0.0188	0.0194	0.0201	0.0208
38	0.0168	0.0174	0.0180	0.0186	0.0238	0.0246	0.0255	0.0263
40	0.0224	0.0232	0.0240	0.0248	0.0288	0.0298	0.0309	0.0319
42	0.0280	0.0290	0.0300	0.0310	0.0338	0.0350	0.0362	0.0374
44	0.0336	0.0348	0.0360	0.0372	0.0388	0.0402	0.0416	0.0430
46	0.0392	0.0406	0.0420	0.0434	0.0438	0.0454	0.0470	0.0485
48	0.0448	0.0464	0.0480	0.0496	0.0488	0.0506	0.0523	0.0541
50	0.0504	0.0522	0.0540	0.0558	0.0538	0.0558	0.0577	0.0596
52	0.0559	0.0579	0.0599	0.0619	0.0588	0.0609	0.0630	0.0652
54	0.0615	0.0637	0.0659	0.0681	0.0638	0.0661	0.0684	0.0707
56	0.0671	0.0695	0.0719	0.0743	0.0688	0.0713	0.0738	0.0762
58	0.0727	0.0753	0.0779	0.0805	0.0738	0.0765	0.0791	0.0818
60	0.0783	0.0811	0.0839	0.0867	0.0788	0.0817	0.0845	0.0873
62	0.0838	0.0868	0.0898	0.0928	0.0838	0.0868	0.0898	0.0928
64	0.0894	0.0926	0.0958	0.0990	0.0888	0.0920	0.0951	0.0983
66	0.0950	0.0984	0.1018	0.1051	0.0938	0.0971	0.1005	0.1039
68	0.1005	0.1041	0.1077	0.1113	0.0988	0.1023	0.1058	0.1094
70	0.1061	0.1099	0.1137	0.1175	0.1037	0.1075	0.1112	0.1149
72	0.1117	0.1156	0.1196	0.1236	0.1087	0.1126	0.1165	0.1204
74	0.1172	0.1214	0.1256	0.1298	0.1137	0.1178	0.1218	0.1259
76	0.1228	0.1271	0.1315	0.1359	0.1187	0.1229	0.1272	0.1314
78	0.1283	0.1329	0.1375	0.1421	0.1237	0.1281	0.1325	0.1369
80	0.1339	0.1387	0.1434	0.1482	0.1286	0.1332	0.1378	0.1424
82	0.1394	0.1444	0.1494	0.1544	0.1336	0.1384	0.1432	0.1479
84	0.1450	0.1502	0.1553	0.1605	0.1386	0.1435	0.1485	0.1534
86	0.1505	0.1559	0.1613	0.1667	0.1435	0.1486	0.1538	0.1589
88	0.1561	0.1616	0.1672	0.1728	0.1485	0.1538	0.1591	0.1644
90	0.1617	0.1674	0.1731	0.1790	0.1535	0.1589	0.1644	0.1699

P.P. Tem +	0°.4	0°.8	1°.2	1°.6	2°.0
Part I.	12	24	35	47	59

P.P. Tem +	0°.4	0°.8	1°.2	1°.6	2°.0
Part II.	10	21	31	42	52

TABLE III.

Logarithms of the Expansion of Mercury.

$\tau-\tau'$	Log. E.	$\tau-\tau'$	Log. E.	$\tau-\tau'$	Log. E.	$\tau-\tau'$	Log. E.
0°	0.000000	20°	0.000868	40°	0.001736	60°	0.002604
1	0.000043	21	0.000911	41	0.001779	61	0.002647
2	0.000087	22	0.000955	42	0.001823	62	0.002691
3	0.000130	23	0.000988	43	0.001866	63	0.002734
4	0.000174	24	0.001042	44	0.001910	64	0.002778
5	0.000217	25	0.001085	45	0.001953	65	0.002821
6	0.000260	26	0.001128	46	0.001996	66	0.002864
7	0.000304	27	0.001172	47	0.002040	67	0.002908
8	0.000347	28	0.001215	48	0.002083	68	0.002951
9	0.000391	29	0.001259	49	0.002127	69	0.002995
10	0.000433	30	0.001302	50	0.002170	70	0.003038
11	0.000477	31	0.001345	51	0.002213	71	0.003081
12	0.000521	32	0.001389	52	0.002257	72	0.003125
13	0.000565	33	0.001432	53	0.002300	73	0.003168
14	0.000609	34	0.001476	54	0.002344	74	0.003212
15	0.000652	35	0.001519	55	0.002387	75	0.003255
16	0.000695	36	0.001562	56	0.002430	76	0.003298
17	0.000739	37	0.001606	57	0.002474	77	0.003342
18	0.000782	38	0.001649	58	0.002527	78	0.003385
19	0.000825	39	0.001693	59	0.002561	79	0.003429

P.P.	·1	·2	·3	·4	·5	·6	·7	·8	·9
+	4	9	13	17	22	26	30	35	39

TABLE IV.

Elastic Force of Aqueous Vapour. Barometer 30 Inches. By the Formula of Mr Ivory.

Temp	Force.	Diff.	Temp	Force.	Diff.	Temp	Force.	Diff.	Temp	Force.	Diff.
	Inch.			Inch.			Inch.			Inch.	
0°	0.0511	22	30°	0.1720	67	60°	0.5145	180	90°	1.3799	436
1	0.0533	23	31	0.1787	70	61	0.5325	187	91	1.4235	448
2	0.0556	24	32	0.1857	72	62	0.5512	193	92	1.4683	461
3	0.0580	25	33	0.1929	75	63	0.5705	199	93	1.5144	473
4	0.0605	26	34	0.2004	78	64	0.5904	205	94	1.5617	487
5	0.0631	27	35	0.2082	80	65	0.6109	211	95	1.6104	500
6	0.0658	28	36	0.2162	82	66	0.6320	217	96	1.6604	513
7	0.0686	29	37	0.2244	85	67	0.6537	224	97	1.7117	528
8	0.0715	30	38	0.2329	88	68	0.6761	232	98	1.7645	542
9	0.0745	31	39	0.2417	92	69	0.6993	238	99	1.8187	556
10	0.0776	33	40	0.2509	95	70	0.7231	246	100	1.8743	572
11	0.0809	34	41	0.2604	98	71	0.7477	253	101	1.9315	587
12	0.0843	36	42	0.2702	102	72	0.7730	261	102	1.9902	602
13	0.0879	37	43	0.2804	105	73	0.7991	268	103	2.0504	618
14	0.0916	38	44	0.2909	108	74	0.8259	277	104	2.1122	634
15	0.0954	39	45	0.3017	112	75	0.8536	285	105	2.1756	652
16	0.0993	40	46	0.3129	115	76	0.8821	293	106	2.2408	668
17	0.1033	42	47	0.3244	119	77	0.9114	302	107	2.3076	686
18	0.1075	43	48	0.3363	124	78	0.9416	311	108	2.3762	704
19	0.1118	45	49	0.3487	128	79	0.9727	320	109	2.4466	723
20	0.1163	47	50	0.3615	132	80	1.0047	330	110	2.5189	742
21	0.1210	49	51	0.3747	137	81	1.0377	339	111	2.5931	758
22	0.1259	50	52	0.3884	141	82	1.0716	348	112	2.6689	777
23	0.1309	53	53	0.4025	145	83	1.1064	358	113	2.7466	797
24	0.1362	55	54	0.4170	150	84	1.1422	369	114	2.8263	819
25	0.1417	56	55	0.4320	155	85	1.1791	379	115	2.9082	838
26	0.1473	58	56	0.4475	160	86	1.2170	390	116	2.9920	860
27	0.1531	60	57	0.4635	165	87	1.2560	402	117	3.0780	883
28	0.1591	63	58	0.4800	170	88	1.2962	412	118	3.1663	905
29	0.1654	66	59	0.4970	175	89	1.3374	425	119	3.2568	926

TABLE V.

Logarithms of the Expansion of Dry Air.

t+t′	Log. E′ Dalton	Log. E′ Gay Lussac	t+t′	Log. E′ Dalton	Log. E′ Gay Lussac	t+t′	Log. E′ Dalton	Log. E′ Gay Lussac	t+t′	Log. E′ Dalton	Log. E′ Gay Lussac
20°	9.983096	9.979624	70°	0.002305	0.002706	120°	0.021514	0.024622	170°	0.040723	0.045486
21	983480	980097	71	002689	003155	121	021898	025050	171	041107	045893
22	983865	980569	72	003073	003604	122	022262	025477	172	041491	046300
23	984249	981042	73	003457	004052	123	022666	025903	173	041875	046707
24	984633	981514	74	003842	004501	124	023050	026329	174	042259	047113
25	985017	981987	75	004226	004948	125	023434	026755	175	042643	047519
26	985401	982457	76	004610	005395	126	023819	027180	176	043028	047924
27	985785	982928	77	004994	005841	127	024203	027604	177	043412	048329
28	986170	983398	78	005378	006288	128	024587	028027	178	043796	048733
29	986554	983869	79	005762	006733	129	024971	028452	179	044180	049137
30	9.986938	9.984339	80	0.006147	0.007179	130	0.025356	0.028876	180	0.044564	0.049541
31	987322	984807	81	006531	007623	131	025740	029299	181	044948	049945
32	987706	985275	82	006915	008068	132	026124	029721	182	045333	050348
33	988090	985743	83	007299	008511	133	026508	030144	183	045717	050751
34	988475	986211	84	007683	008954	134	026892	030566	184	046101	051153
35	988859	986678	85	008067	009396	135	027276	030987	185	046485	051555
36	989243	987148	86	008452	009838	136	027661	031408	186	046869	051956
37	989627	987609	87	008836	010280	137	028045	031829	187	047253	052357
38	990011	988074	88	009220	010723	138	028429	032249	188	047638	052758
39	990395	988540	89	009604	011165	139	028813	032669	189	048022	053159
40	9.990780	9.989005	90	0.009989	0.011606	140	0.029197	0.033089	190	0.048406	0.053559
41	991164	989468	91	010373	012046	141	029581	033508	191	048790	053959
42	991548	989931	92	010757	012486	142	029966	033926	192	049174	054358
43	991932	990394	93	011141	012925	143	030350	034344	193	049558	054757
44	992317	990857	94	011525	013364	144	030734	034762	194	049943	055155
45	992701	991319	95	011909	013802	145	031118	035180	195	050327	055553
46	993085	991779	96	012294	014240	146	031502	035597	196	050711	055951
47	993469	992239	97	012678	014677	147	031886	036013	197	051095	056349
48	993853	992700	98	013062	015115	148	032271	036429	198	051479	056746
49	994237	993160	99	013446	015552	149	032655	036845	199	051863	057143
50	9.994622	9.993620	100	0.013830	0.015988	150	0.033039	0.037260	200	0.052248	0.057539
51	995006	994078	101	014214	016424	151	033423	037675	201	052632	057935
52	995390	994535	102	014599	016860	152	033807	038090	202	053016	058331
53	995774	994993	103	014983	017294	153	034191	038504	203	053400	058726
54	996158	995450	104	015367	017729	154	034576	038918	204	053784	059121
55	996542	995908	105	015751	018162	155	034960	039331	205	054168	059516
56	996927	996364	106	016135	018596	156	035344	039744	206	054553	059910
57	997311	996820	107	016519	019029	157	035728	040157	207	054937	060304
58	997695	997276	108	016904	019462	158	036112	040569	208	055321	060698
59	998079	997732	109	017288	019894	159	036496	040981	209	055705	061091
60	9.998463	9.998187	110	0.017672	0.020327	160	0.036881	0.041393	210	0.056090	0.061484
61	998847	998641	111	018056	020758	161	037265	041804	211	056474	061876
62	999232	999094	112	018440	021189	162	037649	042214	212	056858	062268
63	9.999616	9.999547	113	018824	021620	163	038033	042625	213	057242	062660
64	0.000000	0.000000	114	019209	022050	164	038418	043035	214	057626	063052
65	000384	000452	115	019593	022480	165	038802	043444	215	058011	063443
66	000768	000904	116	019977	022909	166	039186	043853	216	058395	063834
67	001152	001355	117	020361	023338	167	039570	044262	217	058779	064224
68	001537	001806	118	020745	023767	168	039954	044670	218	059163	064614
69	001921	002256	119	021129	024195	169	040338	045078	219	059547	065004

P. P. +		.1	.2	.3	.4	.5	.6	.7	.8	.9
	D.	38	77	115	153	192	230	264	307	345
	G	43	86	128	171	214	257	300	342	385

Table VI.

Logarithmic Values of 1+0.00268 cos. 2 L.

Lat.	Log.	Lat.	Log.	Lat.	Log.	Lat.	Log.
0°	0.001162	23°	0.000808	46°	9.999959	69°	9.999136
1	0.001162	24	0.000778	47	9.999919	70	9.999109
2	0.001160	25	0.000747	48	9.999878	71	9.999084
3	0.001156	26	0.000716	49	9.999838	72	9.999059
4	0.001151	27	0.000684	50	9.999798	73	9.999036
5	0.001145	28	0.000651	51	9.999758	74	9.999014
6	0.001138	29	0.000617	52	9.999719	75	9.998993
7	0.001129	30	0.000582	53	9.999679	76	9.998973
8	0.001118	31	0.000546	54	9.999640	77	9.998955
9	0.001106	32	0.000510	55	9.999602	78	9.998938
10	0.001093	33	0.000473	56	9.999566	79	9.998922
11	0.001078	34	0.000434	57	9.999527	80	9.998907
12	0.001062	35	0.000398	58	9.999490	81	9.998894
13	0.001045	36	0.000360	59	9.999454	82	9.998882
14	0.001027	37	0.000321	60	9.999418	83	9.998871
15	0.001007	38	0.000281	61	9.999383	84	9.998862
16	0.000986	39	0.000242	62	9.999349	85	9.998855
17	0.000964	40	0.000202	63	9.999316	86	9.998849
18	0.000941	41	0.000162	64	9.999284	87	9.998844
19	0.000916	42	0.000122	65	9.999253	88	9.998840
20	0.000891	43	0.000081	66	9.999222	89	9.998838
21	0.000864	44	0.000041	67	9.999192	90	9.998838
22	0.000836	45	0.000000	68	9.999164		

Table VII.

Logarithmic Coefficients.

Height of Lower Bar. in Feet.	Difference of the Heights of the Mercury in the Barometers in Inches.					
	0 Inch.	1 Inch.	2 Inches.	3 Inches.	4 Inches.	5 Inches.
0	4.779272	4.779509	4.779746	4.779984	4.780221	4.780 458
1000	779314	779551	779788	780026	780263	780500
2000	779355	779592	779829	780067	780304	780541
3000	779397	779634	779871	780109	780346	780583
4000	779438	779675	779912	780150	780387	780624
5000	779480	779717	779954	780192	780429	780666
6000	779522	779759	779996	780234	780472	780708
7000	779563	779800	780037	780275	780512	780749
8000	779605	779842	780079	780317	780554	780791
9000	779646	779883	780120	780358	780595	780832
10000	779688	779925	780162	780400	780637	780874
	6 Inches.	7 Inches.	8 Inches.	9 Inches.	10 Inches.	11 Inches.
0	4.780695	4.780933	4.781170	4.781407	4.781644	4.781881
1000	780737	780976	781212	781449	781686	781923
2000	780778	781018	771253	781490	781727	781964
3000	780820	781060	781295	781532	781769	782006
4000	780862	781101	781336	781573	781810	782047
5000	780903	781143	781378	781615	781852	782089
6000	780945	781185	781420	781657	781894	782131
7000	780986	781226	781461	781698	781935	782172
8000	781028	781268	781503	781740	781977	782214
9000	781069	781309	781544	781781	782018	782255
10000	781111	781351	781586	781823	782060	782297

P. P. for H +	100 4	200 8	300 12	400 17	500 21	600 25	700 29	800 33	900 37
P. P. for B +	0.1 24	0.2 47	0.3 71	0.4 95	0.5 110	0.6 142	0.7 166	0.8 190	0.9 214

EXPLANATION OF THE BAROMETRIC TABLES.

1. Table I. contains the depression of mercury in glass tubes, arising from capillary action, and must always be added to the observed height of the mercury in the barometer in those instruments which, from their construction, require it.

2. Table II. contains the corrections arising from temperature, in order to reduce the observed height of the mercury within a moderate range, to what it would be at the freezing-point, and when above that point it is always to be subtracted, but when below it must be added to get the true height at 32° Fahrenheit. When the temperature is below 32°, subtract it from 32°, and the difference added to 32° will be the argument above 32°, to obtain the correction to be added to reduce the height of the mercury in the barometer to that point of temperature when Part I. only is employed, which for general purposes is sufficiently correct.

Part II. was deduced from the following formula:—

Let the observed height on the scale of brass be represented by β, the dilatation of mercury for one degree of the thermometer by Δ, the dilatation of brass for one degree of temperature also by δ, the normal temperature, (the freezing-point), to which all heights of the barometer are to be reduced by τ, then it will be found, that to reduce the height of the barometer, at any temperature t common to the scale and mercury, the correction must be

$$C=\beta\,\frac{\Delta(t-\tau)-\delta(t-\theta)}{1+\Delta(t-\tau)} \qquad . \qquad . \qquad \text{(C)}$$

in which θ is the standard temperature, 62° Fahrenheit, at which the brass scale shows English inches.

The table was calculated on the supposition, that the dilatation from the freezing to the boiling points were

Brass . . 0.0018782, Mercury . . 0.018018*

3. Table III. contains the expansion of mercury in volume, for the purpose of reducing the upper and lower barometers to the same temperature, as shown in the computation of heights by the barometer.

4. Table IV. contains the elastic force of aqueous vapour, and its use is shown in the computations.

Correction for Pressure and Temperature.

Let p be the actual barometric pressure, f the tabular elastic force, t the temperature, and d the dew-point; then if f' be the correct elastic force, $f'=\frac{pf}{30}\left(1+\frac{t-d}{448+d}\right)$

5. Table V. contains the expansions of dry air according to the hypothesis of Dalton, which, from the computations that I have made, seems to give accurate barometric heights. Should any computer prefer that of Dulong and Petit, of an equable expansion, there is given a column for that purpose.

6. Table VI. contains the effects arising from a variation of the force of gravity according to the latitude on the mercurial column.

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* Hence, on this supposition, $E=\frac{0.0001004\,(t-32°)-0.0000098\,(t-62°)}{1+0.0001004\,(t-32°)}$, which would be rather more accurate than Table III. for mercury alone, when very great nicety is required.

7. Table VII. contains the logarithmic coefficients deduced from the principles previously explained according to the situations of the barometers relative to one another, and that of the lower above the level of the sea.

The usual formulæ given by Roy, Shuckburgh, and Laplace may give the height as near the geometrical method in certain cases, such as in a mean state of the atmosphere, as that which we have given, though there is no doubt but that the circumstances which have induced us to give a new method, involving considerations not usually attended to in such measurements, are more conformable to the laws of nature, and will in time become more accurate as those branches of physical science on which they depend are rendered more perfect.

The dew-point is supposed to be found by Daniell's hygrometer. If that instrument is not at hand, the dew-point may be found by two good thermometers, one of which has its ball covered with moistened tissue-paper, as proposed by Mr Anderson, Rector of the Academy of Perth, who also gives a formula for the barometric measurement of altitudes, in which in some of the corrections I have been anticipated.

Let F, the elastic force of vapour by Table IV. be thus reduced to f according to the difference between the naked and covered thermometers, then $f = F - \frac{0.028\delta t \times p}{30} = F - 0.00092\delta t \times p$, in which δt is the difference between the temperatures of the thermometers, and p the barometric pressure.

Now let φ be the elastic force at the dew-point, then

$$\varphi = \frac{f}{1+0.002084(t-d)} = \frac{F-0.00092p\,\delta t}{1+0.0021(t-d)} \text{ nearly} \quad . \quad . \quad (1.)$$

Here d, the temperature of the dew-point, is unknown, but may be determined, first approximately from the numerator of the formula, and then substituted in the denominator, and a second approximation obtained, which will generally be sufficiently correct.

To exemplify this, let the thermometer with the dry ball show 60° F, and that covered with moistened tissue-paper . . $51\frac{1}{2}$

T—t or δt. . . . . $8\frac{1}{2}$

Now if the barometer be at 30.1 inches we have from the numerator of formula (1) $f = 0.524 - 0.00092 \times 8\frac{1}{2} \times 30.4 = 0.524 - 0.238 = 0.286$. This f corresponds, by the table of Dalton, to 42° nearly, which being substituted for d in the denominator of the formula gives $\varphi = \frac{0.286}{1+0.0021(60-42)} = \frac{0.286}{1.0378} = 0.2756$, which finally gives $d = 41°.3$, the dew-point. This is perhaps one of the best methods of determining the point of deposition, as the instruments are not, like the hygrometers of Deluc and Saussure, liable to be deteriorated by time, and, besides, may still answer other purposes which none of the usual hygrometers can.

Cor.—From the same principles may be derived a formula to determine the weight of moisture in 100 cubic inches of air, or $W = \frac{0.6854\,\varphi}{1+0.0021\,(d-32)}$ at the freezing-point. When $\varphi = .2756$ and $d = 41.3$, we get from the expression $W = 0.1837$ grains when the air is completely saturated with humidity. But when the temperatures

are 60° and 41° the $W=\frac{0.1837}{1+0.0021\,(60-41)}=0.1767$ grains in 100 cubic inches. Perhaps this method may be conveniently compared with Mr Daniell's, to show their relative accuracy and consistency.

It may be added, that Mr Dalton states from experiments at moderate heights, that an elevation of 240 feet gives a depression of 1° temperature Fah. and an elevation of 390 feet gives a depression of 1° F. of the dew-point. Hence, if t be the temperature and D the dew-point

$$\Delta t=\frac{\Delta H}{240}, \text{ and } \Delta D=\frac{\Delta H}{390}$$

To determine the height from the given temperature. Let h be the height in feet, and n the change of temperature in degrees of Fahrenheit's thermometer, then $h=\{251.5+[illegible]\,(n-1)\}\,n$, . (1.)

also $n=\frac{h}{251.5+0.005\,h}$ very nearly, . . . (2.)

If t be the temperature, and λ the latitude,

$t=97°.08 \cos^{\frac{5}{8}} \lambda-\left(10°.53+\frac{h}{251.5+0.005\,h}\right)$ . . (3.)

This last formula agrees very well with the continent of America where the latitude is considerable, but gives results generally too great near the equator.

In Europe and Asia $t=81°.5 \cos \lambda$ very nearly, . . (4.)

Method II.

For ordinary heights, such as those usually met with in Britain, the following method, requiring no tables, and easily recollected, is subjoined.

Let B be the barometric altitude at the lower situation, and b that at the upper corrected for the difference of temperature in the usual manner, the atmosphere being in its mean state with regard to aqueous vapour, &c.

Then $H=13100\,\frac{(B+b)(B-b)}{B\,b}\left\{1+0.00244\left(\frac{t+t'}{2}-32°\right)\right\}=$

$\left\{12100+16(t+t')\right\}\frac{(B+b)(B-b)}{B\,b}$ in feet nearly.

	Bar. In.	At. Ther.	Det. Ther.
Ex.—Leith Pier,	29.567	55°¼	54°
Arthur's Seat,	28.704	51 ¾	50 ½

$28.704\times 0.0001\times 3.5=0.010$, and $28.704+0.010=28.714=b$

$12100+(54°+50°.5)16=104.5\times 16+12100=13772$

$H=13772\times\frac{58.281\times 0.853}{29.567\times 28.714}=$ . . . . 806 feet.

Height by levelling 803

Difference . . . . . 3 feet.

Method III.

Mr Alexander Adie, optician in Edinburgh, has lately invented a very convenient and delicate instrument called a sympiesometer, to supply with advantage, in several cases, the mercurial barometer. It consists of a glass tube about 18 inches long, terminated above with a bulb filled with hydrogen gas, and having the lower extrem-

ity bent upwards, and expanding into an oval cistern, open at bottom like the ordinary barometer, and filled with almond-oil coloured red. Close beside this, there is placed, on the right side, a fixed scale, to which a slide is adapted for the purpose of reading the height of the barometer, and also a graduated scale of fathoms for ascertaining heights. On the opposite side is a delicate thermometer to observe the temperature, and the instrument is used in the following manner:—

Observe the temperature by the thermometer, and set the index upon the sliding scale opposite to the degree of temperature upon the fixed scale, then the height of the oil indicated on the sliding scale will be the pressure of the atmosphere in inches of mercury. By the fathom scale also, by subtracting the number of fathoms indicated at the under station from that indicated at the upper, the height of one place above another will be found. This must be corrected for the difference between the mean temperature of the two stations and 32°, or the factor by which the approximate height must be multiplied according to the sum of the two temperatures, may be taken from the following table:—

$t+t'$	Factor.
50°	0.983
60	0.995
70	1.007
80	1.020
90	1.032
100	1.044
110	1.056
120	1.068
130	1.081
140	1.093
150	1.105
160	1.117
P. P.	2° 4° 6° 8°
+	2 4 7 10

Ex. At the top of Arthur's Seat, the sympiesometer stood at . . . 244 fath. 48° tem.

At the bottom at .	134	52
Approximate height,	110	100
Factor to $t+t$=100	1.044	
True height, . .	114.840 fathoms	
Above the Duke's Walk, . .		

EXAMPLES BY THE FRENCH MEASURES.

Observer Humboldt.	Height of Barometer.	Attached Thermometer.	Detached Thermometer.	Dew-Point.	Latitude.
Quindiu, Pac. Oc.	0m.509818 0 .762944	20°.0 cent. 25 .3	18°.75 cent. 25 .30	16°.0 cent. 20 .0	5° 0′ N. H=3543m
Chimb. Pac. Oc.	0m.377275 0 .762000	10°.0 cent. 25 .3	—1°.6 cent. +25.3	0°.0 cent. 20 .0	1° 45′ N. H=5925m

Calculation of the last Example by Method I.

Constant 18393 metres log. . . .	4.264653
$(1.375)^{\frac{11.85}{100}} = \frac{11.85}{100} \times 0.138303 =$	0.016389
Latitude 1° 45′ log. . . .	0.001161
$B - \frac{1}{10}f = 0.759114$ log. 1.880307	
$b - \frac{1}{10}f = 0.377471$ log. 1.576884	
Difference 0.303423 log. . .	9.482048
$1 + \frac{f+f'}{B+b} = 1 + \frac{0.022373}{1.136585} = 1.019686$ . . .	0.008467
H=5925.4 metres . . .	3.772718

Or 19441 English feet, the height of Chimborazo above the level of the Pacific Ocean.

PART II.

SPHERICAL TRIGONOMETRY.

SECTION I.

Definitions, Principles, and General Properties.

1. *Spherical Trigonometry* is that branch of mathematics by which we are enabled, *in all cases*, where three of the six parts of a triangle formed by arcs of great circles in the surface of a sphere are given, to compute or determine the other three.

2. In *plane* trigonometry the knowledge of the three angles is not sufficient for ascertaining the sides; for in that case the *relations* only of the three sides can be obtained, and not their value; whereas, in *spherical* trigonometry, when the sides are circular arcs, whose value depends on their proportion to the whole circle, that is, on the number of degrees they contain, the sides may always be determined when the three angles are known. Among other remarkable differences between plane and spherical triangles are,

(1.) That in the former, two known angles always determine the third; while in the latter they never do.

(2.) The surface of a plane triangle cannot be determined from a knowledge of the angles alone; while that of a spherical triangle always can.

3. A *sphere* or *globe* is a round body formed by the revolution of a semicircle about its diameter, which remains fixed.

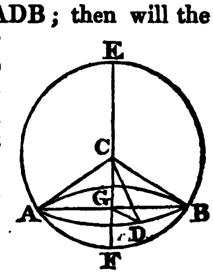
4. The *centre* of the sphere is the same with that of the revolving semicircle.

5. The *axis* of the sphere is the straight line about which the semicircle relolves.

PROPOSITION I.

6. If a sphere be cut by a plane, the section will be a circle.

Let the sphere AEBF be cut by the plane ADB; then will the section ADB be a circle. Draw the chord, or diameter of the section AB, perpendicular to the section ADB, and through the centre C draw the axis of the sphere ECGF, which will (Euc. III. 3.) bisect the chord AB in the point G. Also join CA, CB; and draw CD, GD, to any point D in the perimeter of the section ADB.



Then, because CG is perpendicular to the plane ADB, it must be perpendicular both to GA and GD. Hence CGA and CGD are two right-angled triangles, having

the perpendicular CG common, and the hypotenuse CA equal to the hypotenuse CD, being both radii of the same sphere; therefore their sides GA, GD, are also equal. In like manner, it may be shown, that any other line drawn from G to the circumference of the section ADB, is equal to GA, or GB, and consequently that section is a circle.

Cor.—If a sphere be cut by a plane through the centre, the section is a circle, having the same centre with the sphere, and equal to the circle by the revolution of the half of which the sphere was described. For all the straight lines drawn from the centre to the surface of the sphere are equal to the radius of the generating semicircle. Therefore the common section of the spherical surface, and of a plane passing through its centre, is a line lying in one plane, having all its points equally distant from the centre of the sphere, and is consequently the circumference of a circle, having for its centre the centre of the sphere, and for its radius the radius of the sphere, that is, of the semicircle by which the sphere is described. It is therefore equal to the circle of which that semicircle is a part.

7. Any circle formed from the section of a sphere, by a plane through its centre, is called a *great circle* of the sphere.

Cor.—All great circles of the sphere are equal, and any two of them bisect each other.

They are all equal, because they have all the same radii, as has just been shown, and any two of them bisect one another; for, as they have the same centre, their common section is a diameter of both, and therefore bisects both.*

8. The *pole* of a great circle of the sphere is a point in the surface of the sphere equidistant from every part of the circumference of that circle.

9. A *spherical angle* is an angle on the surface of a sphere contained by the arcs of the two great circles which intersect each other, and is the same as the inclination of the planes of, or tangents at the point of intersection to, these great circles.

10. A *spherical triangle* is a figure on the surface of a sphere formed by the intersection of three arcs of great circles, each of which is less than a semicircle.

11. A *right-angled* spherical triangle has one right angle; the sides about the right angle are called *legs*, and that opposite the right angle is called the *hypotenuse*.

12. A *quadrantal* spherical triangle has one side equal to a quadrant, or 90°.

13. An *oblique*-angled spherical triangle has none of its angles right.

14. Spherical triangles are also called *equilateral*, *isosceles*, or *scalene*, according as they have three sides equal, two sides equal, or all the three sides unequal.

15. Two arcs, or angles, when compared together, are said to be *alike*, or of the *same affection*, when both are less or both are greater than 90°. But when one is less and the other greater than 90°, they are said to be *unlike*, or of *different affections* or *characters*.

16. Every spherical triangle has three sides and three angles;

* Hence the intersections of the circumference of two great circles are two points diametrically opposite to each other.

and if any three of these six parts be given, the other three may be found.

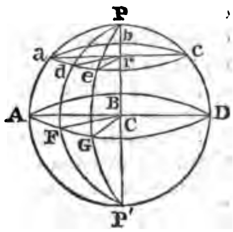
17. A *lune* is a part of the surface of a sphere contained by the semicircumferences of the two great circles.

18. A *small circle* of the sphere is that whose plane does not pass through the centre of the sphere.

19. The small circles of the sphere do not fall under the consideration of spherical trigonometry, but such only as have the same centre with the sphere itself. And hence it is that spherical trigonometry is of so much use in practical astronomy, the apparent heavens assuming the shape of a concave sphere whose centre is the same as the centre of the earth.

20. The *sides* of a spherical triangle are all arcs of great circles, which, by their intersection on the surface of a sphere, constitute that triangle.

21. If ABDG be a great circle of the sphere whose centre is C and PCP′ a diameter of the sphere perpendicular to its plane, the points P, P′ are the poles of that circle. And if the small circle $a, b\ c\ d$ be perpendicular to PP′, we call P, P′ the poles of that small circle also.

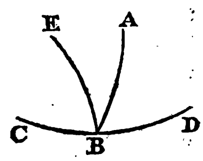


22. The great circles PAP′, PGP′ passing through the poles P, P′ of the great circle ABDG, are called *secondaries* to that circle.

Proposition II.

23. If two arcs of circles meet each other, they make two angles, which are together equal to two right angles.

Let the arc AB meet the arc CD in the point B; then will the two angles ABC, ABD be equal to two right angles. For, suppose the arc BE to be perpendicular to CD, then the angles EBC, EBD are right angles.



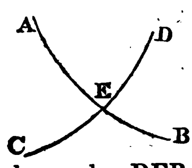
And since the angle EBD is equal to the angles EBA, ABD, the three angles, EBC, EBA, ABD, are equal to the two right angles.

But the two angles EBC, EBA, are equal to the angle ABC; whence the two angles, ABC, ABD, are also equal to two right angles.

Proposition III.

24. If two arcs of a circle intersect each other, the vertical or opposite angles will be equal.

Let the two arcs, AB, CD, intersect each other in E, then will the angle AEC be equal to DEB, and AED to CEB.



For since the arc AE meets the arc CD, the angles AEC, AED are together equal to two right angles, (Prop. II.)

And because the arc DE meets the arc AB, the angles DEB, DEA are also equal to two right-angles.

Taking away from each the common angle AED, and the re-

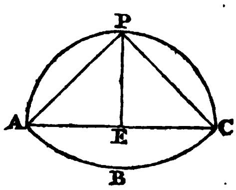
maining angle, AEC will be equal to DEB. In the same manner it may be proved that the angle AED is equal to CEB.

Cor.—Hence if any number of arcs of circles intersect each other, all the angles formed about the point of intersection are together equal to four right angles.

Proposition IV.

25. The arc of a great circle, between the pole and the circumference of another great circle, is a quadrant.

Let ABC be a great circle, and P its pole; if PC, an arc of a great circle, pass through P and meet ABC in C, the arc PC is a quadrant.



Let the circle, of which PC is an arc, meet ABC again in A, and let AC be the common section of the planes of these great circles, which will pass through E, the centre of the sphere: Join PA, PC.

Because AP=PC, (def.), and equal straight lines in the same circle, cut off equal arcs, the arc AP = the arc PC; but APC is a semicircle, therefore the arcs AP, PC, are each of them quadrants.

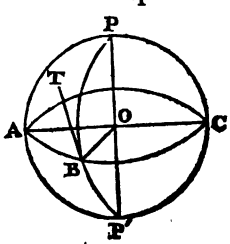
Cor. 1. If PE be drawn, the angle AEP is a right angle; and PE, being at right angles to every line it meets with in the plane of the circle ABC, is at right angles to that plane. Therefore the straight line drawn from the pole of any great circle to the centre of the sphere is at right angles to the plane of that circle; and, conversely, a straight line drawn from the centre of the sphere perpendicular to the plane of any great circle meets the surface of the sphere in the pole of that circle.

Cor. 2. The circle APC has two poles, as has been shown in art. 21., one on each side of its plane, which are the extremities of a diameter of the sphere perpendicular to the plane APC; and no other points but these can be poles of the circle APC.

Proposition V.

26. If the pole of a great circle be the same with the intersection of other two circles, the arc of the first circle intercepted between the other two, is the measure of the spherical angle which the same two circles make with one another.

Let the great circles AP, BP, on the surface of the sphere of which the centre is O, intersect each other in P, and let AB be an arc of another great circle of which the pole is P, AB is the measure of the spherical angle APB.



Join PO, AO, BO; since P is the pole of AB, PA, PB are quadrants, and the angles POA, POB are right; therefore the angle AOB is the inclination of the planes of the circles PA, PB, and is equal to the spherical angle APB; but the arc AB measures the angle AOB, therefore it also measures the spherical angle APB.

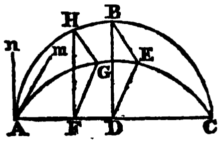
Cor. If two arcs of great circles, PA, PC, which intersect each other in P, be each of them quadrants, P will be the pole of the great circle which passes through A and B, the extremities of those

arcs. For since the arcs PA and PB are quadrants, the angles POA, POB are right angles, and PO is therefore perpendicular to the plane AOB, that is, to the plane of the great circle which passes through A and B. The point P, therefore, is the pole of the great circle which passes through A and B.

Proposition VI.

27. An angle made by any two great circles of the sphere is equal to the angle of inclination of the planes of these circles.

Let BAE be a spherical angle made by two great circles CBA, CEA; then will this angle be equal to the angle of inclination of the planes of those circles. For, take the arcs AB, AE, each equal to 90°, or a quadrant, and through the points B, E draw the arc of the great circle BE, and from D, the centre of the sphere, draw DB, DE.



Then, because AB, AE are quadrants, A and C are the poles of the circle of which BE is a part, and the lines DB, DE are each perpendicular to the common section AC; consequently BDE is the angle of inclination of the planes CBA, CEA. But since DB, DE are equal, being radii of the same sphere, the angle BDE, which is measured by the arc BE, is equal to the angle BAE, which is measured by the arc BE, is equal to the angle BAE, which is measured by the same arc.

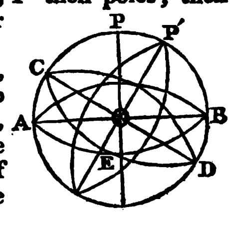
And if FH be drawn in the plane CBA, and FG in the plane CEA, each perpendicular to the common section AC, the angle HFG, which is equal to the angle BDE, will also be equal to the angle BAE.

Cor. The angle BAE made by two great circles of the sphere BA, EA, is equal to the angle *n* A *m*, formed by two tangents drawn from the angular point A, one in each plane, these tangents being each perpendicular to the diameter AC.

Proposition VII.

28. The distance of the poles of any two great circles of the sphere is equal to the angle of inclination of the planes of those circles.

Let AEB, CED be two great circles, and P, P′ their poles; then will the arc PP′ be equal to the angle of their inclination AOC or BOD.



For, since P is the pole of the circle AEB, and P′ of CED, the arc PA will be equal to PC, being each quadrants, or 90°; and if PC, which is common to each, be taken away, the remaining arc, PP′, which is the distance of two poles, is equal to CA, the measure of the angle of inclination AOC.

Proposition VIII.

29. The circumference of a secondary is at right angles to the circumference of its great circle at the point of intersection.

The direction of the circumference of a great circle at any point being the same as the direction of its tangent at that point, the angle OBT, (figure prop. V.), is a right angle, BT being a tangent to BP

at the point B. POB is also a right angle, and the arc PB is in the plane POB, therefore the direction of the circumference PB at B must be parallel to PO. But PO is perpendicular to the circle ABC; therefore the circle PBP′ is at B perpendicular to the circle ABC; hence the arc PB at B is at right angles to AB at B. For the same reason PAB is also a right angle.

Cor. 1.—If a great circle, PBP′, be perpendicular to ABC, and BP, BP′ be taken each equal to a quadrant, or 90°, P, P′ are the poles of the circle ABC.

Cor. 2.—If any two great circles, PAP′, PBP′, be perpendicular to the circle ABC, they meet at the poles P, P′ of that circle.

Proposition IX.

30. In an isosceles spherical triangle the angles at the base are equal.

Let ABE (figure prop. VI.) be a spherical triangle, having the side AB equal to the side AE, the spherical angles ABE, ABE are equal.*

Cor. 1.—Hence, if two of the angles of a triangle be equal, the sides opposite to them are likewise equal.

Cor. 2.—A perpendicular drawn from the vertex of an isosceles spherical triangle to the base, bisects both the base and the vertical angle, except when the two sides are quadrants; in which case there are an indefinite number of perpendiculars.

Proposition X.

31. If the three sides of one spherical triangle be equal to the three sides of another, each to each, the angles which are opposite the equal sides are equal.

Proposition XI.

32. If two sides and the included angle of one spherical triangle be equal to two sides and the included angle in another, these two triangles are equal.

Proposition XII.

33. If from the angles of a spherical triangle, as poles, there be described on the surface of the sphere three arcs of great circles, which, by their intersections, form another spherical triangle, each side of this new triangle will be the supplement of the measure of the angle which is at its pole, and the measure of each of its angles the supplement to that side of the primitive triangle to which it is opposite.

Proposition XIII.

34. If the three angles of one spherical triangle be equal to the three angles of another, each to each, the sides which are opposite to the equal angles are equal.

Proposition XIV.

35. If a side and two adjacent angles of one spherical triangle be

* The demonstrations, which may be seen in Playfair's or Legendre's Geometry, are omitted, as they would swell this work too much, but may perhaps appear in a more complete treatise on trigonometry that has been long meditated.

equal to a side and two adjacent angles of another, each to each, their remaining sides and angles will be equal.

Proposition XV.

36. The sum of any two sides of a spherical triangle is greater than the third side, and the difference of any two sides is less than the third side.

Cor.—The shortest distance between any two points on the surface of a sphere is the arc which passes through these points.

Proposition XVI.

37. The greater side of any spherical triangle is opposite to the greater angle, and the less side to the less angle.

And, in a similar manner, it may be shown that the less side is opposite to the less angle, and the less angle to the less side.

Proposition XVII.

38. The sum of the three sides of any spherical triangle is less than the circumference of a circle, or 360°; and the difference of any two sides is less than 180°.

Proposition XVIII.

39. The sum of the three angles of every spherical triangle is greater than two right angles, or 180°, and less than six, or 540°.

Cor.—The sum of any two angles of a spherical triangle is greater than the supplement of the third angle.

For the angles $A+B+C$, being greater than two right angles, or than $ACB+ACG$, if ACB or C be taken away, the sum of the remaining angles $A+B$, will be greater than ACG.

Proposition XIX.

40. If the sum of any two sides of a spherical triangle be equal to, greater, or less than a semicircle, the sum of their opposite angles will, accordingly, be equal to, greater, or less than two right angles, and conversely.

And, in a similar manner, it may be shown, that if the sum of the two angles B and C be equal to, greater, or less than 180°, the sum of the opposite sides, AB and AC, will also be equal to, greater, or less than 180°.

Cor. 1.—If each side of a spherical triangle be equal to, greater, or less than 180°, each of the angles will, accordingly, be right, obtuse, or acute, and conversely.

Cor. 2.—Half the sum of any two sides of a spherical triangle is of the same kind as half the sum of their opposite angles.

Proposition XX.

41. In any right-angled or quadrantal spherical triangle, the legs or sides are of the same kind or affection as their opposite angles, and conversely.

The same will also hold if the triangle be quadrantal; for its sides and angles being the supplements of the angles and legs of the polar triangle, which in this case is right-angled, the similarity will be the same as before.

PROPOSITION XXI.

42. In any right-angled spherical triangle the hypotenuse is less or greater than 90°, according as the two legs, or the two angles, or a leg and its adjacent angle, are alike or unlike.

SECTION II.

Solution of Spherical Triangles.

HAVING given a view of the general principles and properties of spherical triangles, the solution of the various problems in spherical trigonometry ought necessarily to follow. These problems may be resolved either by geometrical construction or by arithmetical calculation. There are various methods of construction, but the most simple and generally employed is the stereographic, in which all the circles of the sphere are represented by straight lines or circles.

Of the Stereographic Projection of the Sphere.

DEFINITIONS.

I. To *project* an object, as it is commonly called, is to represent every point of that object upon the same plane as it appears to the eye in a certain position.

II. That plane upon which the object is projected is called the *plane of projection*, and the point where the eye is situated, the *projecting point*.

III. The *stereographic projection* of the sphere is that in which a great circle is assumed as the plane of projection, and one of its poles as the projecting point.

IV. The *great circle*, upon the plane of which the projection is made, is called the *primitive*.

V. By the semitangent of any arc is meant the tangent of half that arc.

VI. The *line of measures* of any circle of the sphere is that diameter of the primitive, produced indefinitely, which is perpendicular to the line of common section of the circle and the primitive.

VII. The *projection*, or representation of any point in the sphere, is the point in which the straight line drawn from it to the projecting point intersects the plane of projection.

THEOREM I.

Every great circle of the sphere, which passes through the projecting point, is projected in a straight line, passing through the centre of the primitive; and every arc of it, reckoned from the other pole of the primitive, is projected into its semitangent.*

Cor. 1.—Every small circle, which passes through the projecting point, is projected into that straight line which is its common section with the primitive.

* For the investigation of the properties of this method of projection, see Gregory's or Keith's Treatises of Trigonometry, and West's Mathematics, published under the care of Professor Leslie.

Cor. 2.—Every straight line in the plane of the primitive, and produced indefinitely, is the projection of some circle on the sphere passing through the projecting point.

Cor. 3.—The stereographic projection of any point on the surface of the sphere, is distant from the centre of the primitive by the semitangent of the distance of that point from the pole opposite the projecting point.

Theorem II.

Every circle of the sphere, which does not pass through the projecting point, is projected into a circle.

Cor. 1.—The centres and poles of all circles parallel to the primitive have their projections in its centre.

Cor. 2.—The centre and poles of every circle, inclined to the primitive, have their projections in the line of measures.

Cor. 3.—All projected circles cut the primitive in two points diametrically opposite.

Theorem III.

The centre of the projection of a great circle is distant from the centre of the primitive by the tangent of the inclination of the great circle to the primitive, and its radius is the secant of the same.

Theorem IV.

The centre of projection of a small circle, perpendicular to the primitive, is distant from the centre of the primitive by the secant of the distance of the circle from its nearest pole, and the radius of projection is the tangent of the same.

Theorem V.

The projections of the poles of any circle inclined to the primitive, are in the line of measures distant from the centre of the primitive by the tangent and cotangent of half its inclination.

Theorem VI.

Any two circles upon the sphere, passing through the poles of two great circles, intercept equal arcs upon them.

Theorem VII.

If, from either pole of a projected great circle, two straight lines be drawn to meet the primitive and the projection, they will intercept corresponding arcs of these circles.

Solution of Right-angled Spherical Triangles.

The solution of right-angled spherical triangles may be accomplished by formulæ investigated expressly for that purpose. We are indebted to Napier, however, for a comprehensive rule of great advantage to the memory, by reducing all the theorems employed in the solution of right-angled triangles to two. This is called the *rule of the circular parts*, and is perhaps one of the happiest examples of artificial memory that is known.

Definitions.

1. If in a right-angled spherical triangle the right angle be set

aside, and the five remaining parts of the triangle alone be considered, consisting of the three sides, and the two oblique angles, then the two sides containing the right-angle, and the complements of the other three, namely, of the two angles, and of the hypotenuse, are called the *circular parts*.

II. When, of the five circular parts, any one is taken for the middle part, then, of the remaining four, the two which are immediately adjacent to it on the right and left are called *adjacent parts*; and the other two, each of which is separated from the middle part by an adjacent part, are called *opposite parts*.

This arrangement being made, the solution is obtained by the following

THEOREM.

In any right-angled spherical triangle, the rectangle under the radius, and the sine of the middle part is equal to the rectangle under the *tangents* of the *adjacent* parts; or to the rectangle under the COSINES of the OPPOSITE parts.

This theorem, or rule, may be easily remembered, by remarking, that the first vowels in *sine, tangent, cosine,* are respectively the same as the first in *middle, adjacent, opposite,*

or, R × s*i*n m*i*d = rect t*a*n *a*dj = rect c*o*s *o*p.*

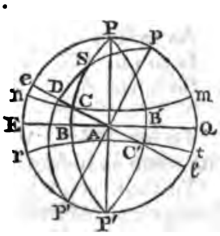
It is usual to convert the equation under consideration into an analogy having the unknown quantity for the last term, though to those acquainted with algebra it would be more convenient to make it alone the first term of an equation, and the remaining terms, combined properly according to the rules of algebra, the last.

PROBLEM I.

Given three of the six parts, as, for example, the hypotenuse and one of the angles of a right-angled spherical triangle, to find the sides and the remaining angle.

On the first of May, 1826, the sun's longitude was $1^s\ 10°\ 32'\ 12''$, and the obliquity of the ecliptic $23°\ 27'\ 40''$; required the right ascension and declination? †

Ans.—R. A. $2^h\ 32^m\ 27^s.3$; dec. $14°\ 59'\ 47''$ N.



Construction.—With the chord of 60° describe the primitive circle EPQP′ on the plane of the solstitial colure, and draw the diameters EQ and PP′ at right angles to one another, then will EQ represent the equator, and PP′ the polar axis. Lay off from the same line of chords E e=23° 27′ 40″, the obliquity of the ecliptic, and draw the diameter $e\,l$ representing the ecliptic, at right angles to which draw $p\,p'$, and p, p' are the poles of the ecliptic. From the line of semitan-

* Should either of the oblique angles, or hypotenuse, be one of the parts, then, instead of the word in the formula, use that derived from its complement, that is, for *sine* read cosine, for *cosine* read sine, and so on.

† For the explanation of these terms, the usual treatises on astronomy may be consulted. To those acquainted with the use of the globes correct ideas relative to these problems may be readily obtained. It may be added, that the sun's longitude and the obliquity of the ecliptic are computed from astronomical tables.

gents, (Theorem I.), lay off the sun's longitude $1^s\ 10^\circ\ 32'\ 12''$, or $40^\circ\ 32'.2$ on the ecliptic, from A to C, then C will be the place of the sun, and by laying off from E to *n*, and from Q to *m* from the line of chords, extents equal to the sun's declination, then will the circle *n c m* passing through these points be a parallel of declination. Through the points PCP′ draw a circle of right ascension, cutting the equator EQ at right angles in B, then will AB be the right ascension, BC the declination, and BCA the remaining angle or angle of position, as it is sometimes called, which, in astronomy, is seldom of much use.

Calculation.—In the triangle ABC there are given AC $= 40^\circ\ 32'\ 12''$, and the angle BAC $= 23^\circ\ 27'\ 40''$, to find BC, the distance of the sun from the equator EQ, or the declination, as it is usually called. Now, since in spherical trigonometry the *sines* of the *sides* are proportional to the *sines* of their *opposite angles*,

Therefore,

As sine ABC or radius		10.000000
Is to sine BAC	23° 27′ 40″	9.600021
So is sine AC	40 32 12	9.812870
To sine BC	14 59 47	9.412891

To find AB we may employ the method of the circular parts.

In the triangle ABC are given AC and the angle BAC, to find AB the right ascension. Now, since the side CA, the angle CAB, and the side AB are all connected, that which stands in the middle or the angle A is called the middle part, and the sides AC and AB adjacent to it on each side are called the adjacent parts.*

Consequently $R \times \cos A = \cot AC \times \mathit{tan}\ AB$; and resolving this into an analogy, as is frequently done in this country, we have,

As cot AC	40° 32′ 12″	10.067939
Is to radius		10.000000
So is cos A	23 27 40	9.962526
To tan AB	$2^h\ 32^m\ 27^s.3$	9.894587

or, since cot : R : : R : tan, or $\tan = \frac{1}{\cot}$ to radius unity (§ 35, page 11.)

As radius		10.000000
Is to tan AC	40° 32′ 12″	9.932061
So is cos A	23 27 40	9.962526
To tan AB	$2^h\ 32^m\ 27^s.3$	9.894587

the same as before.

To those acquainted with algebra, it is better, after the manner of foreign mathematicians, still to retain the form of an equation thus:

$\tan AB = \frac{R \times \cos A}{\cot AC} = \cos A \times \tan AC$, the radius being represented by unity; in which case ten must be rejected in the index.

To log cos A	23° 27′ 40″	9.962526
Add log tan AC	40 32 12	9.932061
Sum tan AB	$2^h\ 32^m\ 27^s.3$	9.894587

* It may be remarked, that if the parts are all connected, that which stands in the middle is called the middle part, and the other two are called the adjacent parts. If two only are connected, and one stands by itself, then this is called the middle part, and the other two are called the opposite parts.

To find the angle ACB, since the parts under consideration are still all connected, AC standing in the middle is assumed as the middle part, and the angles A and C are the adjacent parts, whence $R \times \cos AC = \cot A \times \cot C$, and $\cot C = \frac{\cos AC}{\cot A} = \cos AC \times \tan A$, hence

To log cos AC	40° 32′ 12″	9.880808
Add log tan A	23 27 40	9.637496
Sum = cot C	71 44 42 .2	9.518304

Or the comp 18 15 17 .8, is called properly the angle of position, sometimes useful in computing the parallaxes in solar eclipses and occultations of the fixed stars and planets by the moon.

By assuming different parts of the triangle ABC for the middle part, may be resolved the following

Examples for Exercise.

1. On the first of June, 1827, at noon on the meridian of Greenwich, the sun's longitude will be 2^s 10° 9′ 45″, the obliquity of the ecliptic 23° 27′ 36″; required the right ascension and declination?
Ans.—R. A. 4^h 34^m $7^s.6$; Dec. 21° 59′ 34″ N.

2. August 12th, 1827, the obliquity of the ecliptic being 23° 27′ 36″, the sun's right ascension will be 9^h 25^m $29^s.3$; required his longitude and declination?
Ans.—Longitude 4^s 18° 56′ 28″, Dec. 15° 9′ 32″ S.

3. On the 10th November, 1828, on the meridian of Greenwich, the sun's right ascension will be 15^h 2^m $32^s.7$, and declination 17° 14′ 12″ S.; required the sun's longitude and the obliquity of the ecliptic?
Ans.—Longitude 7^s 18° 6′ 7″, and obliquity of the ecliptic 23° 27′ 34″.

4. On the 2d of March, 1828, when the sun's declination was 7° 5′ 18″ S., and the obliquity of the ecliptic 23° 27′ 35″; required his longitude and right ascension?
Ans.—Longitude 11^s 11° 56′ 34″; R. A. 22^h 53^m 24^s.

Problem II.

When the celestial object is not upon the ecliptic, as the moon, or the planets, and some of the fixed stars, the right ascension and declination are found by the solution of two right-angled triangles.

1. On the 17th of January, 1826, at noon, on the meridian of Greenwich, the moon's longitude was 1^s 11° 5′ 14″, and her latitude 2° 34′ 3″ N.; required her right ascension and declination, the obliquity of the ecliptic being 23° 27′ 40″? To resolve this example it is necessary to employ two right-angled spherical triangles.

In the foregoing figure, the longitude of the moon or any star S, is AD, the latitude DS, the obliquity of the ecliptic BAC, the right ascension AB and the declination BS. Now, supposing a line drawn from A to S, there would be formed the right-angled spherical triangle ADS, right-angled at D, of which AD and DS are given to find the angle DAS and the side AS. If the position S of the star is *without* the ecliptic, then to the obliquity of the ecliptic BAC, add the angle DAS, the *sum* will be the angle BAS; but if S is *within* the ecliptic, that is, between it and the equator, *subtract* the angle

DAS from the obliquity BAC, and the remainder will be the angle BAS. Since the side AS, and the angle BAS, are now known, AB the right ascension, and BS the declination, may be found.

Calculation.—By the rule of the circular parts, first AD and DS are given to find AS, and since the last is separated from the two first by the oblique angles, it will be the middle part, and AD and DS are the opposite parts; therefore, R × cos AS=cos DS × cos AD, or cos AS=cos DS × cos AD to radius unity.

To log cos DS	2°	34′	3″	. . .	9.999564
Add log cos AD	41	5	14	. . .	9.877204
Log cos AS	41	9	11	. . .	9.876768

Again, to find DAS, since the right angle does not separate the parts, DA standing in the middle is called the middle part, and the side DS and the angle DAS are the adjacent parts, hence R × sin DA = tan DS × cot DAS, and therefore, cot DAS $=\frac{\text{sin DA}}{\text{cot DS}}=$ sin DA × cot DS, consequently

To log cot DS	2°	34′	3″	. . .	11.348322
Add log sine DA	41	5	14	. . .	9.817702
Sum=log cot DAS	3	54	12	. . .	11.166024
To this add Ob. Ec.	23	27	40		
Sum=angle BAS	27	21	52		

Hence AS and BAS are now known, to find AB and BS.

First to find AB. In this case the parts are connected; therefore BAS is the middle part, and AB and AS are the adjacent parts, whence

R × cos BAS = tan AB × cot AS, or tan AB= $\frac{\text{cos CAS}}{\text{cot AS}}$, and tan AB=cos BAS × tan AS, hence

To log cos BAS 27° 21′ 52	. . .	9.948462
Add log tan AS 41 9 11	. . .	9.941505
Sum=log tan AB 37° 49′ 5.″5	. . .	9.889967
Or in time R. A. 2^h 31^m $16^s.4$		

To find BS, the angle BAS and side AS are connected, and BS is disjoined, whence R × sin BS = sin AS × sin BAS, or since the sines of the sides are proportional to the sines of their opposite angles.

As sine ABS or radius	. . . . . .	10.000000
Is to sine AS 41° 9′ 11″	. . . . .	9.818274
So is sine BAS 27 21 52	. . . .	9.662426
To sine Dec. BS 17 36 24.″4 N	. .	9.480700

The foregoing method is general and applicable to any part of the ecliptic, provided proper attention be paid to the situation of the celestial object with respect to the ecliptic and equator. As this problem and its converse is of frequent occurrence in practical astronomy, rules and formulæ, and even tables, have been formed for the purpose of facilitating the computations. The following rules, given

by the late Dr Maskelyne, will be found very convenient for this purpose.

Problem II.

Given the right ascension, the declination, and the obliquity of the ecliptic, to find the longitude and latitude.

Let RA denote the right ascension, O the obliquity of the ecliptic, and D the declination; then using logarithms:

Tan D—sin RA=tan A, North or South as the declination is.

Call O in the first six signs of RA South or S., and in the last six signs North or N.

Then A+O=B, regard being had to the algebraic signs.

A being less than 45°, and using logarithms.

Sec A+cosec B+tan RA=tan lon of the same kind as RA, unless B be more than 90°, when the quantity found of the same kind as RA must be taken from twelve signs.

A being more than 45°.

Tan A+cosec A+cos B+tan RA=tan lon of the same kind as RA, unless B be more than 90°, when the quantity found of the same kind as RA must be taken from twelve signs.

Lon being nearer III. and IX. signs than O and VI. signs.

Sin lon+tan B=tan lat of the same name as B.

Lon nearer O and VI. signs, than III. and IX. signs.

Tan lon+cos lon+tan B=tan lat of the same name as B.

Example.

On Monday the 12th of June, 1826, the moon's R A at noon was found by observation to be $10^h\ 39^m\ 31^s$ and her declination 2° 51′ 58″ N.; required her longitude and latitude?

D=	2° 51′ 58″ N.	tan 8.699533				
RA=	$10^h\ 39^m\ 31^s$	sine 9.536560	tan 9.563908			
A	8° 16′ 50″ N.	tan 9.162973	sec 0.004551			
O	23 27 40 S.					
B	15 10 50 S.	cos . .		9.984575	tan	9.433497
Lon	160 20 17	tan . .		9.553034	sine	9.526946
Lat	5 12 59 S.	. . . . .			tan	8.960443

Problem III.

Given the longitude and latitude of a celestial object, and the obliquity of the ecliptic; to find the right ascension and declination.

Tan lat—sin lon=tan A, North or South as the latitude is.

Call O north in the six first signs, and South in the six last signs.

A+O=B, as before.

A being less than 45°, sec A+cos B+tan lon=tan RA of the same kind as the longitude, unless B be more than 90°, when the quantity found of the same kind as the longitude must be subtracted from twelve signs.

A being more than 45°, tan A+cosecant A+cos B+tan lon= tan RA of the same kind as the longitude, unless B be more than 90°, when the quantity found of the same kind as the longitude must be subtracted from twelve signs.

If RA be nearer III. signs and IX. signs, than O and VI. signs, sine RA+tan B=tan Dec of the same name as B.

And RA being nearer O and VI. signs, than III. and IX. signs, tan RA+cos RA+tan B=tan Dec of the same name as B.*

Example.

On the 1st of January, 1820, the mean longitude of the Star Fomalhaut was 11ˢ 1° 19′ 34″, the mean latitude 21° 6′ 45″ S.; required the right ascension and declination, the obliquity of the ecliptic being 23° 27′ 46″?

Lat	21°	6′	45″	S.	tan	9.586721				
Lon	331	19	34		sine	9.681082	tan	9.737901		
A=	38	49	26	S.	tan	9.905639	sec	0.108420		
O=	23	27	46	S.						
B=	62	17	12	S.	cosine	. .		9.667498	tan	10.279585
RA=	341	55	14		tangent	. .		9.513819	sine	9.491831
Dec	30	34	21	S.	. . . . . .				tan	9.771416

Examples for Exercise.

1. The mean longitude of α Arietis, on the 1st January, 1820, was 1ˢ 5° 8′ 48″, and mean latitude 9° 57′ 34″ N. when the obliquity of the ecliptic was 23° 27′ 46″; what was the right ascension and declination?

Ans.—R. A. 1^h 57^m 3^s; Dec 22° 36′ 24″ N.

2. Required the right ascension and declination of Pollux, when the longitude was 3ˢ 20° 43′ 58″, the latitude 6° 40′ 17″ N. the obliquity of the ecliptic being 23° 27′ 46″?

Ans.—R. A. 7^h 34^m 17.5^s; declination 28° 27′ 8″ N.

3. The mean longitude of Spica Virginis is 6ˢ 21° 19′ 50″, latitude 2° 2′ 24″ S. and the obliquity of the ecliptic 23° 27′ 46″; required the right ascension and declination?

Ans. R. A. 13^h 15^m 43.5^s; declination 10° 13′ 4″ S.

4. The mean right ascension of α Aquilæ is 19ʰ 42ᵐ, and declination 8° 24′ 4″ N. the obliquity of the ecliptic being 23° 27′ 46″; required the longitude and latitude?

Ans.—Longitude 9ˢ 29° 14′ 14″, Latitude 29° 18′ 36″ N.

5. Required the longitude and latitude of α Pegasi, of which the right ascension is 22^h 55^m 48^s, declination 14° 14′ 24″, the obliquity of the ecliptic being 23° 27′ 46″?

Ans.—Longitude 11ˢ 20° 58′ 47″, Latitude 19° 24′ 36″ N.

Problem IV.

Given the latitude of the place, and the sun's declination, to find his altitude and azimuth at 6 o'clock.

* These rules may, in general, be depended upon, except in peculiar circumstances, which a consideration of the figure will enable the computer to correct, as when the longitude, or RA, fall upon PP′, or pp′, &c.

See Dr Abram Robertson's paper in the Phil. Trans. for 1816, which for want of room cannot be given here.

1. At Edinburgh, in latitude 55° 57′ 20″ N. on the 21st of June, 1826, the sun's declination was 23° 27′ 36″ N.; required his altitude and azimuth at 6 o'clock in the morning or evening, his declination being supposed to remain the same?

Construction.—Describe the primitive HPON on the plane of the meridian. Let HO represent the horizon, ZN the prime vertical at right angles to the former. Make OP, from a scale of chords equal to the latitude of the place, North in the present instance; draw PP′, the six-o'clock hour circle in this case, and at right angles to it draw the equator EQ; describe the small circle *sm* at the distance of 23° 27′ 36″ from the equator, representing the parallel of declination, and it will cut the six-o'clock hour circle PP′ in F, the sun's place at the given time.

Through Z, F, and N, describe the azimuth circle ZFN cutting the horizon in D, then FD is the altitude, FZ the zenith distance, and the angle FZP, or its measure, the arc DO, is the azimuth; consequently the things given and required fall in either of the triangles FZP, or FDA, which are supplemental to each other. For, since OP is the latitude, PZ is the colatitude, AF is the declination; consequently FP is the polar distance, DF being the altitude, FZ must be the zenith distance.

Calculation.—In the right-angled spherical triangle FPZ, right-angled at P, FP and PZ are given, to find the angle FZP and FZ; or in the triangle ADF, right-angled at D, there are given the angle FAD, equal to the latitude of the place, and AF, the sun's declination, to find DF, the altitude, and the side AD the azimuth.

By the rule of the circular parts FP, PZ, and PZF, are all connected, therefore PZ is the middle part, and PZF and PF are the adjacent parts, where

$$R \times \text{sine } ZP = \tan PF \times \cot PZF, \text{ or}$$
$$R \times \cos \text{lat} = \cot \text{dec} \times \cot \text{azimuth, therefore}$$
$$\cot \text{azimuth} = \frac{\cos \text{lat}}{\cot \text{dec}} = \cos \text{lat} \times \tan \text{dec}$$

To log cos lat	55° 57′ 20″	9.748061
Add log tan dec	23 27 36	9.637472
Sum=log cot az	76 20 38	9.385533

Again, to find FZ the coaltitude, and the same things being given,
$R \times \cos FZ = \cos ZP \times \cos FP$, or sine alt=sine lat $\times$ sine dec.

To log sine lat	55° 57′ 20″	9.918347
Add log sine dec	23 27 36	9.600002
Sum=log sine alt	19 15 40	9.518349

Problem V.

Given the latitude of the place, and the sun's declination, to find the altitude and hour when the sun is due East or West.

Example.

At Edinburgh, on the 21st June, 1826, what was the sun's altitude and hour when due East or West, the declination being 23° 27′ 36″ N?

In the last figure let ZAN meet the parallel n m in K, and suppose a circle to be drawn through the points PKP′, forming the triangle ZKP, right-angled at Z, then ZK is the coaltitude, and ZPK the hour from noon; hence

$$R \times \cos PK = \cos ZP \times \cos ZK, \text{ or}$$

$$\cos ZK = \frac{\cos PK}{\cos ZP} = \cos PK \times \sec ZP = \cos PK \times \text{cosec } PO$$

or sine alt = sine dec × cosec lat

Dec 23° 27′ 36″	sine	9.600002
Lat 55 57 20	cosec	0.081653
Alt 28 42 55	sine	9.681655

$$R \times \cos ZPK = \tan ZP \times \cot PK, \text{ or}$$

cot T = cot lat × tan dec

Lat 55° 57′ 20″	cot	9.829714
Dec 23 27 36	tan	9.637472
Time 4^h 51^m 48^s	cos	9.467186

From noon, that is, at 7^h 8^m 12^s A. M., and 4^h 51^m 48^s P. M.

This problem is of considerable utility to the navigator and practical astronomer, for the purpose of determining time accurately when an altitude instrument is used. As the change of altitude, on which the accuracy of the determination of the time depends, is quickest when the object is on the prime vertical, the most proper time for observing an altitude for that purpose is, therefore, when the object is due East or West, as any small error in the observation has then the least possible effect on the time. Other errors are also in this case in a great degree avoided, or at least considerably lessened, particularly that arising from any small error in the estimated latitude at the time of observation. To facilitate its application, the following table, corresponding to the latitude and declination (which must be of the same name with the latitude), has been given. When the latitude and declination are of different names, the altitude must be as near the horizon as is consistent with accuracy, so far as depends upon the uncertainty of the horizontal refraction. Altitudes under 5° should not be used when great accuracy is required.

Table, showing the Time from Noon and Altitude when a Star whose Declination is less than the Latitude is on the Prime Vertical.

Lat.	Declination of the Star of the same name with the latitude.											
	5°		10°		15°		20°		25°		30°	
	T	A	T	A	T	A	T	A	T	A	T	A
°	h	°	h	°	h	°	h	°	h	°	h	°
5	0	90	4	30	5	20	5	15	5	12	5	10
10	4	30	0	90	3	42	4	31	5	24	5	20
15	5	20	3	42	0	90	3	49	4	38	4	31
20	5	15	4	31	3	49	0	90	3	55	3	43
25	5	12	5	24	4	38	3	54	0	90	2	58
30	5	10	5	20	4	31	3	43	2	58	0	90
35	6	9	5	18	4	27	4	37	3	47	2	61
40	6	8	5	16	5	24	4	32	4	41	3	51
45	6	7	5	14	5	21	5	29	4	37	4	45
50	6	7	5	13	5	20	5	27	4	33	4	41
55	6	6	6	12	5	18	5	25	5	31	4	38
60	6	6	6	12	5	17	5	23	5	30	5	35
65	6	6	6	11	6	17	5	22	5	28	5	33
70	6	5	6	11	6	16	6	21	5	27	5	32
75	6	5	6	10	6	16	6	20	6	26	5	31
80	6	5	6	10	6	15	6	19	6	25	6	30

The change of altitude on the prime vertical in one second of time is $=15'' \times \cos\lambda =$ const log $1.176091 + \log \cos\lambda$, in which λ is the latitude. This gives $8''.4$ at $56°$ N.

PROBLEM VI.

Given the latitude of the place and the sun's declination, required his amplitude and ascensional difference?*

At Edinburgh, on the 21st of June, 1826, from the data given in the last example, on what point, and at what time, did the sun rise and set?

In the triangle ABC, in the last figure, there are given the angle BAC, equal to the colatitude, and BC the sun's declination; to find AC and AB.

$$R \times \text{sine BC} = \text{sine AC} \times \text{sine BAC, or}$$

$$\text{sine AC} = \frac{\text{sine BC}}{\text{sine BAC}} = \text{sine BC} \times \text{cosec BAC.}$$

BC, or dec	23°	27′	36″ N.	sine	9.600002
Latitude,	55	57	20	sec	10.251939
AC,	45	19	33	sine	9.851941
CO,	44	40	27,		

in which case AC is the amplitude reckoned from the East or West, to the North or South, according to the name of the declination, and CO is that reckoned from the meridian, or *from* the North or South, according to the name of the declination.

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* By the ascensional difference is meant the time before or after six o'clock the sun rises or sets. By this problem, therefore, the lengths of day and night are determined, and the variation of the mariner's compass.

Again, in the same triangle AB is the ascensional difference, and R × sine AB=cot BAC × tan BC, or sine AB=tan lat × tan dec

Lat	55° 57′ 20″ tangent,	.	10.170286
Dec	23 27 36 tangent,	.	9.637472
A. D.	$2^h\ 39^m\ 52^s$ sine	. .	9.807758
	6		
	8 39 52=time of setting.		
	3 20 8=time of rising,		

the latitude and declination being of the same name, or if instead of sine we read cosine, then we would get the time of rising if the latitude and declination are of the same name, and the time of setting if of different names.

TABLE OF SEMIDIURNAL ARCS.

Lat.	Declination of the same name with the Latitude.													
	0°		5°		10°		15°		20°		25°		30°	
°	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.
0	6	0	6	0	6	0	6	0	6	0	6	0	6	0
5	6	0	6	2	6	4	6	5	6	7	6	9	6	12
10	6	0	6	4	6	7	6	11	6	15	6	19	6	23
15	6	0	6	5	6	11	6	16	6	22	6	29	6	36
20	6	0	6	7	6	15	6	22	6	30	6	39	6	49
25	6	0	6	9	6	19	6	29	6	39	6	50	7	2
30	6	0	6	12	6	23	6	36	6	49	7	2	7	18
35	6	0	6	14	6	28	6	43	6	59	7	16	7	35
40	6	0	6	17	6	34	6	52	7	11	7	32	7	56
45	6	0	6	20	6	41	7	2	7	25	7	51	8	21
50	6	0	6	24	6	49	7	14	7	43	8	15	8	54
55	6	0	6	29	6	58	7	30	8	5	8	47	9	42
60	6	0	6	35	7	11	7	51	8	36	9	35	12	0
65	6	0	6	43	7	29	8	20	9	25	12	0		
70	6	0	6	56	7	56	9	10	12	0				
75	6	0	7	16	8	45	12	0						
80	6	0	7	59	12	0								
85	6	0	12	0										

This table gives the time of setting of the sun when the latitude and declination are of the *same name*, but that of rising if of different names. However, it is only the approximate time, as no allowance is made for the effects of a change of declination, the horizontal refraction and parallax in the case of the sun and planets. Their joint effect may be obtained by the following rule. First, let the approximate time be found by the foregoing table of semidiurnal arcs. To this time let the declination of the object be reduced. With it find the ascensional difference as formerly. Now find the sum and difference of the natural cosine of the reduced declination and natural sine of the latitude, which may be carried to four places of figures only, these being sufficiently accurate for this purpose, and take half the sum of the logarithms of these quantities, to which add the constant logarithm 7.1761, and the proportional logarithm of the difference between the horizontal parallax and the sum of the

horizontal refraction and dip of the horizon, the sum, rejecting 10 in the index, will be the proportional logarithm of the correction which is to be *subtracted* from the time of rising, or *added* to the time of setting, if the horizontal parallax is *less* than the sum of horizontal refraction and dip, otherwise the correction must be *added* in the first case, and *subtracted* in the second.

Example.

Required the time of rising and setting of the sun on the 1st of April, 1826, in latitude 33° 42′ N., and longitude 16° 20′ W. the height of the eye above the sea being 28 feet?

By the table to latitude 35° N. and declination 5° N. the approximate time of

	Setting is	6^h 15^m nearly.		
	Long. in T.	1 5 W.		
	Reduced T.	7 20	Dec. 4° 33′ 54″ N.	
	Rising,	4 40	Dec. 4 19 46 N.	
Latitude,	33° 42′ 0″	N. tangent	.	9.824072
Declination,	4 33 54	N. tangent	.	8.902238
Ascen. Diff.	0^h 12 13	sine	. .	8.726310
	6			
Appr. T.	6 12 13			
Correction,	+ 3 10			
Setting,	6 15 23			

In a similar manner, the time of rising will be 5^h 45^m 15^s.

Dec. 4° 28′ N.	cos 9969		
Lat. 33 42 N.	sine 5548		
	Sum	15517	log 4.1908
	Diff.	4421	log 3.6455

Dip to 28 feet	— 5′ 16″		7.8363
Hor. refrac.	—34 17		
Parallax	+ 9		3.9181
		const log	7.1761
	—39 24 P. L.	. .	0.6598
Cor. in Time	3^m 10^s P. L.	. .	1.7540

The correction to be subtracted from the time of rising, or added to the time of setting. As the moon's horizontal parallax is in general greater than the effects of dip and refraction, the correction thus obtained would have been applied with a contrary sign. This method of determining time may sometimes be of use when a better cannot be obtained, and in the case of the sun or moon, a mean of the times of appearance of the upper and lower limb may be taken.*

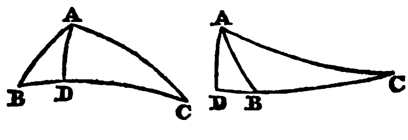
* To find the rising and setting of a star or planet, the transit over the meridian must be first computed as follows:—From R. A. of the star subtract that of the sun for noon, the remainder is the approximate time of transit. Reduce the R. A. of both to this time and the given longitude, and subtract as before, and the remainder will be the true time of transit, which, properly applied to the semidiurnal arc, will give, when corrected for dip, &c., the true time of rising or setting. See explanation of table of sun's declination.

Solution of Oblique-angled Spherical Triangles.

The different cases of oblique-angled spherical triangles may be solved by the following theorems:—

Theorem I.

In every spherical triangle the sines of the sides are proportional to the sines of the angles opposite to them,* or, sin AB : sin AC :: sin C : sin B.



Theorem II.

In oblique-angled spherical triangles, a perpendicular arc being drawn from any of the angles upon the opposite side, the cosines of the angles at the base are proportional to the sines of the segments of the vertical angle, or cos B : cos C :: sin BAD : sin CAD.

Theorem III.

The same things remaining, the cosines of the sides are proportional to the cosines of the segments of the base, or cos AB : cos AC :: cos BD : cos CD.

Theorem IV.

The same construction remaining, the sines of the segments of the base are reciprocally proportional to the tangents of the angles at the base, or sin BD : sin CD : : tan C : tan B.

Theorem V.

The same construction remaining, the cosines of the segments of the vertical angles are reciprocally proportional to the tangents of the sides, or cos BAD : cos CAD : : tan AC : tan AB.

Theorem VI.

If, from an angle of a spherical triangle, there be drawn a perpendicular to the opposite side or base, the tangent of half the sum of the segments of the base is to the tangent of half the sum of the two sides of the triangle, as the tangent of half the difference of those sides to the tangent of half the difference of the segments of the base, or tan $\frac{1}{2}$ (BD + CD) : tan $\frac{1}{2}$ (AB + BC) :: tan $\frac{1}{2}$ (AB $\backsim$ AC) : tan $\frac{1}{2}$ (BD $\backsim$ CD).

When the three sides or the three angles are not the given parts of the triangle, to have sufficient *data* for the solution of the problem, the perpendicular must be so drawn, that two of the given things in the oblique-angled triangle may be known in one of the resulting right-angled triangles.

Theorem VII.

If a perpendicular be drawn from an angle of a spherical triangle to the opposite side or base, the sine of the sum of the angles at the

* See Playfair's Geometry, article Spherical Trigonometry, Prop. XXIV., or Legendre's Geometry, article LXXVI., and the following in order.

base is to the sine of their difference as the tangent of half the base is to the tangent of half the difference of its segments: and the sine of the sum of the two sides is to the sine of their difference, as the cotangent of half the angle contained by the sides is to the tangent of half the difference of the angles which the same sides make with the perpendicular,* or sin (B+C) : sin (B$\backsim$C) :: tan $\frac{1}{2}$ BC : tan $\frac{1}{2}$ (BD$\backsim$CD). And sin (AB+AC) : sin (AB$\backsim$AC) :: cot $\frac{1}{2}$ A : tan $\frac{1}{2}$ (BAD$\backsim$CAD).

Theorem VIII.

The sine of half the sum of any two angles of a spherical triangle is to the sine of half their difference, as the tangent of half the side adjacent to these angles is to the tangent of half the difference of the sides opposite to them. And the cosine of half the sum of the same angles is to the cosine of half their difference as the tangent of half the side adjacent to them is to the tangent of half the sum of the sides opposite, or sin $\frac{1}{2}$ (A+B) : sin $\frac{1}{2}$ (A$\backsim$B) :: tan $\frac{1}{2}$ AB : tan $\frac{1}{2}$ (BC$\backsim$AC). And cos $\frac{1}{2}$ (A+B) : cos $\frac{1}{2}$ (A$\backsim$B) :: tan $\frac{1}{2}$ AB : tan $\frac{1}{2}$ (BC$\backsim$AC).

Corollary.—The sine of half the sum of any two sides of a spherical triangle is to the sine of half their difference as the cotangent of half the angle contained between them is to the tangent of half the difference of the angles opposite to them: And the cosine of half the sum of these sides is to the cosine of half their difference as the cotangent of half the angle contained between them is to the tangent of half the sum of the angles opposite to them,† or sin $\frac{1}{2}$ (AB+AC) : sin $\frac{1}{2}$ (AB$\backsim$BC) :: cot $\frac{1}{2}$ A : tan $\frac{1}{2}$ (B$\backsim$C); cos $\frac{1}{2}$ (AB+AC) : cos $\frac{1}{2}$ (AB$\backsim$BC) :: cot $\frac{1}{2}$ A : tan $\frac{1}{2}$ (B+C).

Theorem IX.

It will be sometimes more easy in practice to compute an angle from the three given sides by the following formulæ and rules, than by any of those already given: thus, suppose A, B, C are the angles as before, and a, b, c the sides opposite; then

$$\text{Sin}\ \tfrac{1}{2}\text{A} = \sqrt{\frac{\sin\{\frac{1}{2}(a+b+c)-c\}\ .\ \sin\{\frac{1}{2}(a+b+c)-b\}}{\sin b \sin c}} \qquad (1.)$$

$$\text{Cos}\ \tfrac{1}{2}\text{A} = \sqrt{\frac{\sin\frac{1}{2}(a+b+c)\ \sin\{\frac{1}{2}(a+b+c)-a\}}{\sin b \sin c}} \qquad (2.)$$

$$\text{Tan}\ \tfrac{1}{2}\text{A} = \sqrt{\frac{\sin\{\frac{1}{2}(a+b+c)-b\}\ .\ \sin\{\frac{1}{2}(a+b+c)-c\}}{\sin\{\frac{1}{2}(a+b+c)-a\}\ .\ \sin\{\frac{1}{2}(a+b+c)\}}} \qquad (3.)$$

Rules in Words.

I. From half the sum of the three sides subtract each of the two sides which contain the required angle. Then to the cosecants of the sides which contain the required angle add the sines of the two remainders; half the sum of these foregoing logarithms will be the sine of half the required angle.

II. Find the difference between half the sum of the three sides,

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* This theorem forms Proposition XXX. in Playfair's Spherical Trigonometry, where it is partly erroneous. See the French édition of Cagnoli's Trigonometry, § 1088, 1108 and 1109.

† Legendre, § LXXXIII.

and the side opposite the required angle. Then to the cosecants of the two containing sides add the sines of the half sum and difference; half the sum of these four logarithms will be the cosine of half the required angle.

III. To the cosecant of half the sum of the three sides add the cosecant of half that sum diminished by the side *opposite* the required angle, and the sines of the same half sum diminished by each of the sides *containing* the required angle; half the sum of these four logarithms will be the tangent of half the required angles. See remarks annexed to Case III., Plane Trigonometry.

Theorem X.

Given two sides and the contained angle to find the side opposite that angle.

To twice the log. sine of half the contained angle add the sines of the two containing sides, and from half the sum of these three logarithms subtract the sine of half the difference of the sides; the remainder will be the tangent of an arc, the sine of which being subtracted from the half sum of the three logarithms already found, leaves the *sine* of half the required side.

Theorem XI.

The two sides and contained angle being given, the third side may be found in the following manner.

To twice the sine of half the contained angle add the sines of the two containing sides; half the sum of these three logarithms, after rejecting 20 in the index, will be the cosine of an arc. Also find half the difference of the two containing sides.

To the sine of the sum of these two last arcs add the sine of their difference; half the sum of these two logarithms will be the *cosine* of half the required side.

It may be remarked, that when the side is not greater than 90°, theorem X. may be used; when it is greater than 90°, theorem XI. may be employed when great accuracy is required.

Theorem XII.

The three angles of a spherical triangle being given, to find the sides.

From half the sum of the three angles subtract each of the angles *next* the required side, then to the cosecants of the adjacent angles add the cosines of the two remainders; half the sum of these four logarithms will be the *cosine* of half the required side.

Theorem XIII.

The same things being given; from half the sum of the three angles subtract the angle *opposite* the required side, then to the cosecants of the adjacent angles add the cosine of half the sum and the cosine of the difference; half the sum of these four logarithms will be the *cosine* of half the required side.

Either of these theorems may be employed which will give the more accurate result.

Having stated the theorems on which the solutions in oblique-angled spherical triangles depend, it is necessary to illustrate them by

examples, which will chiefly consist of those applicable to the usual cases that occur in practical astronomy and navigation.

Before proceeding to the application of these theorems, it will be necessary to show the method of correcting astronomical observations obtained by the usual instruments.

Of the Method of correcting Astronomical Observations for the Effects of Refraction, Parallax, Semidiameter, and Dip of the Horizon.

The method of computing the refraction is shown in the explanation of Table XVII. It is always to be added to the apparent zenith distance, or subtracted from the altitude. The parallax of the sun answering to the time of the year, and Z. D. or altitude may be taken from Table XVI. and is always to be subtracted from the Z. D. or added to the altitude. If the horizontal parallax of the moon or a planet be known, the parallax in altitude may be found by adding to the log. secant of the altitude the proportional logarithm of the horizontal parallax, the sum will be the prop. log. of the parallax in altitude. The semidiameter of the sun may be taken from Table XV., that of the moon from the Nautical Almanac, and they must be applied by addition or subtraction according to the limb observed, in order to reduce an observation to the centre.

Problem I.

Given the latitude of the place, the sun's altitude and declination, to find the time and the azimuth.

At the Observatory of Edinburgh, on the Calton-hill, in latitude 55° 57′ 21″ N., on the third of June, 1826, the following observations of the sun's lower limb were taken in the morning; required the time and azimuth, the barometer being at 29.56 in., and the thermometer at 64° F.?

Times by Watch.			Altitudes.		
7h	1m	20s	26°	51′	20″
	2	18	26	59	30
	3	25	27	7	15
	4	30	27	15	40
	5	27	27	23	45
5	17	0	35	37	30
Means, 7	3	24	27	7	30 lower limb.

Or observed Z. D.	62 52 30	
Z. D. 62° 52′.5 log. ✱		2.05522
Thermometer 64° F. log.		9.98751
Barometer 29.56		9.99358
Thermometer 64.0 F.		9.99940
r=108″.6=1′ 48″.6 log.		2.03571

Z. dist.	=62°	52′	30″
Refraction	+	1	48.6
True Z. D.	62	54	18.6 of the lower limb.
Semidiameter	—	15	47.5
Parallax	—		7.6
True Z. D.	62	38	23.5 of the centre.

Approximate time, June 2d,	19^h 4^m		
Longitude in time add	+ 13 West.		
Estimated Greenwich time	19 17	D. L.	0.09503
Daily variation of dec.	7′ 42″	P. L.	1.36878
Prop. part. to 19^h 17^m	+ 6 11	P. L.	1.46381
Dec., June 2d,	22° 9 38 N.		
Reduced declination	22 15 49 N.		
Polar distance	67 44 11		

1. Now in the figure, (page 87), there are given OP the latitude, and consequently ZP the colatitude, PK the polar distance, and ZK the zenith distance, the place of the sun being K near the prime vertical, as being most advantageous to determine the time with accuracy, or the three sides of the triangle KPL; to find the angle ZPK the time, and the angle PZK the azimuth from the southern meridian PEP′. This, therefore, is solved by means of theorem IX.

The polar distance in most problems in astronomy and navigation is reckoned from the elevated pole of the same name with the latitude.

Now the latitude being 55° 57′ 21″, the colatitude is 34° 2′ 39″

Z. D.	62° 38′ 24″		
Colatitude	34 2 39	cosec	0.251942
Polar Dist.	67 44 11	cosec	0.033647
Sum	164 25 14		
Half	82 12 37		
First rem.	48 9 58	sine	9.872205
Second rem.	14 28 26	sine	9.397833
			19.555627
	2^h 27^m 20^s		
	2 .8	sine	9.777814
Time from noon 3d	4 54 41 .6		
	12		
App. time, A. M.	7 5 18 .4		
Time by watch	7 3 24 .0		
Watch slow	1 54 .4 for apparent time.		
Again app. time	7 5 18 .4		
Equation of time	— 2 22 .2		
Mean time	7 2 56 .2		
Time by watch	7 3 24 .0		
Watch fast	27 .8 for mean time.		

2. To find the azimuth or the angle KZP, the point K being that in which the circles $n\ m$ and ZIN cut each other, there are given the three sides of the triangle KPZ.

KP, or polar dist.	67° 44′ 11″		
PZ, or colatitude	34 2 39	cosec .	0.251942
ZK, or Z. dist.	62 38 24	cosec .	0.051572
Sum . . .	164 25 14		
Half . . .	82 12 37	sine .	9.995974
Difference . .	14 28 26	sine .	9.397833
			19.697321
	45 6 31	cos .	9.848660
	2	.	
	N. 90 13 2	E. .	
	44 53 29	sin. or . .	
	2		
	S. 89 46 58	E.	

or reckoned from the South in north latitude, or from the North in south latitude.

This problem is very useful in navigation for the purpose of finding the variation of the compass, which is the difference between the true and observed amplitude or azimuth.

To determine this, let the observer be supposed to look directly from the centre of the card towards the point representing the true azimuth; then if the observed azimuth is to the *left* of the true azimuth, the variation is *easterly*, but if to the right it is westerly to the amount of the difference between them.

Thus let the true azimuth be S.	89° 47′	E.
Observed	62 25	
Variation . .	27 22	West.

Or about $2\frac{1}{2}$ points westerly.

Ex. 3. Required the time of rising and setting of the sun on the top of the Calton-hill at Edinburgh, in latitude 55° 57′ 20″ N. and longitude 3° 10′ W. on the 1st of June, 1828, the height of the eye above the sea being 350 feet?

To latitude 56° N. and declination 22° N. nearly, the approximate time of setting by the table is about $8^h\ 28^m$, and rising $32^h\ 3^m$.

Approximate time of rising	$3^h\ 32^m$	Setting . .	$8^h\ 28^m$
Long. in time, West ..	+ 13	. . .	+ 13
Reduced time, A. M. .	3 45	P. M. . .	8 41
Dec. at rising 22° 2′ 53″ N.		Setting 22° 8′ 34″ N.	

The height of the eye being greater than any in the table (XI.) the dip must be computed by the rule, page 40.

Constant logarithm . . .		3.53441	
Height of the eye, 350 feet log.		2.54407	
Sum . . .		6.07848	
Half or log. dip . .		3.03924=1095″=18′ 15″	
Horizontal refraction . . .			+34 17
Horizontal parallax			— 9
Semidiameter			—15 48
Correction for upper limb . . .			+36 35
			90 0 0
Reduced zenith distance . . .			90 36 35

R. Z. D. .	90° 36′ 35″		
P. D. rising	67 57 7	cosecant . .	0.032981
Colatitude	34 2 40	cosecant . .	0.251939
Sum	192 36 22		
Half	96 18 11	sine . .	9.997366
Diff.	5 41 36	sine . .	8.996528
			19.278814
	1^h 43 $22\frac{1}{2}$	sine . .	9.639407
	2		
Rising	3 26 45 of upper limb.		

In the same manner, the time of setting of the sun's upper limb will be found to be 8^h 33^m 40^s.

If the semidiameter had been omitted, the computed time would have been that of the centre. The exact time of the rising and setting of the centre may be found by taking the mean times of the rising and setting of the upper and lower limbs, which, with a sea horizon, may serve to find the error of a clock when a better cannot be obtained.

These results for time and variation have been deduced strictly from the solution of the spherical triangle formed by the data, but they may be found more readily by rules derived from it, as may be seen in various books on navigation and nautical astronomy.

When tables which have proportional parts annexed to them are used, the following method may be advantageously employed for determining the time.

Rule.—When the latitude of the place and the declination are of the *same name*, let their *difference*, but, if of contrary names, let their sum be taken. Under this difference or sum place the zenith distance, and let the half sum and half difference of these be taken; then add together the secant of the latitude, the secant of the declination, the sine of the half sum, and the sine of the half difference; half the sum of these four logarithms will be the sine of half the hour angle or time from noon, from which the apparent and mean time may be obtained as formerly.

				65
Latitude	55° 57′ 21″ N.	secant	. .	0.251877
Declination	22 15 49 N.	secant	.	0.033605
				42
Difference	33 41 32			
Zenith dist.	62 38 24			
				109
Sum	96 19 56 half 48° 9′ 58″	sine	.	9.872095
Difference	28 56 52 half 14 28 26	sine	.	9.397621
				212
				19.555626
	$2^h\ 27^m\ 20^s$	sine	.	9.777813
	0.75 P. P.			781
	2 27 20.75		43	32
	2			0.75
	4 54 41.50			
	24			

June 2d, 19 5 18.50 P. M., the same as before.

In the above computation the several proportional parts are set down and summed all together, which renders the operation somewhat more easy when our tables are employed.

For the practice of these problems, the following rules derived from the foregoing principles are subjoined.

1. *To find the Time.*—Set down the true altitude, polar distance, and latitude. Find half the sum of these three, and the difference between the altitude and half sum. Then to the log. cosecant of the polar distance add the log. secant of the latitude, the cosine of the half sum, and the sine of the difference or remainder, half the sum of these four logarithms will be the log. sine of half the hour-angle from the meridian.

2. *To find the Azimuth.*—Set down the polar distance, the true altitude and latitude, and find half their sum, and the difference between this half sum and the polar distance. Now to the log. secant of the altitude add the secant of the latitude, the cosine of the half sum, and the cosine of the difference, half the sum of these four logarithms will be the log. sine of half the azimuth from the meridian to be reckoned from the *south* in *north* latitude, and from the north in south latitude. When a table of reduced versines is given, the sum of these four logarithms will be the log. versine of the hour-angle or azimuth respectively.

In the case of determining the time, it would be convenient to estimate it according to the astronomical method of reckoning, namely, from noon to noon, through the whole 24 hours. In using the table of reduced versines, therefore, the time must be taken from the top of the page if the observation be made in the afternoon, but from the bottom if in the forenoon.

3. *To find the Time by an Altitude of a Star.*—To the time of observation add the longitude in time if west, but subtract it if east, and the result will be the estimated Greenwich time. To this time find the sun's right ascension, and the star's right ascension and de-

clination. Deduce the horary angle, or the star's distance from the meridian. Add this distance to the star's right ascension, if at the time of observation it is west of the meridian, but subtract it if east, and the sum or remainder will be the right ascension of the meridian. From the right ascension of the meridian, increased by 24 hours if necessary, subtract the sun's right ascension, reduced to the time of observation, and the remainder will be the apparent time. If to this time the equation of time be applied, the result will be the mean time. If the table of reduced versed sines be used, the horary angle must be taken from the top of the page if the star be west of the meridian, but from the bottom if to the east, which being added to the stars R. A. gives the R. A. of the meridian.

EXAMPLES TO THE PRECEDING RULES.

Edinburgh, 30th March, 1829.

In latitude 55° 57′ 20″ N., longitude 3° 12′ W., the following series of observations were made on the star Arcturus, east of the meridian, and on Aldebaran, west of the meridian, in succession, to determine the error of the chronometer, the barometer being at 29.56 inches, and the thermometer at 54° Fahrenheit.

	Arcturus. Times.	Double Alts.		*Aldebaran.* Times.	Double Alts.
	8h 33m 56s	50° 10′ 30″		8h 43m 34s	45° 30′ 20″
	34 55	50 17 40		44 35	45 22 30
	35 58	50 25 20		45 34	45 14 20
	36 57	50 33 10		46 33	45 6 30
	37 59	50 41 20		47 34	44 59 45
	29 45	128 0		27 50	73 25
	8 35 57	50 25 36		8 45 34	45 14 41
Half		25 12 48	Half		22 37 20.5
Refr.		— 2 1	Refr.		— 2 16.2
True Altitude		25 10 47	True Altitude		22 35 4.3

Stars T. Alt.	25°	10′	47″		
Pol. D.	69	55.	35	cosecant	0.027218
Latitude	55	57	20	secant	0.251939
Sum	151	3	42		
Half	75	31	51	cosine	9.397695
Difference	50	21	4	sine	9.886473
	h	m	s		
East	19	2	14.6	R. Versine	9.563325
Star's R. A.	14	7	53.8		
Comp. O's R. A.	23	23	40.3		
Sum — 48 h =	8	33	48.7 = App. Time.		
Eq. of time	+	4	28.4 or Supp. to Naut. Almanac.		
Mean time	8	38	17.1		
Chro. time	7	35	57.0		
Chro. slow		2	20,1		

Star's T. Alt. .	22°	35′	4″		
Pol. D. . .	73	50	40	cosecant .	0.017498
Latitude . .	55	57	20	secant . .	0.251939
Sum . .	152	23	4		
Half . .	76	11	32	cosine . .	9.377789
Difference .	53	36	28	sine . .	9.905782
	h	m	s		
West . . .	4	53	39.6	R. Versine .	9.553008
Star's R. A. .	4	26	7.5		
Comp. O's R. A.	23	23	39.6		
Sum — 24ʰ = .	8	43	26.7 = App. Time.		
Eq. of time .	+	4	28.3 or Supp. to N. Al.		
Mean time .	8	47	55.0		
	8	45	34.0		
Chro. slow .		2	21.0		
		2	20.1		
Mean error .		2	20.5 slow.		

By observing an object both to the east and west about the same time, and having the same altitude nearly, the errors of reading, the glasses, the latitude, &c. will be very much lessened, if not entirely destroyed, and are consequently nearly equivalent to equal altitudes in point of accuracy, and are more convenient, as there is less risk of losing corresponding altitudes.

Several variations may be made on the six things here proposed, that may serve as a useful exercise, which, by a reference to the theorems and rules already given, will be easily performed.

Problem II.

Given the latitude of the place and the sun's declination, to find the time when twilight begins and ends.

At what time will twilight begin and end at London in latitude 51° 32′ N., on the second of May, 1827, the sun's declination being 15° 14′ N.?

In figure, (page 87), suppose a parallel $n\,m$ to the equator EQ to be drawn at the distance of 15° 14′ above it, while another parallel to the horizon HO is drawn at the distance of 18° below it, these two would cut one another somewhere between o and m in S, forming the triangle ZPS, in which ZP, PS, and ZS, are given to find the angle ZPS, the angle between the meridian PEP and another meridian passing through the sun at the time he is 18° degrees below the horizon, his situation when twilight begins and ends.

Z s or zenith distance	108° 0′		
P s or polar distance	74 46	cosecant	0.015534
PZ or colatitude	38 28	cosecant	0.206168
Sum . . .	221 14		
Half . . .	110 37	sine .	9.971256
Difference . . .	2 37	sine	3.659475
			18.852433
	4ʰ 58ᵐ 6ˢ	cosine	9.426216
	2		443
Time from noon	9 56 12 in the evening	.	227
Or at . .	2 3 48 in the morning.		

Problem III.

Given the right ascensions and declinations, or the longitudes and latitudes of two celestial objects, to find their angular distance.

In this problem there are given two sides and the contained angle to find its opposite side. The contained angle is the difference between their right ascensions or longitudes, and the containing sides are the complements of the declinations or latitudes. If the sun be one of the objects, as his latitude is very small, he may be supposed to be always in the ecliptic; then the triangle so formed will be right-angled if the longitudes and latitudes are used, and the computation becomes more simple. By means of this problem the lunar distances in the nautical almanac are computed.

On the 1st of June, 1828, required the distance between the moon and α Pegasi, at noon, on the meridian of Greenwich, the moon's right ascension being 295° 23′ 46″, and declination 16° 11′ 45″ S., the star's right ascension being $22^h\ 56^m\ 13^s.85$, or 344° 3′ 28″, and north polar distance 75° 43′ 2″, or declination 14° 16′ 58″ N.

344° 3′ 28″—295° 23′ 46″=48° 39′ 42″ the angle at the pole. Instead, however, of following the operation derived from the spherical triangle, a more simple practical rule may be derived from it according to theorem X.

To twice the sine of half the contained angle add the cosines of the moon and star's declinations, and take half the sum of these three logarithms. From this half sum subtract the sine of half the *sum* of the declinations if they are of *contrary names*, or that of half their difference if of the same name, the remainder will be the tangent of an arc, the sine of which being subtracted from half the sum of the three logarithms already found will give the sine of half the required distance.

Diff. of R. A. . . .	48° 39′ 42″		
Half	24 19 51	sine × 2=	19.229804
Moon's declination .	16 11 45	S. cos	9.982413
Star's declination .	14 16 58	N. cos	9.986364
			39.198581
Sum	30 28 43		
			19.599291 (*a*)
Half	15 14 21½	sine	9.419717
Arc	56 31 18	tan	10.179574
Same arc		sine	9.921215 (*b*)
Half distance . .	28 27 29	sine	9.678076 (*a*—*b*)
	2		
True distance . .	56 54 58		

Examples for Exercise.

1. Required the distance between the moon and sun on July 2d, 1828, at noon, on the meridian of Greenwich, the longitude of the

sun being 3ˢ 10° 28′ 44′, the longitude of the moon 11ˢ 17° 59′ 39″, and latitude 2° 51′ 40″ N.?

Ans.—112° 27′ 19″ east of her.

2. Required the distance between the moon and sun on the 20th January, 1828, at noon, the sun's longitude being 9ˢ 29° 29′ 39″, that of the moon 11ˢ 17° 54′ 42″, and latitude 3° 24′ 28″?

3. Required the distance between the moon and α Aquilæ, at noon, on the 10th of May, 1828, the right ascension of the moon being 6° 58′ 43″, the declination 4° 44′ 48″ N., the right ascension of α Aquilæ in time, being 19ʰ 42ᵐ 25ˢ.62, and north polar distance 81° 34′ 41″?

Ans.—70° 54′ 51″ west of her.

4. Required the distance between the moon and Aldebaran, at midnight, on the 16th of December, the moon's R. A. being 32° 31′ 30″, the declination 11° 18′ 11″ N., the R. A. of Aldebaran being 4ʰ 26ᵐ 8ˢ.67, and N. P. D. 73° 50′ 37″.4?

Ans.—33° 21′ 10″ west of her.

PROBLEM IV.

On finding the Latitude by Observation.

The most simple practical method of finding the latitude is from the meridian altitude of a celestial body whose declination is known.

Should the object be the sun, moon, or some of the planets, the altitude or zenith distance of the lower or upper limb, or both, are observed, and by the application of several corrections, that of the centre is obtained.

When reflecting instruments, such as the sextant, repeating circle, &c. with an artificial horizon, are employed, the arc read off must, from the principles of optics, be halved before the other corrections are applied.*

A meridian altitude of the sun, moon, or a planet taken at land, must be corrected for refraction, parallax, and semidiameter, and at sea, in addition to these, for the dip of the horizon.† When great accuracy is required, a number of observations are taken near the meridian, and reduced to it by formulæ, or tables, such as XXVIII., to which the index error, when necessary, must be applied.

Having found the true altitude, take its complement to 90°, which gives the zenith distance, denominated north or south, according as the observer is north or south of the object.

Now, if the zenith distance and declination are of the *same name*, their *sum* is the latitude; if of *contrary* names, their *difference* is the latitude of the same name with the greater.

Ex. 1.—Edinburgh Observatory, March 28th, 1825, with an artificial horizon and one of Troughton's best sextants, the vernier of which showed 10″, Captain Pringle Stokes, R. N., found the meridian altitude of the sun's lower limb to be 73° 32′ 15″, the index error being +2′ 26″, the barometer standing at 29.66 inches, and Fahrenheit's thermometer 56°; what was the latitude, employing the refractions in the table in the nautical almanac?

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* See explanation of Table XXV.

† Tables XIII. and XIV. combining the whole in one, have been computed for the sun and stars expressly with this view. They are sufficiently accurate for practice at sea, and diminish that liability to error which the corrections taken out separately are likely to occasion.

Observed altitude	73° 32′ 15″
Index error	+ 2 26
Sum	73 34 41
Half	36 47 20
Refraction to 29.66 and 56° F.	— 1 15
Parallax	+ 8
Semidiameter	+ 16 2
True altitude	37 2 15
Zenith distance	52 57 45N.
Declination	2 59 43N.
Latitude	55 57 28

Ex. 2.—On the same evening the altitude of Polaris to the north, when at the upper transit, was 115° 7′ 44″, the index error was + 2′ 26″, barometer 29.7 inches, the thermometer 50° Fah.; required the latitude?

Observed altitude	115° 7′ 44″ N.
Index error	+ 2 26
Sum	115 10 10
Half	57 35 5
Refraction	— 37
True altitude	57 34 28
Polar distance	1 37 16
Latitude by Polaris	55 57 12 N.
Latitude by Sun	55 57 28
Mean	55 57 20 N.

When the sextant is employed to determine the latitude, an object having the same altitude nearly should be chosen both to the north and south, in order to destroy the errors of the glasses, divisions, and reading, as far as possible. In these examples the mean is very near the truth, though either separately is erroneous to the amount of about 8″.

Edinburgh, 10th January, 1826.

Ex. 3.—On the Calton-hill, near the Observatory, with one of Troughton's reflecting circles on a stand, and an artificial horizon, the author, at about ten o'clock, P. M., observed the following double altitudes of the polar star, when the sympiesometer stood at 29.86 inches, and thermometer at 42° Fahrenheit; required the latitude of the place of observation?

	Siderial Time.		*Double Altitudes.*
	After Transit.		With Art. Horizon.
	$4^h\ 22^m\ 30^s$	. .	113° 10′ 50″
	4 23 35	. .	113 10 55
	4 24 30	. .	113 10 55
	4 25 40	. .	113 10 50
	4 26 45	. .	113 11 0
Means	4 24 36	. .	113 10 54
App alt or half		. . .	56 35 27
			90 0 0
App zenith dist or comp		. .	33 24 33

Now by tables 17, 18, 19, and 20, compute the refraction.

Zenith dist = 33° 24′ 33″ log $\mathfrak{M}$ (17) .	1.5860
Thermometer 42° Fah. (18) . .	0.0073
Barometer 29.86 inches (19) .	9.9980
Thermometer 42° Fah. (20) . .	0.0003
Log r=39″.05	1.5916
App zenith distance	33° 24′ 33″
Refraction	+ 0 39.05
True zenith distance	33 25 12.05

Now $4^h\ 24^m\ 36^s$ gives M=72″.973, and N=+0° 0′ 0″.57
Then 72″.973 × —3′.3 × 0.02=—4″.816=3′.3 × ·02 M.
And 72″.973—4″.816=68″.157 log 1.833510
Cot Z=33° 25′ 12″ . . 0.180535

Natural number .	103″·29=2.014045= —	0 1 43.29
Cos t=$4^h\ 24^m\ 36^s$.	9.606751	
p 96.7 log . . .	1.985426	
Natural number .	39′.19=1.592177 = +	0 39 11.40
Sum (Table XXV.)	+	0 37 28.68
Z		33 25 12.05
ψ or colatitude		34 2 40.73
Latitude		55 57 19.27N.

From a trigonometrical measurement he also found the latitude 55° 57′ 18″.6 N., supposing the latitude of the flag-staff in Leith Fort to be 55° 58′ 41″ N.

Ex. 4.—To determine from the observations of Captain Basil Hall, R. N., taken June 4th and 6th, 1822, the latitude of San Blas, that by estimation being about 21° 32½′ N., and longitude 105° 15′ W.= $7^h\ 1^m$ in time.

o

To compute the sun's declination, June 4, 1822.

Longitude in time	7^h	1^m	D.L.	0.53408
Daily variation	6′	56″	P.L.	1.41433
Prop. part to 7 1^m	2′	1″.6	P.L.	1.94841
Eq. to sec. diff.—23″ and 7^h +		2 .4		
Correct prop. part	2	4 .0		
Declination at noon G. 22°	24	41 .0		
Sun's true dec. 22	26	45 .0 N.		

To compute the refraction, the barometer being at 29.75 inches, and the thermometer 86° Fahrenheit, to merid. alt. *l. l.* 88° 50′, or

Z. D. 1° 10′ log. *θ*	0.0755
Ther. 86° Fah.	9.9686
Bar. 29.75	9.9963
Ther. 86°	9.9984
r 1″.1	0.0388

Parallax 0″.2 (Table XVI.)

Face of the circle west.

Readings { 1st vernier	88° 50′ 0″
Readings { 2d vernier	50 10
Obs. merid. alt. sun's *l. l.*	88 50 5
Sun's semidiameter	+ 15 47.2
Refraction	— 1.1
Parallax	+ 0.2
True alt. sun's centre	89 5 51.3
	90
Zenith dist.	0 54 8.7 S.
Declination	22 26 45.0 N.
Latitude with face west	21 32 36.3 N.

To compute the sun's declination, June 6, 1822.

Longitude in time	7^h 1^m	D. L.	0.53408
	6 9″	P. L.	1.46640
Prop. part to 7^h 1^m,	1′ 48″	P. L.	2.00048
Eq. to sec. diff.—24″ and 7^h	+ 2.5		
Correct prop. part	+ 1 50.5		
Dec. at noon G.	22 38 10.0		
True dec. at S. B.	22 40 0.5 N.		

To compute the refraction, the barometer being 29.8 inches, and the thermometer 85° Fah., the meridian Z. D. being 1° 23′.5 nearly.

Z. D.	1° 23′.5 log	0.1926	Parallax 0″.21
Ther. 85°	. log .	9.9694	
Bar. 29.8	. . .	9.9971	
Ther. 85°	. . .	0.9085	
r 1″.44	. . .	0.1576	

Face of the circle east.

Readings	1st vernier . . .	1° 23′ 30″
	2d vernier . . .	25
Obs. zenith dist. sun's *l. l.*	. .	1 23 27.5
Sun's semidiameter	. . . . .	— 15 47.0
Refraction	. . . . .	— 1.4
Parallax	. . . . . .	+ 0.2
True mer. Z. D.	. . . . .	1 7 39.3 S.
Declination	. . . . .	22 40 0.5 N.
Latitude, face east	. . .	21 32 21.2 N.
face west	. . .	21 32 36.3
Mean latitude by sun	. . .	21 32 28.75

When the latitude is determined by an astronomical circle, an observation is not supposed to be complete, till the observer has reversed the circle, by this means combining two sets of observations, with the face or graduated limb of the instrument alternately, as in this example, towards the east and west, thereby destroying the errors of graduation, collimation, &c.

Ex. 5. San Blas, 20th May, 1822, the barometer being at 29.78 inches, Fahrenheit's thermometer 83°, the chronometer too fast for mean time $4^h\ 4^m\ 45^s$, Polaris on the meridian below the pole by chronometer at $1^h\ 8^m\ 41^s$ and its true apparent N. P. D. 1° 38′ 28″.46.

Face of instrument.	Chronometer.	Time from the Merid.	Reduction to Merid.	Obs. Z. D. and Alts.	Altitudes.
	h m s	m s	I.	° ′ ″	° ′ ″
East	1 6 5	2 36	13″.27	70 3 34.5	19 56 25.5
	1 7 51	0 50	1 .36	3 34.0	56 26.0
	1 8 41	0 0	0 .00	3 35.0	56 25.0
West	1 14 3	5 22	56 .55	19 56 19.0	56 19.0
	1 16 11	7 30	110 .44	56 18.0	56 18.0
	1 18 35	9 54	192 .41	56 20.5	56 20.5
		6	374 .03		19 56 22.33
			62 .34		

To compute the correction of altitude on account of the distance of the star from the meridian.

λ	21° 32′ 30″	cosine . .	9.968553
δ	28 21 30	cosine . .	8.457118
Alt.	19 56 22	secant .	0.026814
m	62″.34 log	. . . . .	1.794767
Cor.—1.77	log	. . . . .	0.247252

The correction for part II. is in this case insensible.

To compute the refraction.

Z. D.	70° 3′.6	log θ . .	2.20325
Ther.	83 F.	log . .	9.97112
Bar.	29.78	log . .	9.99445
Ther.	83	log . .	9.99857
r 147″.02		log . .	2.16739

Or 2′ 27″.02

Observed altitude	19° 56′ 22″.33
Refraction	— 2 27 .02
Correction	— 1 .77
True altitude below the pole . .	19 53 53 .54 N.
Polar distance	1 38 28 .46 N.
Latitude from Polaris	21 32 22 .00 N.
from Sun . . .	21 32 28 .75
Mean	21 32 25 .37

Ex. 6. At Maranham, August 28, 1822, Captain Sabine took the following observations of the star α Lyræ with a repeating circle of six inches in diameter, the barometer being $29^{in}.95$, the thermometer 80° Fahrenheit, the chronometer, No 423, fast $2^h\ 55^m\ 59^s$; the star, whose right ascension was $18^h\ 30^m\ 57^s.4$, was on the meridian, at $8^h\ 4^m\ 35^s$ mean time, and at $11^h\ 1^m\ 34^s$ by the chronometer.

Chronometer.	Horary Angles.	Reduction. P. I.	Reduction. P. II.	Level.			Readings.
h m s	m s	″	″	d	d		
10 49 6	12 28	305.09	0.23	+2	+ 1	Previous.	First Vernier 167° 11′ 50″
10 52 40	8 54	155.51	0.06	—2	— 4		Second . 11 30
10 55 50	5 44	64.54	0.01	—4	— 6		Third . . 12 10
10 57 44	3 50	28.85	0.00	—1	— 2		Fourth . 11 40
11 0 49	0 45	1.10	0.00	—8	—10		
11 3 42	2 8	8.94	0.00	+4	+ 2		Mean 167 11 47 .5
11 6 52	5 18	55.15	0.01	—8	—10		
11 9 17	7 43	116.91	0.03	+7	+ 5	Final.	First Vernier 136° 35′ 0″
	8	736.07	0.34	—	16.5		Second . 34 30
							Third . 35 30
Means .		92.01	0.0425				Fourth . 35 0

A division of the level is 10″.8
Hence 10″.8 × —16.5 = —2′ 58″.2

		Mean .	136 35 0
		Index +	192 48 12 .5
		Level —	2 58 .2
			8)329 20 14 .3

ζ 41° 10′	log $\delta\zeta$ 1.70804		
Ther. 80 F. .	9.97367		
Bar. 29.95 .	9.99926	Obs. Z. D.	41 10 1 .8
Ther. . .	9.99870	r .	+ 47 .8
		Cor. .	— 1 49 .0
r 47″.83	1.67967		
		True Z. D.	41 9 0 .62
		Star's dec.	38 37 37 .60
		Latitude	2 31 23 .01

λ 2° 31′ 25″ cos	9.999578	
δ 38 37 38 cos	9.892776	
z 41 10 2 cosec	0.181603 (a) cot . .	0.058278
	0.073957 × 2 = . .	0.147914
m 92″.01 log	1.963835 n 0.0425 log .	8.628389
1st cor.—109″.09	2.037792 0.068	8.834581
or — 1′ 49 .09	426	
2d cor. + 0 .07		
	366	
— 1 49 .02		

It is unnecessary to repeat the operation in this case, as the difference in the result would be insensible.

Ex. 7. On the 13th of May, 1819, the following set of observations was made on polaris, near the time of its *lower* culmination, at Shanklin in the Isle of Wight, on each side of the meridian, of which the mean gave the zenith distance corrected for refraction, 41° 2′ 5″.71. The assumed latitude of the place, 50° 37′ 23″ N. the apparent declination of polaris by the Nautical Almanac 88° 20′ 28″.87 N. The chronometer showed mean solar time, having a losing rate of 1.8 daily. Polaris passed the meridian at 9h 37m 32s by the chronometer, the difference between which and the times indicated in the second column will give the time from the meridian when each observation was made; required the latitude?

STAR UPON MERIDIAN AT $9^h\ 37^m\ 32^s$.

No.	Time of Observation.			Time from Meridian.		m	n
	h	m	s	m	s	"	"
1	9	33	55	3	37	25.68	0.00
2	9	36	44	0	48	1.26	0.00
3	9	39	55	2	23	11.15	0.00
4	9	42	27	4	55	47.46	0.01
5	9	45	40	8	8	129.87	0.04
6	9	48	56	11	24	255.12	0.15
					6	470.54	0.20
				Means		78.42	0.03

$\lambda = 50° \; 37' \; 23''$ cosine 9.802377
$\delta = 88 \; 20 \; 29$ cosine 8.461561

$\lambda + \delta = 138 \; 57 \; 52$
$\left.\begin{matrix}180° - (\lambda + \delta) \\ \text{or } z\end{matrix}\right\} = 41 \; 2 \; 8$ cosec 0.182748 (a) cotang 0.060293

8.446686 × 2 = 6.893372
$m = 78''.42$ log 1.894371 $n = 0''.03$ log 8.477121

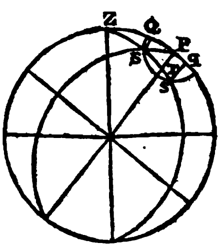
Cor. for mean solar time . 0.002372 $0''.00003$ log 5.430786
Cor. for rate of chronometer 0.000018 or 2*d* cor. insensible.

1*st* cor. $2''.205$ log. . . 0.343447

Hence $41° \; 2' \; 5''.71 + 2''.21 = 41° \; 2' \; 7''92 = z$, and the latitude, or $\lambda = 180° - (z + \delta) = 50° \; 37' \; 23''.22$ N.

To compute the Latitude by the Pole Star.

Let Z be the zenith of the place of observation, P the pole, S, *s* the place of the star in its diurnal revolution round P. From S let fall the perpendicular SQ upon ZP. Then let ZS$=z$ the zenith distance of the pole star, PS$=p$ the north polar distance of the star; the angle ZPS$=t$ the hour-angle; ZP$=\psi$ the colatitude of the place; and QP$=u$ the segment formed by the perpendicular. Now z, p, and t being given, we have, by spherical trigonometry, (Playfair, prop. XXI.) in the triangle QPS, cos QPS : R :: tang PQ : tan PS, or tan PQ$=$cos QPS × tan PS, or tan $u = \cos t \, . \tan p$. Also (prop. XXVI.) from the triangle PSZ, cos ZQ : cos QP :: cos ZS : cos PS or



$$\cos ZQ = \frac{\cos QP \times \cos ZS}{\cos PS} = \cos PQ \times \cos ZS \times \sec PS, \text{ or}$$

$$\cos (\psi \pm u) = \cos u \, . \cos z \, . \sec p \quad . \quad . \quad . \quad (1.)$$

Again, since the angle QPS is the complement to SPT, (prop. XVIII.) tan PT$=$cos SPT × tan PS, or if PT$=r$, then

$$\tan r = \sin t \, . \tan p \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

and consequently the azimuth may be determined by the pole star along with the latitude, which is frequently necessary in finding the bearing of the sides of triangles with respect to the meridian in trigonometrical surveying.

TABLE for finding the Latitude by the Pole-Star.—By CAPTAIN KATER.

Polar Distance.	Tangent.	P. P. +	Secant.	Polar Distance.	Tangent.	P. P. +	Secant.
1° 33′ 0″	8.432315		0.000159	1° 35′ 0″	8.441560		0.000166
10	8.433093	1″ = 77	0.000159	10	8.442322	1″ = 75	0.000166
20	8.433870	2 = 154	0.000160	20	8.443082	2 = 151	0.000167
30	8.434645	3 = 231	0.000161	30	8.443841	3 = 226	0.000168
40	8.435419	4 = 308	0.000161	40	8.444599	4 = 302	0.000168
50	8.436191	5 = 385	0.000162	50	8.445355	5 = 377	0.000169
1° 34′ 0″	8.436962	6 = 462	0.000162	1° 36′ 0″	8.446110	6 = 453	0.000169
10	8.437732	7 = 539	0.000163	10	8.446864	7 = 528	0.000170
20	8.438500	8 = 616	0.000163	20	8.447616	8 = 604	0.000170
30	8.439267	9 = 693	0.000164	30	8.448368	9 = 679	0.000171
40	8.440033		0.000165	40	8.449117		0.000172
50	8.440797		0.000165	50	8.449866		0.000172

CORRECTION OF THE ALTITUDE OF THE POLE-STAR FOR DIURNAL ABERRATION.

Lat.	Time after Superior Transit.						
	0^h — 12^h +	1^h — 13^h +	2^h — 14^h +	3^h — 15^h +	4^h — 16^h +	5^h — 17^h +	6^h — 18^h +
°	″	″	″	″	″	″	″
0	0.00	0.08	0.15	0.22	0.27	0.30	0.31
5	0.00	0.08	0.15	0.22	0.26	0.29	0.30
10	0.00	0.08	0.15	0.21	0.26	0.29	0.30
15	0.00	0.07	0.14	0.20	0.25	0.28	0.29
20	0.00	0.07	0.14	0.20	0.25	0.28	0.28
25	0.00	0.07	0.13	0.19	0.24	0.27	0.27
30	0.00	0.07	0.13	0.19	0.23	0.26	0.26
35	0.00	0.06	0.12	0.18	0.21	0.24	0.25
40	0.00	0.06	0.12	0.17	0.20	0.23	0.23
45	0.00	0.05	0.11	0.16	0.18	0.21	0.22
50	0.00	0.05	0.10	0.14	0.17	0.19	0.20
55	0.00	0.04	0.09	0.12	0.15	0.17	0.18
60	0.00	0.04	0.08	0.11	0.13	0.15	0.15
65	0.00	0.03	0.07	0.09	0.11	0.12	0.13
70	0.00	0.03	0.05	0.07	0.09	0.10	0.11
75	0.00	0.01	0.03	0.04	0.05	0.05	0.05
80	0.00	0.01	0.03	0.04	0.05	0.05	0.05
85	0.00	0.01	0.01	0.02	0.02	0.05	0.03
90	0.00	0.00	0.00	0.00	0.00	0.02	0.00
	24^h + 12^h —	23^h + 11^h —	22^h + 10^h —	21^h + 9^h —	20^h + 8^h —	19^h + 7^h —	18^h + 6^h —

Ex. 8. At York Gate, Regent's Park, London, on the 22d of February, 1826, at $7^h\ 42^m\ 49^s$, mean time, the altitude of the pole star was observed by Captain Kater to be 51° 58′ 18″.1; required the latitude?

First to find the mean solar time when the star was upon the meridian.

*'s App. R. A.	0^h	53^m	$15^s.2$	App. alt.	51°	58′	18″	.1
⊙'s R. A. at noon,	22	21	18 .3	Refrac. —			45	.4
				D. Aber. —			0	.1
	2	36	56 .9	True alt.	51	57	32	.6
Difference from Table XXXI. —			25 .7	z =	38	2	27	.4
	2	36	31 .2					
Equation of time for noon, +		13	50 .7					
* Upon the Meridian,	2	50	21 .9					
Time of Observation,	7	42	49 .0					
Distance of Star from the meridian in mean time, W.	4	52	27 .1	=73° 18′ 47″ = t by Table XXXII.				

p = 1° 36′ 48″. tan	8.449716	secant	0.000172
t = 73 18 47 . cos	9.458097		
u = 0 27 48 .2 *tan	7.907813	cosine	9.999986
	z=38° 2′ 27.4″	cosine	9.896290
(ψ—u)=38 0 51 .2		cosine	9.896448
ψ =38 28 39 .4			
λ =51 31 20 .6			
p= 1° 36′ 48″ tan			8.449716
t =73 18 47 sin			9.981315
Const. log.			5.314425
$\frac{1}{3}$ secant			53
5565″.6 log.			3.745509

=1° 32′ 45″.6 = r = the azimuth W. from the pole.

It seems unnecessary to extend our remarks farther with regard to these observations, more especially if the examples in the explanation of the table XXVIII. be consulted.

If the observations are taken at sea with a reflecting instrument, on the principles of Hadley's quadrant, a correction must be made for the dip in addition to these already given. This may be taken from Table XI.; or the true altitude may be still more readily

* Found by precept, page 10 of Explanation of the Tables.

Were the author permitted to add any thing to what Captain Kater approves, it would be to employ the constant log 5.314425, the log of an arc = R″, Table LXIII., and the sum of these logs would be the log u in seconds, which would save the trouble of finding the value of the log tangent of small arcs, or even by taking the log of p in seconds, the errors thence arising, not much exceeding half a second, when the star is in its most unfavourable position, or when t is about 90°.

In the application of u, attention must be paid to the sign of the arc t, according to its situation in the circle which the star describes round the pole, in its diurnal revolution. If t is in the first or fourth quadrant, it is additive; but if in the second or third, it is subtractive.

found from Table XIII. or XIV., sufficiently correct for all the usual purposes at sea.

Correction of the Moon's Altitude.

Moon's Alt.	Moon's Horizontal Parallax. +							Diff. for Alt.
	54′	55′	56′	57′	58′	59′	60″	
	′	′	′	′	′	′	′	′
5°	43.8	44.8	45.8	46.8	47.8	48.8	49.8	+4.0
10	47.9	48.9	49.8	50.8	51.8	52.8	53.8	+0.7
15	48.6	49.6	50.5	51.5	52.5	53.4	54.4	—0.6
20	48.1	49.1	50.0	50.9	51.9	52.8	53.7	—1.4
25	46.9	47.8	48.7	49.6	50.5	51.4	52.3	—0.9
30	45.1	46.0	46.8	47.7	48.6	49.4	50.3	—2.4
35	42.9	43.7	44.5	45.3	46.1	47.0	47.8	—2.6
40	40.2	41.0	41.7	42.5	43.3	44.1	44.8	—3.2
45	37.2	37.9	38.6	39.3	40.0	40.8	41.5	—3.5
50	33.9	34.5	35.2	35.8	36.5	37.1	37.8	—2.8
55	30.3	30.9	31.4	32.0	32.6	33.2	33.7	—4.1
60	26.4	26.9	27.4	27.9	28.4	28.9	29.4	—4.3
65	22.4	22.8	23.2	23.6	24.1	24.5	24.9	—4.5
70	18.1	18.5	18.8	19.1	19.5	19.8	20.2	—4.8
75	13.7	14.0	14.2	14.5	14.7	15.0	15.3	—4.8
80	9.2	9.4	9.5	9.7	9.9	10.1	10.2	—4.9
85	4.6	4.7	4.8	4.9	5.0	5.1	5.2	—5.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
Co —	Height of Eye.	ft. 5 2′.2	ft. 10 3′.1	ft. 15 3′.8	ft. 20 4′.4	ft. 25 5′.0	ft. 30 5′.5	ft. 35 5′.9

Ex. 1.—May 1st, 1825, in longitude 64° 25′ W., the observed meridian altitude of the sun's *l. l.* was 48° 34′ 30″, the zenith being north of the sun, and the height of the eye 14 feet; what was the latitude?

May 1st at ship, time . .	$0^h\ 0^m$	Dec 1st	15° 4′ 19″ N.
Long. in time . . .	4 18	P. P.	+ 3 14
Gr. time, May 1st . .	4 18	R. D.	15 7 33 N.
Observed Altitude			48° 34′.5
Cor. to 48½°, 14 feet, and May			+ 11.5
True alt.			48 46.0
Z. D.			41 14.0 N.
Declination			15 7.6 N.
Latitude			56 21.6 N.

It is unnecessary to push the calculations nearer than tenths of a minute, as any observation taken at sea is, from the indistinctness of the horizon and the uncertainty of the horizontal refraction, unless a dip-sector be used, liable to an error of at least one minute.

Examples for Exercise.

2. On the 1st of September, 1824, in longitude 54° W., the meridian altitude of the sun's lower limb was 79° 44′ 15″ S., the height of the eye being 24 feet; what was the latitude?

Ans.—18° 30′.9 N.

3. On the 1st of January, 1826, the meridian altitude of the star Arcturus was 60° 41′ S., the height of the eye being 24 feet; what was the latitude?

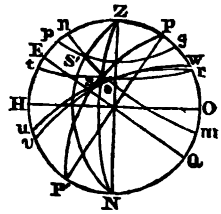
Ans.—49° 29′.8.

4. On the 14th September, 1827, in longitude 103° 18′ E., let the meridian altitude of the moon's lower limb be 51° 4′ N., and the height of the eye 20 feet; required the latitude?

Ans.—19° 48′.4 S.

4. On the 29th September, 1827, in longitude 20° 40′ W., if the observed meridian altitude of the moon's upper limb be 83° 6′ N., and the height of the eye 16 feet; required the latitude?

Ans.—21° 25′.7 S.



As the meridian altitude may, by the interposition of clouds, or other causes, be lost at sea when a knowledge of the latitude is necessary for the safety of the ship, recourse must be had to other methods, particularly to that of double altitudes, and the time between them, as being the most practicable.* This method requires solutions in three spherical triangles. In the triangle ZPS there are given PS the sun's polar distance at the time of the first observation, PS′ that at the second, and the angle S′PS measured by the elapsed time; to find the side S′S and the angle PS′S.† Again in the triangle ZS′S there are given the zenith distance ZS at the time of the observation, ZS′ that at the second, and the side S′S already found to determine the angle ZS′S. But PS′S being already computed, ZS′P may be obtained. Whence there are in the triangle ZS′P, the sides ZS′, and PS′, and the contained angle ZS′P; to find the side ZP the colatitude. This is the regular method by spherical trigonometry; but if the polar distance PS be supposed to remain the same, that at the middle time, between the observations, or, as Professor Lax seems to think preferable, the same as at the time of the greater altitude, and, by combining the solutions of the several triangles in one, the operation becomes more simple. In order to render this method still more easy to practical seamen, Douwes proposed an approximate method by introducing the latitude by account, which, when properly restricted according to the rules of Maskelyne or the tables of Lax, will generally give the desired result sufficiently correct for nautical purposes, and the computations may be very readily performed by the tables of Lynn.

When the common tables are used, Mr Ivory's solution is the best, particularly in the form of that Mr Riddle has given it, which we shall adopt here.

Find the sun's declination for the time of the greater altitude, and the true altitudes, reducing the less if necessary for the ship's run to what it would have been had it been taken at the same place with the greater. This is accomplished by observing the sun's bearing by compass, at the time of taking the less altitude, and, finding the

* On the authority of a very distinguished practical navigator, I am informed, that double altitudes are not of such importance as is generally supposed; for, if double altitudes can be successfully taken, meridian altitudes of the sun, moon, a planet, or a fixed star, may be generally obtained.

† A circle is supposed to pass through PS′ P′ similar to PSP′.

angle contained between that and the ship's course by compass, corrected for lee-way if she makes any, in the interval between the observations. With this angle as a course enter a traverse table, and the difference of latitude, answering to the distance run during the elapsed time, will be the reduction of altitude.

If the less altitude be observed in the forenoon, the reduction of altitude must be added to it, if the angle between the ship's course and the sun's bearing be less than eight points; but if that angle be greater than eight points, the reduction is to be subtracted from the less altitude. If the less altitude be observed in the afternoon, the reduction is to be subtracted from it, if the angle between the ship's course and the sun's bearing is less than eight points; but if greater, the reduction is to be added to the less altitude. With the corrected altitudes, the elapsed time, and the declination, the latitude at the time of the observation of the greatest altitude will be found, which may be reduced to noon by means of the dead reckoning.

1. Take half the interval between the observations, and call it the *half elapsed time.*

2. To the *sine* of the half elapsed time add the *sine* of the sun's polar distance, the sum, rejecting always ten in the index, will be *arc first.*

3. To the *secant* of arc first add the *cosine* of the polar distance, the sum will be the *cosine* of *arc second,* which will be of the same affection or character as the polar distance.

4. To the *cosecant* of arc first add the *cosine* of half the sum of the true altitudes, and the *sine* of half their difference, the sum will be the *sine* of *arc third.*

5. Add together the *secant* of arc first, the *sine* of half the sum of the true altitudes, the *cosine* of half their difference, and the *secant* of arc third, the sum will be the *cosine* of *arc fourth.*

6. The *difference* of *arc second* and *arc fourth* is *arc fifth,* when the zenith and the elevated pole are on the same side of the great circle, passing through the places of the sun at the times of observation, otherwise their *sum* is *arc fifth.**

7. To the *cosine* of *arc third* add the *cosine* of *arc fifth,* and the sum will be the *sine* of the *latitude.*

Ex. 1.—On the 6th of June, 1828, in latitude 58° N., and longitude 48° W., by account at $10^h\ 53^m\ 20^s$ A. M. per watch, the altitude of the sun's lower limb was 52° 20′, and at $1^h\ 17^m\ 8^s$, the altitude of the same limb was 52° 54′, and the bearing per compass S. W. by W. The ship's course during the elapsed time was S., the wind E. S. E., and hourly rate of sailing 8 knots, and the ship making $1\frac{1}{2}$ pts. of lee-way. Required the true latitude at the time of observation of the greatest altitude, the height of the eye being 16 feet?

* Should there be any doubt whether the zenith and elevated pole are on the same side of the great circle, passing through the places of the sun, the latitude may be computed on both suppositions, which, being compared with that by account, the true latitude will, in general, be readily discovered with little additional trouble, for it is only arc fourth and its cosine that will require alteration.

Ship's apparent course	S. or $0^{pts.}$
Lee-way . . .	$1\frac{1}{2}$
Ship's true course .	S. by W. $\frac{1}{2}$ W. = $1\frac{1}{2}$ pts. S. W.
Sun's bearing at 2d obs.	S. W. by W. = 5 pts. S. W.
Contained angle	$3\frac{1}{2}$

Interval between the observations $= 2^h\ 23^m\ 48^s = 2^h.4$

Distance run $= 2^h.4 \times 8 = 19.2$ miles.

Now to course $3\frac{1}{2}$ points and distance 19′.2, the difference of latitude is 14′.84, and since the least altitude was observed in the afternoon, and the angle between the ship's course and sun's bearing is less than eight points, this reduction is *subtractive.*

First observed alt	53° 20′	Second observed alt	51° 54′
Cor. Table XIII.	+ 11.2		+ 11.2
1. True alt .	53 31.2	Reduction .	− 14.8
		2 True alt .	52 50.4

1. True alt . .	53° 31′.2	
2.	52 50.4	
Sum	106 21.6	half 53° 10′.8 = 53° 10′ 48″
Difference . .	0 40.8	half 0 20.4 = 0 20 24

Times	$10^h\ 53^m\ 20^s$	Time .	$10^h\ 53^m\ 20^s$ A.M.
	13 17 8	Long W. .	3 12
Elapsed t.	2 23 48		14 5 20 A.M.
		on 6th at . .	2 5 20 P.M.
H. E. T.	1 11 54		

App. time	$2^h\ 5^m\ 20^s$	D. L. . . .	1.06030
Daily variation	5′ 55″	P. L. . . .	1.48320
Prop. part	0 31″	.	2.54350
Dec. at noon or 6th .	22° 41′ 17″ N.		
Reduced dec. . . .	22 41 48 N.		
Polar dist.	67 18 12		

9.489404 sin $1^h\ 11^m\ 54^s$ H. E. T.
9.965055 sin 67 18 12 pol dist cos 9.586422

9.454459 sin 16 32 37 arc 1st sec 0.018362 cosec . . 0.545624

66 15 52 arc 2d cos 9.604784

0.018362 sec arc 1
9.903374 sin 53° 10′ 48″ cos 9.777646
9.999993 cos 0 20 24 sin 7.773187

0.000034 sec 3d 0 42 56 sin 8.096457

33 22 8 arc 4. cos 9.921763 3d . cos 9.999966
32 52 44 arc 5. cos 9.924104

Latitude 57 5 51 N. arc 6. sine 9.924070

In this example the computation is carried to seconds of arc, but such a degree of accuracy is unnecessary at sea.

2. On the 6th of March, 1827, in latitude 60° N. by account, and longitude 105° E., the altitude of the sun's lower limb was observed to be 19° 42′ at 40^h 4^m 20^s in the forenoon, his centre bearing S. S. E. by compass, and at 1^h 32^m 36^s afternoon it was 21° 8′. The ship's course during the elapsed time was N. W. by N., sailing at the rate of 9 knots per hour, and the height of the eye 16 feet. Required the ship's latitude at the time of taking the greater altitude?

Ans.—60° 37′ N.

3. August 31, 1827, in altitude 12° 40′ S. by account and longitude 165° E. at 11^h 13^m 30^s A. M., the altitude of the sun's lower limb was 66° 9′ 30″, and at 1^h 15^m 12^s P. M. it was 62° 0′ 15″, bearing at the same time N. W. ½ W. During the elapsed time the ship was sailing S. W. by W. at the rate of 4 knots per hour, and the height of the observer's eye was 28 feet. Required the latitude at the time of taking the first altitude?

Ans.—11° 37′ S.

PROBLEM VI.

On finding the Longitude.

I. BY LUNARS.

Since the rotation of the earth about its axis is performed in a day, the sun appears to pass over 360° in 24 hours, and, consequently, over 15° in one hour; therefore it is obvious, that the difference of time between any two places will give the difference of longitude between those places.

A variety of methods have been proposed for determining the longitude of a place, but almost all of them depend upon one general principle, the comparison of the relative times under two different meridians; so that, if the time on two different meridians be known, the difference of these times turned into degrees, at the rate of 15° to an hour, will give the difference of longitude between these meridians.

As the sun apparently moves from the east towards the west, it is evident, that all places lying to the eastward of any meridian will have noon, or any other hour, sooner, or if westward, later, by the precise time the sun takes to pass from the meridian of the one place to that of the other. Hence, if the time on the meridian of Greenwich, the place from which our longitude is reckoned, and that of any other place at the same instant be known, the longitude of the latter place from Greenwich is also known, by turning the difference of time into degrees, at the rate of 15° to an hour.

Among the heavenly bodies which frequently present themselves for observation, there is none whose apparent velocity is so rapid with regard to the sun, planets, and fixed stars near the ecliptic, as that of the moon; the diurnal motion of that object being at a mean rate about 13° 11′. Hence, her distance from these bodies is continually changing in proportion to the time, and an error of 2″ in the distance between the moon and any of these bodies will produce an error of about 1′ only of longitude. Of all the various modes, then, which have been proposed to determine the longitude at sea, it is probable the method by lunar observations will continue to be

the most practicable. It appears also from the numerous observations lately made by several of our most distinguished navigators, that a series of lunars taken at land with good instruments, will, when great nicety in the requisite observations and calculations is attended to, give the longitude with singular accuracy.

The instruments generally employed are a good chronometer for connecting observations taken at different times with one another, two good quadrants for obtaining the altitudes, and a sextant or reflecting circle for taking the distance. These instruments are all described in our usual treatises on navigation and nautical astronomy.

If the sun or star be at a sufficient distance from the meridian at the time of taking the distance, the true altitude of either of these objects will serve to compute the apparent time at the ship, and this compared with the Greenwich time, derived from the lunar distance, will give the longitude. The same thing may be obtained from the moon's altitude, but less readily, as her right ascension and declination must be very accurately computed by applying the equation of second difference. If neither of these objects be in a proper situation for determining the time, the error of the chronometer must be found when either of them or some other object is in a convenient position before or after taking the lunar distances. In correcting altitudes observed at sea, the dip should properly be first applied, and then the refraction computed for the altitudes thus corrected.

On the Minute Corrections of Lunar Distances.

In lunar observations the corrections for the spheroidal figure of the earth are sometimes made by diminishing the equatorial horizontal parallax by the reduction for the latitude only; but unless the latitude and altitudes are in like manner reduced, which leads to a complex calculation, the results are still inexact. The method here proposed is similar to that of Mendoza Rios,* requiring only a small table to facilitate its application. The table has been computed by my ingenious friend, Mr Thomas Henderson, for an ellipticity of $\frac{1}{300}$, which seems to agree well with the latest measures, and to the mean horizontal parallax 57′, which is sufficiently accurate for practical purposes, as the greatest error can hardly exceed 1″, and at a mean not above half that quantity. This is within the limits of uncertainty arising from an error in the ellipticity, which seems to vary between $\frac{1}{295}$ and $\frac{1}{305}$ even from the best measures, the mean between which, $\frac{1}{300}$, has been here adopted. No doubt such refinements are unnecessary in the usual sea-practice, and in that

* See his Collection of Nautical Tables, published in 1801, for this method. Mendoza's Table XIII. (Edition 1805,) corresponds to our Table I., and his Table XIV. is merely the logs of twice the number in Table XIII., to save the constant log 0.30103. Consequently those who compute by Mendoza's Tables can easily employ his Tables XII., XIII., and XIV., by means of our rule.

case may be omitted; but as the lunar method, which is still capable of improvement, can be practised with great success at land, it was thought necessary to correct an erroneous rule, which I believe has been generally acted upon.

Rule.—When computing the parallax in altitude; to the logarithm of the earth's radius (Table XXIII.) add the secant of the moon's apparent altitude, and the proportional logarithm of the moon's equatorial horizontal parallax, the sum of these will be the proportional logarithm of the moon's parallax in altitude to be employed in computing the true distance. Now from half the sum of the moon's polar distance, the sun's or star's polar distances, and the distance of the moon from the sun or star, subtract the moon's polar distance, and the distance from the sun or star respectively. Then to the constant logarithm 0.30103, add the cosecant of the moon's distance from the sun or star, the sines of the two remainders, and the logarithm of the number from the table (III.) here given; the sum of these is the logarithm of the number of seconds to be *always* subtracted from the computed distance, while the number from the table itself is always to be added to it to give the true distance on the hypothesis of the earth being a spheroid of $\frac{1}{300}$ of ellipticity.

In general the correction of lunar distances for the earth's ellipticity is small, seldom amounting to 10″ of distance or 5′ of longitude, in most cases likely to occur in practice; and in any place within the tropics, the results on the spherical hypothesis may be considered almost perfectly correct. Within the polar circles, when the moon's equatorial parallax is diminished by the reduction for the latitude, the results will also be nearly correct.

On this subject Mr Henderson has remarked to me, that "the method prescribed by most authors, of allowing for the effects of the earth's spheroidal figure upon the lunar distances, by diminishing the equatorial parallax, is not altogether exact, but leaves an error uncorrected, which, at its *maximum* under any particular latitude, is nearly *one-sixtieth* of the reduction of latitude, or angle of the vertical with the radius. The greatest error therefore which can possibly happen in any part of the globe is under the parallel of 45°, where it may amount to 12″. Under the equator and poles the error is nothing.

"If the equatorial parallax be employed in the computation of the true distance, the result is liable to a greater error. The *maximum* error under any particular latitude may be expressed by the hypothenuse of a right-angled plane triangle, in which one side is equal to the *sixtieth* part of the reduction of latitude, and the other to the correction of the equatorial parallax. Under the parallel of London, the maximum error is 14″."

General Formula for taking a complete Set of Lunar Distances.

March 3, 1818, . . . Height of the eye 20 feet.
Latitude 15° 10′ 30″ S. Longitude 177° 10 E.
Barometer 28° 6 inches. Thermometer 72° Fahrenheit.

		h. m.
Ship time	.	18 24
Long. in time	.	11 48
Est. Greenwich time		6 36

	Time by Chronometer.	Obs. Alts. of Sun's lower limb.	Obs. Alts. of Moon's lower limb.	Obs. Dist. between the nearest limbs of Sun and Moon.
	h m s	° ′ ″	° ′ ″	° ′ ″
	18 20 11	6 59 0	45 31 0	40 5 10
	21 15	7 13 15	45 45 0	40 5 0
	22 25	7 24 45	45 56 15	40 4 50
	23 29	7 37 0	46 8 15	40 4 45
	24 35	7 48 30	46 19 30	40 4 40
	111 55	37 2 30	229 40 0	24 25
	18 22 23	7 24 30	45 56 0	40 4 53
Errors	+ 1 33	+ 2 15	+ 1 45	— 1 5
Correct	18 23 56	7 26 45	45 57 45	40 3 48
Dip to 20 feet . .		— 4 26	— 4 26	
App. alt. of lower limbs		7 22 19	45 53 19	

Sun's semidiameter	16′ 9″		
Moon's semidiameter	15 56	hor. par. 58′ 25″	
Augment.	+ 12		
	32 17	. . . .	32 17

To find the contraction of semidiameters:

d = 40	gives	A = 19 . .	19	
s = 8				
s' = 46				
94				
h = 47	. .	83 . .	83	
$h-s$ = 39	. .	80		
$h-s'$ = 1	. .	. .	924	
To s = 8 and	.	182 .	26	

The cor is — 1″ To s = 46 And 26 cor = 0

Correction arising from the contraction of the semidiameters of sun and moon . . — 1

Apparent central distance . . . 40 36 4

Should the farthest limb of either object be observed, then the semidiameter of that object must be subtracted.

This method will be rendered familiar by the following examples.*

Ex. 1.—September 24, 1827, in latitude 48° 50′ south, and longitude by account 120° west, at 8^h 18^m 30^s A. M., the following observations were made to obtain the true longitude; the height of the eyes of the observers being 30 feet above the surface of the sea, the angular instruments being perfectly adjusted when the English barometer stood at 29.4 inches, and Fahrenheit's thermometer at 60°.

The mean of five distances between the moon and sun's nearest limbs was 44° 33′ 45″, the altitude of the sun's lower limb 22° 4′ 15″, and the altitude of the moon's upper limb 6° 6′ 0″.

Time at ship	23^d 20^h 18^m 30^s	To this time by estimation.	
Longitude in time	8 0 0	the sun's semidiameter is	15′ 59″
		the moon's	16 2
Ext. Green. time	24 4 18 30	augmentation	2
Obs. dist. *n. l.*	44° 33′ 45″	hor. parallax	58 49
Sun's semidia.	+ 15 59		
Moon's semidia.	+ 16 2		
Augmentation	+ 2		
Cor. oblique *s. d.*	0		
App. cent. dist.	45 5 48		

$d = 45°$	gives A = 15		15	(Table I.)
$s = 6$				
$s = 23$				
74				
$h = 37$		90	90	
$h - s = 31$		71		
$h - s' = 14$			38	
Contr. 0		176	143 cont. 0	(T. II.)

Alt. sun's *l. l.*	22° 4′ 15″		alt. moon's *u. l.*	6° 6′ 0″
Dip to 30 feet	— 5 27			— 5 27
	21 58 48			6 0 33
Z. D.	68 1 12	log *t* 2.15762		83 59½ l. *t* 2.70689
Thermometer	60°.0 F.	9.99104		
Barometer	29 .4 E.	9,99123		
Thermometer	60 .0 F.	9.99957		
$r = 137''.86$		9.98184		9.98184
Or 2′ 17″.86		2.13946	$r = 488''.34$	2.68873
For the sun			or $= 8' 8''.34$	

$-0.104 \times (60-50) = -\cdot104 \times 10 = -1\ .04$

$+0.15 \times (30-29.4) = \cdot15 \times \cdot6 = +0\ .09$

True refraction for the moon $= 8\ 7\ .39$

* The necessary computations are readily and accurately performed, according to the rules of spherical trigonometry from the tables contained in this work. There are several collections of tables, such as those of Mendoza Rios, Lax, Lynn, and Thomson, which, for general practice at sea, by abating something of rigorous accu-

Alt. sun's *l. l.*	22° 4′ 15″	Alt. moon's *u. l.*	6° 6′ 0″
Dip to 30 feet	— 5 27	. .	— 5 27
	21 58 48		6 0 33
Semidiameter	+ 15 59	Semidiameter augm.	—16 4
App. alt.	22 14 47	App. altitude .	5 44 29
Refraction	— 2 18	Refraction .	— 8 7
		Parallax . .	+ 58 38
Parallax	+ 3		
		Correct alt. .	6 35 0
Sun's T. alt.	22 12 32		

Latitude	48° 50′ 0″	Log rad. .	9.99918
App. alt. moon	5 44 29	. .	Secant 0.00218
Hor. par. .	58 49	. .	P. L. 0.48577
Par. in alt. .	58 38	. .	P. L. 0.48713

The reduction of the apparent to the true distance is effected by the solution of two spherical triangles. First the angle at the zenith is found from the triangle formed by the apparent zenith distances and apparent distance. Next the true distance is computed from the angle at the zenith and the true zenith distances, and these two may be combined in the following manner :—

App. dist.	45° 5′ 48″		
App. alt. ☉	22 14 47	secant	0.033593
App. alt. ☽	5 44 29	secant	0.002184
Sum .	73 5 4		
Half . .	36 32 32	cosine	9.904942
Difference	8 33 16	cosine	9.995141
True alt. ☉	22 12 32	cosine	9.966522
True alt. ☽	6 35 0	cosine	9.997127
Sum .	28 47 32		19.899509
Half .	14 23 46		
Arc .	17 1 58	cosine	9.949754
Sum	41 25 44	sine	9.820662
Difference	12 38 12	sine	9.339984
			19.160646
½ Dist.	22 21 46½	sine	9.580323
	2		
	44 43 33.		
Cor.	— 4		
True dist.	44 43 29		

racy, render the calculations more simple. Some of them, however, are rather bulky and expensive.

☽'s Pol. dist.	74° 22′			
⊙'s P. D.	89 40			
App. dist.	45 6	cosecant		0.14976
Sum .	209 8	const. log		0.30103
Half .	104 34			
First rem.	30 12	sine		9.70159
Sec. rem.	59 28	sine		9.93517
No from Table III.	+ 16″.6	log		1.22011
	— 20 .3	log .		1.30766
Correction	— 3 .7			
True dist.	44 43 29			
Dist. at 3^h	44 1 21	0° 42′ 8″	P. L.	0.63065
6	45 38 36	1 37 15	P. L.	0.26738
Time past 3^h	$1^h\ 17^m\ 59^s$.		P. L.	0.36327
Preced. time	3			
Approx. time	4 17 59			

	1st Diff.	*2d Diff.*		*Mean.*
Dist. at noon	42° 24′ 13″			
		1° 37′ 8″		
3^h	44 1 21		7″	
		1 37 15		+7″
6	45 38 36		7	
		1 37 22		
9	47 15 58			

Equation of second difference to $1^h\ 18^m$ and +7″ is 1″ in Table IV. Now to var. 1° 37′ and 1′ the cor. from Table V. is $+2^s$.

This correction must always be applied according to the sign of the second difference.

To the approximate time	4^h	17^m	59^s
Add. cor. from sec. diff.	+		2*
True T. at Greenwich 24^d 4		18	1

Computation of the time derived from the figure in page 87, Theorem IX., page 93, after the examples in page 100.

Sun's T. alt.	22° 12′ 33″		
Sun's pol. dist.	89 40 33	cosecant .	0.000007
Latitude	48 50 0	secant .	0.181608
Sum .	160 43 6		
Half . .	80 21 33	cosine	9.223941
Diff. .	58 9 0	sine	9.929129
			19.334685

* The equation of second difference happens to be small in this example. It may amount to 6 seconds of distance, 12 seconds of time, or 3′ of longitude in some cases. The correction for second difference is obtained by Tables IV. and V. immediately following this article, which have been computed by the author expressly for this purpose.

			19.334685
½ Time from noon	$1^h\ 50^m\ 48^s.6$	sine*	9.667343
	2		
Time from noon	3 41 37 .2		
	24		
App. time	23^d 20 18 23 .0		
App. T. Green.	24 4 18 1 .0		
Lon. in time	7 59 38 .0=119° 54′ 30″ West.		

Ex. 2.—On September 12th, 1823, in latitude 26° 30′ N., longitude by account 26° 30′ W. at $5^h\ 34^m$ P. M. by watch, the altitude of the sun's lower limb was 7° 37′, that of the moon's lower limb was 35° 35′, the distance of their nearest limbs 95° 19′ 58″, the barometer being 30.28 inches, and the thermometer 72°.4 Fahrenheit, and the height of the eye 25 feet; what was the longitude?

Time per watch .	$5^h\ 34^m$		
Longitude 26½ W. in time	+ 1 46		
Estimated Greenwich time	7 20 P. M.		
Moon's semidiameter at noon	14′ 52″	parallax	54′ 31″
Correction for Greenwich time	— 2	correc. for $7^h\ 20^m$	— 5
	14 50	equatorial par.	54 26
Augmentation .	+ 8		
True semidiameter	14 58		

Alt. of sun's *l. l.*	7° 37′ 0″		Moon's	35° 35′ 0″	
Dip to 25 feet	— 4 58			— 4 58	
	7 32 2			35 30 2	
Z. D. .	82 28 log *ϑ*	2.61313	Z. D.	54 30	log *ϑ* 1.91236
	P.P.8′	693			
Thermometer	72°. 4 log	9.98020			9.98020
Barometer	30 . 28 log	0.00289			0.00289
		114			114
Thermometer	72 . 4 log	9.99902			9.99902
r	401″.2	2.60331	*r* 78″.6=1′ 18″.6		1.98561
Or	6′ 41 .2	lat.		26° 30′ log R.	9.99971
—.062 × +22.4=—	1 .4	A. alt. moon		35 45 secant	0.09067
+·09 × +·28 =+	0 .0	hor. par.		54′ 26″ P. L.	0.51941
r .	6 39 .8	parallax in alt.		44 12 P. L.	0.60979

* When the time to be computed is in the morning, read cosine here instead of sine, and the result being [illegible] will give the time reckoned astronomically from [illegible].

Alt. sun's *l. l.*	7°37′ 0″	alt. of moon's *l. l.*	35° 35′ 0″
Dip to 25 feet	— 4 58	dip to 25 feet .	— 4 58
			38 30 2
	7 32 2	semidiameter .	+ 14 58
Semidiameter	+ 15 56	app. altitude .	35 45 0
		refraction . .	— 1 18
App. alt.	7 47 58	parallax .	+ 44 12
Refraction	— 6 40		
Parallax	+ 9	moon's true alt.	36 27 54
Sun's true alt.	7 41 27		

Observed distance . . .	95° 19′ 58″
Sun's semidiameter . . .	+ 14 58
Moon's semidiameter . . .	+ 15 56
Correction from contraction of semidiameter . .	— 2
App. central dist.	95 50 50

d = 96°	A .	0 .	0
s = 36			
s' = 8			
140			
h = 70 =	.	53 .	53
$h-s$ = 34 =	.	75	
$h-s'$ = 62 =	.	. .	95
Moon's cor.=0		128 ☉'s=—2″	148

Hence 2″ must be subtracted from the sun's semidiameter, or from the central distance; but, to prevent mistakes, it would be the better way to subtract it always from the semidiameter.

App. dist.	95° 50′ 50″		
Sun's app. alt	7 47 58	secant .	0.004036
Moon's app. alt.	35 45 0	secant	0.090672
Sum	139 23 48		
Half .	69 41 54	cosine .	9.540283
Difference .	26 8 56	cosine .	9.953108
Sun's true alt.	7 41 28	cosine .	9.996075
Moon's true alt.	36 27 54	cosine .	9.905376
Sum .	44 9 22		19.489550
Half .	22 4 41		
Arc . .	56 14 52	cosine .	0.744775
Sum .	78 19 33	sine .	9.990922
Difference .	34 10 11	sine	9.749463
			19.740385

			19.740385
Half dist.	47° 52′ 16″	sine	9.870192
	2		
	95 44 32		
Correction .	+ 1		
True dist.	95 44 33		

☽'s Pol. dist.	116° 1′				
☉'s P. D.	85 43				
App. dist .	95 50	cosecant			0.00226
Sum .	297 34	const. log			0.30103
Half .	148 47				
First rem. . .	32 46	sine .			9.73337
Second rem. . .	52 57	sine . .			9.90206
No from Table III.	+ 9″.4	log .			0.97313
	— 8.2	log			0.91185
Correction	+ 1.2				
True distance	95° 44′ 33″		0° 36′ 16″	P. L.	0.69576
Dist. at 6^h	95 8 17		1 21 56	P. L.	0.34181
9	96 30 13				
Time past 6^h			$1^h\ 19^m\ 40^s$	P. L.	0.35395
Time of first distance .			6		
Approximate app. time at Greenwich			7 19 40		

To find the correction for second difference.

	1st Diff.	*2d Diff.*	*Mean.*
Dist. at 3^h 93° 46′ 14″			
6 95 8 17	1° 22′ 3″		
8 96 30 13	1 21 56	—7″	—7″
12 97 52 2	1 21 49	—7″	

To second difference —7″ and $1^h\ 20^m$ the equation of second difference is about 1″ to which, and variation 1° 22′ in 3^h the correction is —2^s.

From approximate apparent time . .	$7^h\ 19^m\ 40^s$
Subtract equation now found . .	— 2
True apparent time at Greenwich .	7 19 38

To find the apparent time at the place of observation.

The reduced declination is found as in the explanation of Table IX. and XXVII., then

Latitude	26° 30′ 0″ N.		secant . .	0.048209
Declination	4 17 6 N.		secant . .	0.001216
Difference	22 12 53			
Zenith dist.	82 18 33			
Sum	104 31 26	half 52 15 43	sine .	9.898076
Difference	60 5 40	half 30 2 50	sine .	9.699589

					19.647090
	2^h	47^m	4^s	sine	9.823545
			2		
App. time	5	34	8		
Greenwich time	7	19	38		
Longitude in time	1	45	30 W. = 26° 22′ 30″ W.		

Or about two miles more than Mr E. Riddle makes it in his treatise on navigation,—a very useful work, combining theory with practice, a method too much neglected in the present plan of nautical instruction.

Ex. 3.—On the 14th of June, 1827, in latitude 28° 31′ 10″ N., and longitude 144° W. by account, at about $20^h\ 32^m$, the distance between the sun and moon was observed to be 97° 22′ 40″; when the altitude of the sun's lower limb was 44° 36′ 40″, the altitude of the moon's upper limb was 35° 38′ 20″, the height of the eye being 20 feet; required the longitude, the barometer being at 29.68 inches, and Fahrenheit's thermometer at 68°?

Estimated time, June 14th		$20^h\ 32^m$
Longitude 144° W. in time	+	9 36
Approximate Greenwich time, June 15th .		5 8

To this time moon's semidiameter is 15′ 37″ hor. par. 57′ 18″
Augmentation to 36° alt. . . + 9

Correct semidiameter . . 15 46

Alt. of sun's *l.l.*	44° 36′ 40″	.	moon's *u.l.*	35° 38′ 20″	
Dip to 20 feet	— 4 26	. . . .		— 4 26	
	44 32 14			35 33 54	
Zenith dist.	45 27.8 lo.	1.77318	Z.D.	54 26.1 lo.	1.91230
Thermometer	68	9.98401	. . .		9.98401
Barometer	29.7	9.99563	. . .		9.99563
Thermometer	68	9.99922	. . .		9.99922
r 56″.5		1.75204,	*r* 1′ 17″.8 . .		1.89116

		Latitude	28 31	log R.	9.99967
Sun's semidiameter	15′ 46″	moon's alt.	35° 34	sec	0.08968
Parallax in alt.	6	hor. par.	57′ 18	P.L.	0.49712
		par. in alt.	46 38	P.L.	0.58647

Alt. sun's *l. l.*	44° 36′ 40″	moon's alt. *u. l.* .	35° 38′ 20″
Dip. to 20 feet	— 4 26	. . . . . .	— 4 26
Semidiameter	+ 15 56	semidiameter .	— 15 46
App. alt.	44 48 0	app. alt. . .	35 18 8
Refraction	— 56	refraction . .	— 1 18
Parallax	+ 6	parallax . .	+ 46 38
True alt.	44 47 10	true alt. . .	36 3 28

Observed distance of nearest limbs . . .	97° 22′ 40″
Sun's semidiameter	+ 15 46
Moon's semidiameter	+ 15 46
Correction for contraction of S. D. . .	— 2
Apparent central distance . . .	97 54 10

Now to compute the correction of the oblique semidiameters, by Dr Young's method, there are given,

d = 97°	which by Table I. gives A =	0	0
s = 45			
s' = 36			
178			
h = 89		=924	924
$h-s$ = 44		= 84	
$h-s'$ = 53		. . . .	90
		8	14

As these give in Table II., 1″ for the sun and 1″ for the moon, or 2″ in all, it is necessary to subtract them from the apparent or even true distance when they are so small.

Apparent distance	97° 54′ 10″		
Sun's app. altitude	44 48 0	secant	0.149004
Moon's app. altitude	35 10 8	secant	0.088248
Sum	178 0 18		
Half	89 0 9	cosine	8.240768
Difference . .	8 54 1	cosine	9.994739
Sun's true alt. .	44 47 10	cosine	9.851100
Moon's true altitude	36 3 28	cosine	9.907633
Sum	80 50 38		18.231492
Half	40 25 19		
Arc	82 30 3	cosine	9.115746
Sum	122 55 22	sine	9.923972
Difference . .	42 4 44	sine	9.826174
			19.750146
Half dist. . . .	48° 35′ 32″	sine	9.875073
	2		
	97 11 4		
Correction .	+ 4		
	97 11 8		

☽'s pol. dist.	91° 18′		
☉'s pol. dist.	66 41		
App. dist.	97 53	cosecant .	0.00412
Sum . .	255 52	const log .	0.30103
Half . . .	127 56		
First rem. . .	36 38	sine . .	9.77575
Second rem. . .	30 3	sine . .	9.69963
Num. from Table III.	+ 10.9	log . .	1.03743
	− 6.6	log . .	0.81796
Correction . .	+ 4.3		

True dist. .		97 11 8		
Dist. at .	6^h	97 18 52	0° 7′ 44″ P.L.	1.36691
Dist. at	9^h	95 47 0	1 31 52 P.L.	0.29211
			$0^h\ 15^m\ 9^s$ P.L.	1.07480
			6	
			6 45 9	

Dist. at 3^h 98° 51′ 7″			
6 97 18 52	1° 32′ 15″	— 23″	Mean.
9 95 47 0	1 31 52	— 23	— 23″
12 94 15 31	1 31 29		

To 15^m and 23″ the equation of second difference is 1″, which, for a variation of 1° 32′ nearly, gives 2^s of time to be subtracted, whence the true time is $6^h\ 15^m\ 7^s$ of the 25th of June, or $30^h\ 15^m\ 7^s$ after the noon of the 14th.

To compute the time.

True altitude . .	44° 47′ 10″		
Polar distance . . .	66 41 10	cosecant	0.036992
Latitude . . .	28 31 20	secant	0.056193
Sum	139 59 40		
Half	69 59 50	cosine	9.534111
Difference . . .	25 12 40	sine	9.629364
			19.256660
Half	$4^h\ 19^m\ 24^s.7$	cosine	9.628330
	2		78
App. time 14th + 12 =	20 38 49.4		66 | 48.0
App. time at Greenwich	30 15 7.0		0.7
Longitude	9 36 17.6 = 144° 4′ 24″ W.		

Ex. 4. On the 29th of March, 1826, in latitude 56° 12′ S., and longitude by account 30′ E. at about $1^h\ 32^m$ A. M., the observed distance between the moon's nearest limb and the star Fomalhaut, east of the

meridian, was, from a mean of five sets of observations, 61° 56′ 30″; the observed altitude of the moon's lower limb was 32° 4′; the observed altitude of the star 6° 16′; the barometer being 29.2 inches, the thermometer 42° F., and the height of the eye 20 feet; what was the true longitude?

Est. time	$13^h 32^m$	moon's equatorial hor. par.	58′ 14″
Long. in time	0 2 E.	moon's semidiameter .	15 52
		augmentation to 32° .	+ 9
Est. G. time	13 30	augm. semidiameter .	16 1

Now to correct the oblique semidiameter by Dr Young's method from Tables I. and II. we have

d	= 62° gives A =		5 in Table I.
s	= 6		
s'	= 32		
	100		
h	= 50	= 81	
$h-s$	= 44	= 84	
Sum		170 give Cor. = 0 in Table II.	

Observed distance	61° 56′ 30″
Moon's aug. semidiameter . . .	+ 16 1
App. central distance	62 12 31

Alt. of star	6° 16′ 0″	alt of moon's $l.\ l.$	32° 4′ 0″		
Dip. to 20 feet	− 4 26	. . . .	− 4 26		
	6 11 34		31 59 34		
Z. D.	83 48.5 log s	2.69563	Z. D.	58 0.5 log s	1.96969
Thermometer	42° F.	. . .		0.00730	
Barometer	29.2 in	. . .		9.98826	
Thermometer	42°	. . .		0.00034	
		9.99590	.	9.99590	9.99590
r'	= 491″.5	2.69153	.	r=1′ 32″4 log	1.96559
Or	=8′ 11″.5		Latitude 56° 12′ log R.		9.99900
−0″.1 × −8	=+ 0 .8		Moon's alt. 32° 0′ secant		0.07158
0″.14 × .8	=+ 0 .1		Hor. par. 58′ 14″ P.L.		0.49010
r	=8 12 .4		Par. in alt. 49′ 30″ P.L.		0.56068
Alt. of star	6° 16′ 0″		alt. of moon's $l.\ l.$		32° 4′ 0″
Dip. to 20 feet	− 4 26		dip. to 20 feet		− 4 26
App. alt.	6 11 34				31 59 34
Refraction	− 8 12		semidiameter		+ 16 1
True alt. of star	6 3 22		app. alt. centre		32 15 35
			refraction		− 1 32
			par. in alt.		+ 49 30
			true alt. centre		33 3 33

App. dist.	62°	12′	31″		
Star's app. alt.	6	11	34	secant	0.002542
Moon's app. alt.	32	15	35	secant	0.072816
Sum	100	39	40		
Half	50	19	50	cosine	9.805067
Diff.	11	52	41	cosine	9.990600
Star's true alt.	6	3	22	cosine	9.997569
Moon's true alt.	39	3	34	cosine	9.923298
Sum	39	6	56		19.791892
Half	19	33	28		
Arc	38	5	56	cosine	9.895946
Sum	57	39	24	sine	9.926783
Diff.	18	32	28	sine	9.502407
					19.429190
	31	13	10	sine	9.714595
			2		
	62	26	20		
Cor.	+		8		
True dist.	62	26	28		

Moon's pol. dist.	68°	59′		
Star's pol. dist.	59	28		
App. dist.	62	13	cosecant	0.05320
Sum	190	40	const. log	0.30103
Half	95	20		
First rem.	26	21	sine	9.64724
Second rem.	33	7	sine	9.73747
No from Table V.	+ 18″.9		log	1.27646
	− 10 .4		log	1.01540
Correction	+ 8 .5			

True dist.		62	26	28			
Dist. at	12^h	63^m	10^s	41	0° 44′ 13″	P. L.	0.60969
Dist. at	15	61	41	45	1 28 56	P. L.	0.30621
					1^h 29^m 30^s	P. L.	0.30348
Preceding hour					12		
Approximate app. time					13 29 30 at Greenwich.		

64° 40′ 19			
63 10 41	1° 29′ 38″	— 42″	— 43″.5 or — 44″ nearly.
61 41 45	1 28 56	— 45	
60 13 35	1 28 11		

Now to approximate time $1^h\ 30^m$, and second difference — 44″, the equation of second difference is 5″.5, to which and variation 1° 29′ nearly in 3 hours, the final equation in time is about 11^s to be subtracted. Whence from $13^h\ 29^m\ 30^s$ this equation of 11^s being subtracted, the true apparent time is $13^h\ 29^m\ 19^s$ at Greenwich.

To compute the apparent time at ship.

Star's true alt.	6° 3′ 22″		
Polar dist.	59 27 40	cosecant . .	0.064853
Latitude	56 12 0	secant . . .	0.254694
Sum .	121 43 2		
Half .	60 51 31	cosine . .	9.687499
Diff. .	54 48 9	sine :	9.912312
*'s merid. dist.	$15^h\ 14^m\ 28^s.56$	Reduced versine	9.919358
*'s R. A.	22 48 1.47		
Comp. ☉'s R. A.	23 27 33.88		
App. time	13 30 3.91 — 48^h		
Greenwich time	13 29 19.00		
Long. in time	44.91=11′ 15″ E.		

If the operations had been performed by the ordinary tables without attention to the smaller equations, the longitude would be about 30″ E. and the error on that account 10′ 45″.

Ex. 5.—Let the example to the general formula, page 120, be solved as an exercise.

TABLE I.

CORRECTION FOR THE OBLIQUE SEMIDIAMETER.

For Argument A.

For h $h-s$ d				For h $h-s$ d				For h $h-s$ d			
°	A	°	A	°	A	°	A	°	A	°	A
89	924	1	176	59	71	31	29	29	94	61	6
88	954	2	146	58	72	32	28	28	95	62	5
87	972	3	128	57	74	33	26	27	95	63	5
86	984	4	116	56	75	34	25	26	95	64	5
85	994	5	106	55	76	35	24	25	96	65	4
				54	77	36	23	24	96	66	4
84	2	6	98	53	78	37	22	23	96	67	4
83	9	7	91	52	79	38	21	22	97	68	3
82	14	8	86	51	80	39	20	21	97	69	3
81	19	9	81	50	81	40	19	20	97	70	3
80	24	10	76								
79	28	11	72	49	82	41	18	19	98	71	2
78	32	12	68	48	83	42	17	18	98	72	2
77	35	13	65	47	83	43	17	17	98	73	2
76	38	14	62	46	84	44	16	16	98	74	2
75	41	15	59	45	85	45	15	15	98	75	2
74	44	16	56	44	86	46	14	14	99	76	1
73	47	17	53	43	86	47	14	13	99	77	1
72	49	18	51	42	87	48	13	12	99	78	1
71	51	19	49	41	88	49	12	11	99	79	1
70	53	20	47	40	88	50	12	10	99	80	1
69	55	21	45	39	89	51	11	9	99	81	1
68	57	22	43	38	90	52	10	8	100	82	0
67	59	23	41	37	90	53	10	7	100	83	0
66	61	24	39	36	91	54	9	6	100	84	0
65	63	25	37	35	91	55	9	5	100	85	0
64	64	26	36	34	92	56	8	4	100	86	0
63	66	27	34	33	92	57	7	3	100	87	0
62	67	28	33	32	93	58	7	2	100	88	0
61	69	29	31	31	93	59	7	1	100	89	0
60	70	30	30	30	94	60	6	0	100	90	0

In this table h is half the sum of the apparent distance and altitudes of the moon, and the sun or a fixed star s, the altitude of the moon, s' that of the sun, or a star and d the apparent distance.

TABLE II.

CORRECTION FOR THE OBLIQUE SEMIDIAMETER.

DIMINUTION OF THE SEMIDIAMETER.

Argument $A\,(h)+A\,(h-s)+A\,(d)$.

	Altitude.													
Sum of A	5°	6°	7°	8°	9°	10°	11°	12°	14°	16°	18°	20°	30°	45°
0″	25″	19″	14″	11″	9″	8″	6″	5″	4″	3″	3″	2″	1″	1″
20	24	18	14	11	9	7	6	5	4	3	2	2	1	0
40	23	17	13	10	8	7	6	5	4	3	2	2	1	0
60	21	16	12	9	8	6	5	5	3	3	2	2	1	0
70	20	15	12	9	8	6	5	5	3	3	2	2	1	0
80	19	14	11	8	7	6	5	4	3	2	2	2	1	0
90	17	13	10	9	7	6	5	4	3	2	2	2	1	0
100	16	12	9	7	6	5	4	4	3	2	2	1	1	0
110	14	10	8	6	5	4	3	3	2	2	1	1	1	0
120	11	9	7	5	4	3	2	2	2	1	1	1	0	0
130	9	7	5	4	3	3	2	2	1	1	1	1	0	0
135	7	6	4	3	2	2	2	1	1	1	1	0	0	0
140	6	5	4	3	2	2	1	1	1	1	1	0	0	0
145	5	4	3	2	2	1	1	1	1	0	0	0	0	0
150	3	3	2	2	1	1	1	1	0	0	0	0	0	0
155	3	2	2	1	1	1	1	0	0	0	0	0	0	0
160	1	1	1	0	0	0	0	0	0	0	0	0	0	0
170	0	0	0	0	0	0	0	0	0	0	0	0	0	0
178	1	1	1	0	0	0	0	0	0	0	0	0	0	0
180	2	1	1	1	1	1	0	0	0	0	0	0	0	0
182	3	2	2	1	1	1	1	0	0	0	0	0	0	0
184	4	3	2	2	1	1	1	1	1	1	0	0	0	0
186	5	4	3	3	2	2	1	1	1	1	1	1	0	—
188	7	6	4	3	2	2	2	1	1	1	1	1	1	—
190	9	7	5	4	3	3	2	2	1	1	1	1	1	—
191	10	8	6	4	4	3	3	2	2	1	1	1	1	—
192	11	9	7	5	4	4	3	3	2	2	1	1	1	—
193	12	9	7	5	5	4	3	3	2	2	2	1	1	—
194	14	10	8	6	5	4	4	3	2	2	2	2	1	—
195	15	11	9	6	6	5	4	4	3	2	2	2	—	—
196	17	13	10	7	6	6	5	4	3	3	2	2	—	—
197	19	14	11	8	7	6	5	5	3	3	2	2	—	—
198	21	16	12	9	8	7	6	5	3	3	3	—	—	—
199	23	17	13	10	8	8	6	5	4	—	—	—	—	—
200	25	19	14	11	9	—	—	—	—	—	—	—	—	—
Alt......	5°	6°	7°	8°	9°	10°	11°	12°	14°	16°	18°	20°	30°	45°

The correction from this table is always to be subtracted from the semidiameter found from the Nautical Almanac.

TABLE III.—*Correction for Mean Horizontal Parallax, to be added to the Lunar Distances on account of the Spheroidal Figure of the Earth, its Ellipticity being* $\frac{1}{300}$.

BY MR HENDERSON.

Lat.	Moon's Declination. 0°	5°	10°	15°	20°	25°	30°	Lat.	Moon's Declination. 0°	5°	10°	15°	20°	25°	30°
	″	″	″	″	″	″	″		″	″	″	″	″	″	″
0°	0.0	0.0	0.0	0.0	0.0	0.0	0.0	46°	16.4	16.3	16.1	15.8	15.4	14.9	14.2
2	0.8	0.8	0.8	0.8	0.8	0.7	0.7	48	16.9	16.9	16.7	16.4	15.9	15.4	14.7
4	1.6	1.6	1.6	1.5	1.5	1.4	1.4	50	17.5	17.4	17.2	16.9	16.4	15.9	15.2
6	2.4	2.4	2.4	2.3	2.2	2.2	2.1	52	18.0	17.9	17.7	17.4	16.9	16.3	15.6
8	3.2	3.2	3.1	3.1	3.0	2.9	2.7	54	18.5	18.4	18.2	17.8	17.3	16.7	16.0
10	4.0	3.9	3.9	3.8	3.7	3.6	3.4	56	18.9	18.8	18.6	18.3	17.8	17.1	16.4
12	4.7	4.9	4.7	4.6	4.4	4.3	4.1	58	19.3	19.3	19.0	18.7	18.2	17.5	16.7
14	5.5	5.5	5.4	5.3	5.2	5.0	4.8	60	19.7	19.7	19.4	19.1	18.6	17.9	17.1
16	6.3	6.2	6.2	6.1	5.9	5.7	5.4	62	20.1	20.1	19.8	19.5	18.9	18.3	17.5
18	7.0	7.0	6.9	6.8	6.6	6.4	6.1	64	20.5	20.4	20.2	19.8	19.3	18.6	17.8
20	7.8	7.8	7.7	7.5	7.3	7.1	6.7	66	20.8	20.8	20.5	20.1	19.6	18.9	18.1
22	8.5	8.5	8.4	8.2	8.0	7.7	7.4	68	21.2	21.1	20.9	20.5	19.9	19.2	18.4
24	9.3	9.2	9.1	8.9	8.7	8.4	8.0	70	21.5	21.4	21.1	20.7	20.2	19.5	18.6
26	10.0	10.0	9.8	9.7	9.4	9.1	8.7	72	21.7	21.6	21.4	21.0	20.4	19.7	18.8
28	10.7	10.6	10.5	10.3	10.0	9.7	9.3	74	21.9	21.8	21.6	21.2	20.6	19.9	19.0
30	11.4	11.3	11.2	11.0	10.7	10.3	9.9	76	22.1	22.1	21.8	21.4	20.8	20.1	19.2
32	12.1	12.0	11.9	11.7	11.4	11.0	10.5	78	22.3	22.3	22.0	21.6	21.0	20.2	19.3
34	12.7	12.7	12.5	12.3	12.0	11.5	11.0	80	22.5	22.4	22.1	21.7	21.1	20.4	19.5
36	13.4	13.3	13.2	12.9	12.6	12.1	11.6	82	22.6	22.5	22.2	21.8	21.2	20.4	19.5
38	14.0	14.0	13.8	13.6	13.2	12.7	12.2	84	22.7	22.6	22.3	21.9	21.3	20.5	19.6
40	14.7	14.6	14.4	14.2	13.8	13.3	12.7	86	22.7	22.6	22.4	22.0	21.4	20.6	19.7
42	15.3	15.2	15.0	14.7	14.3	13.8	13.2	88	22.7	22.6	22.4	22.0	21.4	20.6	19.7
44	15.8	15.8	15.6	15.3	14.9	14.3	13.7	90	22.8	22.7	22.5	22.1	21.5	20.7	19.8

TABLE IV.—*Equations of Second Difference for three Hours.*

Time.				Second Difference. 10″	20″	30″	40″	50″	60″	70″	80″	90″	100″	1′	2″	3″	4″	5″	6″	7″	8″	9″	M.	
h.	m.	h.	m.	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	′	′
0	0	3	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	60
	3		57	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	1	59
	6		54	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.5	1.6	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	2	58
	9		51	0.2	0.5	0.7	1.0	1.2	1.5	1.7	1.9	2.1	2.4	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	3	57
	12		48	0.3	0.6	0.9	1.2	1.6	1.9	2.2	2.5	2.8	3.1	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.3	4	56
	15		45	0.4	0.8	1.1	1.5	1.9	2.3	2.7	3.1	3.4	3.8	0.0	0.1	0.1	0.2	0.2	0.2	0.3	0.3	0.3	5	55
	18		42	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1	4.5	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	6	54
	21		39	0.5	1.0	1.5	2.1	2.6	3.1	3.6	4.1	4.6	5.2	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	7	53
	24		36	0.6	1.2	1.7	2.3	2.9	3.5	4.0	4.6	5.2	5.8	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.5	0.5	8	52
	27		33	0.6	1.3	1.9	2.6	3.2	3.8	4.5	5.1	5.7	6.4	0.1	0.1	0.2	0.3	0.3	0.4	0.4	0.5	0.6	9	51
0	30	2	30	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3	6.9	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.6	10	50
	33		27	0.7	1.5	2.2	3.0	3.7	4.4	5.2	6.0	6.7	7.5	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.7	11	49
	36		24	0.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2	8.0	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.6	0.7	12	48
	39		21	0.8	1.7	2.5	3.4	4.2	5.1	5.9	6.8	7.6	8.5	0.1	0.2	9.3	0.3	0.4	0.5	0.6	0.7	0.8	13	47
	42		18	0.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	8.9	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.7	0.8	14	46
	45		15	0.9	1.9	2.8	3.8	4.7	5.6	6.6	7.5	8.4	9.4	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8	15	45
	48		12	1.0	2.0	2.9	3.9	4.9	5.9	6.8	7.8	8.8	9.8	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	16	44
	51		9	1.0	2.0	3.0	4.1	5.1	6.1	7.1	8.1	9.1	10.2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	17	43
	54		6	1.1	2.1	3.2	4.2	5.3	6.3	7.4	8.4	9.5	10.5	0.1	0.2	0.3	9.4	0.5	9.6	0.7	0.8	1.0	18	42
	57		3	1.1	2.2	3.2	4.3	5.4	6.5	7.6	8.7	9.7	10.8	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0	19	41
1	0	2	0	1.1	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.0	11.1	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0	20	40
	3		57	1.1	2.3	3.4	4.5	5.7	6.8	8.0	9.1	10.2	11.4	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0	21	39
	6		54	1.2	2.3	3.5	4.6	5.8	7.0	8.1	9.3	10.5	11.6	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0	22	38
	9		51	1.2	2.4	3.5	4.7	5.9	7.1	8.3	9.5	10.6	11.8	0.1	0.2	0.4	0.5	0.6	0.7	0.8	0.9	1.1	23	37
	12		48	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8	12.0	0.1	0.2	0.4	0.5	0.6	0.7	0.8	1.0	1.1	24	36
	15		45	1.2	2.4	3.6	4.9	6.1	7.3	8.5	9.7	10.9	12.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	25	35
	18		42	1.2	2.5	3.7	4.9	6.1	7.4	8.6	9.8	11.1	12.3	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	26	34
	21		39	1.2	2.5	3.7	5.0	6.2	7.4	8.7	9.9	11.1	12.4	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	27	33
	24		36	1.2	2.5	3.7	5.0	6.2	7.5	8.7	10.0	11.2	12.4	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	28	32
	27		33	1.2	2.5	3.7	5.0	6.2	7.5	8.7	10.0	11.2	12.5	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	29	31
1	30	2	30	1.3	2.5	3.8	5.0	6.3	7.5	8.8	10.0	11.3	12.5	0.1	0.3	0.4	0.5	0.6	0.7	0.9	1.0	1.1	30	30

TABLE V.

CORRECTION OF APPARENT TIME FOR EQUATION OF SECOND DIFFERENCE.

Varia. in 3 hours.	Equation of Second Difference.																	
	″ 1	″ 2	″ 3	″ 4	″ 5	″ 6	″ 7	″ 8	″ 9	″ 10	″ 0.1	″ 0.2	″ 0.3	″ 0.4	″ 0.5	″ 0.6	″ 0.7	″ 0.8	″ 0.9
° ′	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.	s.
1 0	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7
2	2.9	5.8	8.7	11.6	14.5	17.4	20.3	23.2	26.1	29.0	0.3	0.6	0.9	1.2	1.5	1.7	2.0	2.3	2.6
4	2.8	5.6	8.4	11.2	14.1	16.9	19.7	22.5	25.3	28.1	0.3	0.6	0.8	1.1	1.4	1.7	2.0	2.2	2.5
6	2.7	5.5	8.2	10.9	13.6	16.4	19.1	21.8	24.5	27.3	0.3	0.6	0.8	1.1	1.4	1.6	1.9	2.2	2.5
8	2.7	5.3	7.9	10.6	13.2	15.9	18.5	21.2	23.8	26.5	0.3	0.5	0.8	1.1	1.3	1.6	1.9	2.1	2.4
10	2.6	5.1	7.7	10.3	12.9	15.4	18.0	20.6	23.1	25.7	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.1	2.3
12	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3
14	2.4	4.9	7.3	9.7	12.2	14.6	17.0	19.5	21.9	24.3	0.2	0.5	0.7	1.0	1.2	1.5	1.7	2.0	2.2
16	2.4	4.7	7.1	9.5	11.8	14.2	16.6	18.9	21.3	23.7	0.2	0.5	0.7	1.0	1.2	1.4	1.7	1.9	2.1
18	2.3	4.6	6.9	9.2	11.5	13.8	16.2	18.5	20.8	23.1	0.2	0.5	0.7	0.9	1.2	1.4	1.6	1.9	2.1
20	2.2	4.5	6.7	9.0	11.2	13.5	15.7	18.0	20.2	22.5	0.2	0.5	0.7	0.9	1.1	1.4	1.6	1.8	2.0
1 22	2.2	4.4	6.6	8.8	11.0	13.2	15.4	17.6	19.8	21.9	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.8	2.0
24	2.1	4.3	6.4	8.6	10.7	12.9	15.0	17.1	19.3	21.4	0.2	0.4	0.6	0.9	1.1	1.3	1.5	1.7	1.9
26	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.7	18.8	20.9	0.2	0.4	0.6	0.8	1.1	1.3	1.5	1.7	1.9
28	2.0	4.1	6.1	8.2	10.2	12.3	14.3	16.4	18.4	20.5	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
30	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
32	2.0	3.9	5.9	7.8	9.8	11.7	13.7	15.6	17.6	19.5	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
34	1.9	3.8	5.7	7.7	9.6	11.5	13.4	15.3	17.2	19.1	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.5	1.7
36	1.9	3.7	5.6	7.5	9.4	11.2	13.1	15.0	16.9	18.7	0.2	0.4	0.6	0.8	0.9	1.1	1.3	1.5	1.7
38	1.8	3.7	5.5	7.3	9.2	11.0	12.9	14.7	16.5	18.4	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.7
40	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2	18.0	0.2	0.4	0.5	0.7	0.9	1.1	1.3	1.4	1.6
1 42	1.8	3.5	5.3	7.1	8.8	10.6	12.4	14.1	15.9	17.6	0.2	0.4	0.5	0.7	0.9	1.1	1.2	1.4	1.6
44	1.7	3.5	5.2	6.9	8.7	10.4	12.1	13.8	15.6	17.3	0.2	0.4	0.5	0.7	0.9	1.0	1.2	1.4	1.6
46	1.7	3.4	5.1	6.8	8.5	10.2	11.9	13.6	15.3	17.0	0.2	0.3	0.5	0.7	0.9	1.0	1.2	1.4	1.5
48	1.7	3.3	5.0	6.7	8.3	10.0	11.7	13.3	15.0	16.7	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5
50	1.7	3.3	4.9	6.5	8.2	9.8	11.5	13.1	14.7	16.4	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5
52	1.6	3.2	4.8	6.4	8.0	9.6	11.2	12.9	14.5	16.1	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.4
54	1.6	3.2	4.7	6.3	7.9	9.5	11.1	12.6	14.2	15.8	0.2	0.3	0.5	0.6	0.8	1.0	1.1	1.3	1.4
56	1.6	3.1	4.6	6.2	7.8	9.3	11.0	12.4	14.0	15.5	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4
58	1.5	3.1	4.6	6.1	7.6	9.2	10.7	12.2	13.7	15.2	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4
2 0	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5	15.0	0.1	0.3	0.4	0.6	0.7	0.9	1.0	1.2	1.3

In the practice of lunars four persons are frequently employed in making the observations, the first to take the distance, the second to take the altitude of the sun or star, the third to take the altitude of the moon, and the fourth to write down the observations. One person, however, may make the whole himself, according to the following method, which was obligingly communicated by that distinguished practical navigator, Captain Basil Hall. Speaking of his own practice, he says,—" I always take all my altitudes and distances with the same instrument. First the altitudes of the sun, then those of the moon, then several distances; next the altitudes of the moon, then those of the sun, and interpolating by proportional logarithms for the altitudes at the mean time of the distances.* At night I never take an altitude, unless it be about twilight, when it can be done with accuracy and ease."

" The method which I use to connect lunars and chronometers is

* This is similar to the method given in Norie's Navigation.

not very general, but infinitely the best, and ought to be universally adopted, as it renders all allowance for the distance run in the interval of little or no consequence."

"The use of lunars at sea I conceive is, in a great degree, to check the chronometers: the method by lunars being infallible, though not very nice; that by chronometers being fallible, but as nice as possible. So that a number of lunars are necessary to check a chronometer, and the object is to bring the whole of such lunars to bear rigorously on the chronometer without making use of the logboard."

"This will be best illustrated by an example. At noon, or any other hour during the day most convenient for taking a lunar, I observe a set or half dozen sets of lunars with the sun, carefully noting what the chronometer shows, but without taking any account of the actual time. At any other hour when the sun is near the prime vertical, or most suitable for determining the time, I take altitudes expressly with this view, from which I discover the error of the same chronometer used for the lunars. Again, during the night I take lunar distances with the stars, on both sides of the moon if possible, at the moments most favourable, but never mind the exact time, only carefully recording what the chronometer shows. Now by the sights for absolute time I ascertain what was the error of the chronometer on apparent time at that meridian, and this same error, corrected for rate during the interval, I apply to each of the different times by the chronometer when the lunars were taken. By this means I get the apparent times due to the meridian, on which the absolute time sights were taken, with as much accuracy as if the whole, lunars and all, had been taken at that fixed meridian. The distances give the several times at Greenwich, and thus they all concur in settling the difference of time, between the first meridian and that chosen for taking the time, with a view of seeing what longitude the chronometer gives. Hence, if there had been an unseen current of some miles an hour of which no account could possibly be taken, still the result would not be vitiated thereby, but all the lunars would be found to contribute to the same end, thus making, according to Dr Wollaston's simile, the moon serve the purpose of a great Greenwich clock in the heavens. After having determined the true longitude and error of the chronometers when within a few days' sail of the land, I run the remainder of the voyage, in a great degree, by the chronometers alone."

These remarks may be illustrated by the following observations, communicated by Lieutenant-general Sir Thomas Macdougal Brisbane, K. C. B., taken on his passage from New South Wales to Britain.

With Troughton's Circle. Clear.

12th May, 1825. Chr. + A. T. $3^h\ 1^m\ 19^s$

(1.)	Chr. Time.	A. Dist. Sun from Moon.	Moon's App. Alt.		Sun's App. Alt.
	$5^h\ 13^m\ 53^s$	$62°\ 57'\ 1''$	$54°\ 25'\ 2''$		$55°\ 47'\ 50''$
	5 0 9	Moon's hor. par. $55'\ 46''$		T. C.	$5^h\ 13^m\ 53^s$
G+	13 44	Moon's cor.	31 46	A. T.	3 1 19
True dist.		$62°\ 29'\ 30''$			
Greenwich time		$5^h\ 4^m\ 4^s$		Diff.	2 12 34
Ship time		2 12 34			
Long. in time		2 51 30 = $42°\ 52'\ 30''$ W.			

s

(2.) Chr. Time.	App. Dist.	Moon's App. Alt.	Sun's App. Alt.
5^h 18^m 40^s	62° 58′ 14″	55° 17′ 52″	54° 24′ 30″
5 4 54	Moon's hor. par. 55′ 46″	T. C.	5^h 18^m 40^s
G + 13 46	Moon's cor.	31 5	3 1 19
True dist.	62° 31′ 49″		
Greenwich time	5^h 18^m 49^s		2 17 21
Ship time	2 17 21		
Long. in Time	2 51 28=42° 52′ 0″ W.		

(3.) Chr. Time.	Distance.	Moon's Alt.	Sun's Alt.
5^h 23^m 38^s	62° 59′ 53″	56° 5′ 52″	53° 28′ 50″
5 9 57	Moon's hor. par. 55′ 46″		5^h 23^m 38^s
G+ 13 41	Moon's cor.	30 27	3 1 19
True dist.	62° 34′ 16″		
Greenwich time	5^h 13^m 52^s		2 22 19
Ship time	2 22 19		
Long. in time	2 51 33=42° 53′ 15″ W.		

Long. by 1st series	42° 52′ 30″ W.	C+G. M. T.	13^m 44^s
2d	42 52 0		13 46
3d	42 53 15		13 41
Mean long.	42 52 35 W.	Mean+	13 44 Error chr.

Mean long. by lunars 42° 52′ 35″ W.
by chro. 42 35 0

Diff. 17 35

By proceeding in the same manner in the evening, and taking observations from objects on both sides of the moon, a very great degree of precision may be obtained.

On finding the Longitude.

II. BY CHRONOMETERS.

The foregoing method of finding the longitude by lunars is very valuable at sea, on account of the frequent opportunities which occur for observation. About the time of new moon, and in unsteady weather, the necessary observations for the practice of this method cannot be obtained, and the dead reckoning is not to be depended on for any length of time, therefore recourse must be had to other methods.

On account of the very high degree of perfection to which chronometers have been brought, the longitude determined by a mean of three or four of these delicate machines merits great confidence. If the rate of a chronometer be determined on shore, or rather perhaps on board in the situation it is intended to occupy during the voyage, where the various causes which act upon it, and are likely to alter its rate, are in operation, it is likely this rate will remain pretty uniform for some time, and the amount of the gain or loss, being allowed for on the time indicated by it at any future period, the true time may be obtained at the meridian of the place where its rate and original error was determined, with as much accuracy as if it had been adjusted to go accurately to mean solar time on that me

ridian. Hence, it is obvious, that if the original error, and the gain or loss in 24 hours, called the daily rate, of a chronometer, be known, on any meridian, such, for example, as that of Greenwich; by making proper allowance for these, the mean time at Greenwich may be readily known to such a degree of accuracy as the going of the chronometer will warrant.

It is now only necessary to find the apparent time at ship, by an altitude of any celestial body properly situated by some of the methods already given; to which the equation of time being taken from the Nautical Almanac and properly applied, the result will be the mean time to be compared with that at the given meridian to show the longitude of the ship.

The rate of a chronometer is readily obtained by observing daily, if possible, the altitude of one or more celestial objects near the prime vertical, from which the mean time may be accurately determined, and, being compared with that shown by the chronometer, its gain or loss in 24 hours, and also its error on the day of the last observation, called the original error, will become known.*

Ex. 1.—May 1, 1824, near Falmouth, in latitude 50° 8′ 48″ N., and longitude $20^m\ 10^s$ W., at about $18^h\ 47^m\ 20^s$, the following altitudes of the sun's lower limb were taken, with an artificial horizon, in order to ascertain the daily rate of a chronometer previously set to Greenwich time. The observations were made with a sextant of which the index error was + 1′ 30″, the barometer 29.6 inches, and the thermometer 56° Fahrenheit.

	Times by Chronometer.		*Double Alt.*	alt. 19° 3′	
					281
	$19^h\ 10^m\ 35^s$		37° 48′ 45″	Z. D. 70 57 log s	2.22150
	12 45		38 4 30	ther. 56° log	9.99460
	14 58		38 20 15	bar. 29.6 log	9.99417
				ther. 56° log	9.99974
3	8 18	3	13 30		
				r 163″.4	2.21282
Means	19 12 46		38 4 30	= 2′ 43″.4	
		I. E.	+ 1 30	sun's parallax 8″.1	
		2	38 6 0		
			19 3 0		

* These would be more accurately performed on shore by using an artificial horizon and the method of equal altitudes. In this case a pocket-chronometer should be employed, to be compared with those on board, which ought to be as numerous as possible. I am also of opinion, that the rate of a chronometer to be used at sea should not be ascertained in an observatory. At least three of the chronometers should be properly secured in their respective situations on board, and by a good pocket-chronometer a comparison of these should be made daily with an observatory clock, or by direct observation, in order to determine accurately the *sea-rate*.

Time at Falmouth	$18^h\ 47^m\ 20^s$		
Longitude in time	+ 20 10		
Greenwich time	19 7 30	D. L.	0.09861
Daily variation	17′ 58″	P. L.	1.00080
Prop. part to $19^h\ 7\frac{1}{2}^m$	14 19	P. L.	1.09941
Dec. at noon, May 1st	15 8 49		
Sun's reduced declination	15 23 8		

Observed alt. sun's *l. l.*				19° 3′ 0″
Semidiameter				+ 15 53
Refraction				— 2 43 .4
Parallax				+ 8 .1
True altitude				19 16 18
Sun's true dec.	15° 23′ 8″	N.	secant	0.015850
Latitude	50 8 48	N.	secant	0.193260
Difference	34 45 40			
Zenith dist.	70 43 42			
Sum	105 29 22		half 52° 44′ 41″ sine	9.900884
Difference	35 58 2		half 17 59 1 sine	9.489599

Apparent time at Falm.	$18^h\ 47^m\ 12^s.4$	Red. versine	9.599593
Mean time app. noon	23 56 49 .1	Supp. to N. Al.	
Mean time at Falm.	18 44 1 .5		
Time by chronometer	19 12 46 .0		
Chronometer for Falm.	28 44 .5 fast		

Again, on the 11th of May, 1824, the altitude of the sun's lower limb taken with the same instruments as before, the index error being constant, was 19° 9′ 50″, when the chronometer showed $18^h\ 57^m\ 56^s$. This gives the mean time at Falmouth $18^h\ 30^m\ 23^s.5$, and the error of the chronometer for the meridian of the place $27^m\ 32^s.5$.

Whence, on May 1st, the error was	$28^m\ 44^s.5$
11th	27 32 .5
The loss in ten day is	1 12
Or in one day it is	7 .2
Hence the daily rate is	— 7 .2

It is to be observed, that the altitudes should be taken nearly at the same time of the day, otherwise an allowance must be made for the rate during the interval.

1. On the 22d of May, 1824, in latitude 32° 36′ N., and longitude by account 16° 40′ W., the altitude of the sun's lower limb at sea was 37° 24′, when the chronometer showed $5^h\ 12^m\ 24^s.5$, the height of the eye being 20 feet; required the longitude?

Time per watch	$5^h\ 12^m\ 24^s.5$	Daily rate . .	$7^s.2$
			$11\frac{1}{4}$
Original error	— 28 44.5	Loss in $11\frac{1}{4}$ days	79.2 / 1.8
	4 43 40.0		
Loss in 11 days 5^h	+ 1 21	Or . .	60 | 81.0
Greenwich M. time	4 45 1		1 21
Alt. sun's *l. l.*	39° 26′	dec. . .	20° 26′ N.
Cor. Table XIII.	+ 10	cor. for 5^h . .	+ 2
True alt.	39 36	cor dec. . .	20 28 N.
		Z. D. . .	69 32
True alt. .	39° 36′		
Pol. dist. .	69 32	cosecant . .	0.028318
Latitude .	32 36	secant . .	0.074455
Sum . .	141 44		
Half . .	70 52	cosine . .	9.515566
Diff. .	31 16	sine . .	9.715186
App. time	$3^h\ 41^m\ 18^s$	Red. versine .	9.333525
Mean time app. N.	23 56 20	Supp. to Naut. Almanac.	
Mean time at ship	3 37 38		
Mean time at Green.	4 45 1		
Long. in time	1 7 23 = 16° 51′ W.		

For the usual computations at sea it is unnecessary to push the calculations farther than the nearest minute of a degree.

2. On the 11th of October, 1824, at noon, on the meridian of Greenwich, a chronometer was $11^m\ 19^s.4$ fast, and the daily rate was $+4^s.1$. On the 21st of October, at $6^h\ 42^m\ 10^s$ A. M. by the same chronometer, the observed altitude of the sun's lower limb was 42° 17′ 20″, and the height of the eye 20 feet; required the longitude?

Ans.—33° 25′ E.

3. On the 16th August, 1828, in latitude 38° 20′ S., the mean of several altitudes of Antares west of the meridian was 14° 29′, the height of the eye being 12 feet, and the mean of the times per watch $11^h\ 41^m\ 38^s$ P. M., which had been compared with mean time at the Cape of Good Hope on the 22d of June, and was found to be $1^h\ 10^m\ 28^s$ too slow, and gaining $3^s.54$ a day; required the longitude of the ship?

Ans.—17° 36′ E.

EQUATION TO EQUAL ALTITUDES.

The equation of equal altitudes is a correction for the change of declination of a celestial body during the interval of observation, to be applied to the middle time between the instants shown by a chronometer, at which, on a given day, that body has equal altitudes; to find the true time by the chronometer when the object was upon the meridian.

In ordinary cases the error and rate of a chronometer may be determined by single altitudes; but when great accuracy is required

equal altitudes are very superior, especially when a transit instrument cannot be obtained. On this account various tables have been computed to facilitate this operation, though it is believed few of them afford great advantage in practice. By reason of the inconvenience of taking proportional parts, it is often better to give an easy practical rule, requiring the use of the ordinary tables, where neither double entries, different signs, nor proportional parts are necessary.

*Rule.**

To the log cosine of half the interval between the times of observation add the cotangent of the latitude, the sum, rejecting 10 in the index, will be the tangent of *arc first*, the difference between which and the polar distance will be *arc second*. (6.744727 (constant)

Now to the constant logarithm (5.364517) add the cotangent of half the elapsed time, the cosecant of arc first, the cosecant of the polar distance, the sine of arc second, the logarithm of the elapsed time in minutes, the logarithm of the daily variation of the declination in seconds,† the sum will be the logarithm of the equation of equal altitudes in seconds of time, which, when applied to NOON, is *additive* if the polar distance is *increasing*, and subtractive if it is decreasing. If the equation is applied to MIDNIGHT, it is *additive* if the polar distance is *decreasing*, and subtractive if the polar distance is increasing.‡

Ex. 1.—On the 23d of March, 1809, at Pisa, in latitude 43° 43′ 11″ N. equal altitudes of the planet Venus were taken before and after transit, the elapsed time between which was $8^h\ 50^m$; required the equation of equal altitudes when her declination was 20° 42′ 40″ N., and her daily variation +20′ 5″ or +1205″ increasing, and consequently the polar distance *decreasing*?

Latitude	43° 43′	cot 0.019462	C. L.	5.364517
H. E. T.	$4^h\ 25^m$	cos 9.605032	cot	9.643463
Arc 1.	22° 50′	tan 9.624494	cosec	0.411110
Pol. dist.	69 17	cosecant	. .	0.029030
Arc 2.	46 27	sine	. . . .	9.860202
Elap. time	$8^h\ 50^m = 530^m$ log.		. .	2.724276
Daily var. dec.	20′ 5″ = 1205″ log.		. .	3.080987
Eq. E. Alts. — $12^s.99$		. . . .	.	1.113585

Or subtractive, because the polar distance is decreasing, and is to be applied to *noon*.

Ex. 2.—On the afternoon of the 17th of September, 1810, altitudes of the sun were observed at Marseilles, in latitude 43° 17′ 50″ N.,

* See Dr Mackay's or Mr Riddle's Navigation for a similar rule, analogous in principle, though perhaps in the detail somewhat less simple.

† Half the sum of the variations for the given and preceding days should properly be employed, if the equation of equal altitudes be required for the *noon* of the given day; but the variation for the given day simply, if for *midnight*, the longitude not differing much from Greenwich.

‡ By polar distance in the computation is meant the distance of the object from the *elevated* pole, which may be either referred to the north or south pole, according to the name of the latitude. (See page 96.)

and equal altitudes were taken on the forenoon of the 18th, after an interval of 21ʰ 50ᵐ, the sun's declination for the 17th at midnight being 2° 14′ 23″ N., and daily variation of declination — 23′ 14″ = — 1394″; required the equation of equal altitudes?

Ans.—Equation of equal altitudes — 136ˢ.70. = — 2ᵐ 16ˢ.7. Or subtractive, for the polar distance is *increasing*, and is to be applied to *midnight*.

Ex. 3.—At Florence, in latitude 43° 46′ 40″ N., on the 8th of April, 1809, equal altitudes of the planet Mars were taken at an interval of 8ʰ 20ᵐ when his declination was 5° 9′ 40″ S., decreasing at the rate of 6′ 38″ daily; required the correction for the planet's superior passage?

Ans.—Equation of equal altitudes — 5ˢ.196.
Or subtractive, because the polar distance is *decreasing*, and is to be applied to the *superior* transit.

TO FIND THE ERROR OF A CHRONOMETER BY EQUAL ALTITUDES.

By the Sun.—The sun is in general the most convenient object for determining the error of a chronometer by equal altitudes, and the forenoon and afternoon of the same civil day are often preferred, though the evening and succeeding morning may sometimes be employed with advantage.

In the morning, when the sun is more than two hours distant from the meridian, in mean latitudes, let a set of observations be taken with the corresponding times by a chronometer. In the afternoon observe the instants when the sun comes to the same alitude, writing each time down opposite its corresponding altitude.

Now half the sum of any two times, answering to the same altitude, will be the approximate time of noon. Find the mean of all the times of noon in this manner from each corresponding pair of observations; to which the equation of equal altitudes being applied, the result will be the time of apparent noon, or the instant that the sun's centre is on the meridian by the chronometer. The difference between this and noon is the error of the chronometer, which will be fast or slow according as the time of noon thereby is greater or less than twelve hours.

Ex. 1.—On the 29th of January, 1826, in latitude 57° 9′ N., longitude 2° 8′ W., the following equal altitudes of the sun were observed; required the error of the chronometer?

Altitudes.	*Times A. M.*		*Times P. M.*
8° 5′	21ʰ 35ᵐ 8ˢ		2ʰ 55ᵐ 43ˢ
8 10	36 8		54 42
8 15	37 9		53 42
8 20	38 9		52 41
8 25	39 10		51 40
	35 44		15 8
Means	21 37 8.8		2 53 41.6
	2 53 41.6		21 37 8.8
Elapsed time	5 16 32.8	Sum	24 30 50.4
H. E. T.	2 38 16.4	Half	12 15 25.2

Sun's declination at noon, on merid. Greenwich				17° 59′ 15″ S.
Daily variation or decrease of polar distance				— 16 15 N.
Latitude	57° 9′	cot 9.810025	C. L.	5.364517
H. E. T.	$2^h\ 38^m$	cos 9.887406	cot	0.083896
Arc 1.	26 29	tan 9.697431	cosec	0.350726
Pol. dist.	107 59	cosecant		0.021753
Arc 2.	81 30	sine		9.995203
El. time	5^h 16 .5 = $316^m.5$ log			2.500374
Daily var. dec.	16′ 5.5″ = 965″.5 log			2.984752
Eq. equal alts — $20^s.0$				1.301221

Half sum or approximate time of noon	$12^h\ 15^m\ 25^s.2$
Equation of equal altitudes	— 20 .0
Time of apparent noon by chronometer	12 15 5 .2
Equation of time with contrary sign	— 13 27 .9
Time of mean noon by chronometer	12 1 37 .3

Hence the chronometer was $15^m\ 5^s$ fast for apparent noon, and $1^m\ 37^s.1$ fast for mean noon.

Ex. 2.—On the 24th of July, 1822, at Pendennis castle, near Falmouth, in latitude 50° 8′ 48″ N., Dr Tiarks, with a sextant of ten inches radius by Mr Troughton, and an artificial horizon, together with a chronometer by Morice, found the double alitude of the sun's upper limb to be 69° 47′ 20″, at $8^h\ 29^m\ 13^s$ A. M., and $4^h\ 25^m\ 5^s.3$ P. M.; required the time of apparent noon by the chronometer?*

Time after noon 23d per chronometer	$20^h\ 29^m$ 13 0
24	4 25 5 . 3
Sum	24 54 18 . 3
Half sum, or approximate noon	12 27 9 .15
Difference, or elapsed time	7 55 52 .30
Half elapsed time	3 57 56 .15

The declination of the sun, at noon 24th, is 19° 58′ nearly.
Daily variation 12′ 39″ S., or increasing the polar distance.

* See a Report on Chronometrical Observations to ascertain the Longitude of the Island of Madeira, by J. L. Tiarks, 1822. Indeed the variation of the declination of the sun 12^h preceding and 12^h following the time, whether noon or midnight, ought to be taken; but, unless very great accuracy be required, the sun's variation for the given day as it stands in the Nautical Almanac will be sufficiently correct.

Latitude	50° 9′	cot 9.921503	C. L.	5.364517
H. E. T.	3^h 58^m	cos 9.705469	cot	9.770148
Arc 1.	22° 57′	tan 9.626972	cosec	0.409016
Pol. dist.	70 2	cosecant		0.026922
Arc 2.	47 5	sine		9.864716
Elap. time	7^h 56^m = 476^m	log		2.677607
Daily var. dec.	12′ 29″.5 = 749″.5	log		2.874772
Eq. eq. alts. +9^s.72		log		0.987698
To approximate noon				12^h 27^m 9^s.15
Add the equation of equal altitudes				+ 9.72
Apparent noon				12 27 18.87

Ex. 3.—On the 24th of July, 1822, at 8^h 5^m 38^s.7 P. M., and 25th July, at 9^h 49^m 5^s.7 A. M. at the same place, the double altitude of the sun's upper limb was 93° 40′; required the apparent time of midnight by the chronometer?

Time after noon, July 24th	3^h 5^m 38^s.7
24	21 49 59.7
Sum	24 55 38.4
Half sum, or approximate midnight	12 27 49.2
Elapsed time	18 44 21.0
Half elapsed time	9 22 10.5

Declination at midnight 19° 52′ N., daily variation 12′ 39″ S.

Or increasing the polar distance, and the equation is therefore negative for midnight.

Latitude	50° 9′	cot 9.921503	C. L.	5.364517
H. E. T.	9^h 22^m	cos 9.887406	cot	0.083896
Arc 1.	32 47	tan 9.898909	cosec	0.266431
Pol. dist.	70 8	cosecant		0.026648
Arc 2.	37 21	sine		9.782961
Elap. time	18^h 44^m = 1124^m	log		3.050766
Daily varia. dec.	12′ 39″ = 759″	log		2.880242
Eq. eq. alts. —28^s.54		log		1.455461
From approximate midnight				12^h 27^m 49^s.20
Subtract the equation of equal altitudes				— 28.54
Apparent midnight				12 27 20.66

Proceeding in this manner till a considerable number of observations are made, the error of a chronometer may be determined with great accuracy. If this chronometer be compared with any given number of them, all their errors and rates may be found.

The same thing may be done by the stars, though rather less conveniently.

The following method of comparing a chronometer with mean time by Dr. Tiarks, communicated by Captain Basil Hall, R. N., will be found very useful.

The difference of a chronometer from the mean time at a place being known at three different instants, to find that difference for any intermediate instant, with a proper regard to the change of rate

which may have taken place between the first and second, and between the second and third times.

Let the difference at the

times $o \;= a$

$t' \;= a+b$

$t'' = a+b+c$

So that b is the difference between the first and second states of the chronometer, and c the difference between the second and third states of the same chronometer, the state of a chronometer, (namely, its difference from the mean time of a given place), at the moment t will be

$$+\left\{\frac{t\,(t-t'')}{t'(t'-t'')}+\frac{t\,(t-t')}{t''(t''-t')}\right\}b+\frac{t\,(-t')}{t''(t''-t')}c\text{; or}$$

$$+\,a\left\{\frac{t}{t'.t''}(t''+t'-t)\right\}b+\frac{t\,(t-t')}{t''(t''-t')}c=\text{correction}$$

If t is *less* than t', $\frac{t\,(t-t'')}{t\,(t'-t'')}$ is *positive* and $\frac{t\,(t-t')}{t''\,(t''-t')}$ is *negative*, and if t is *greater* than t', both are *positive*.

Example.

The difference of a chronometer from the mean time of a certain place was known on the following days:

	Differences.	*Days.*
August $9^d.5243$		
—— 31 .4146	21.8903	Difference between 1st and 2d $= 21.8903$
Sept. 4 .9250	4.5104	1 and 3 $= 26.4007$

Hence $o = 0.0$

$t' = 21.8903$

$t'' = 26.4007$

$t'+t'' = 48.2910$

It is now required to find the state of the chronometer for August 17th, at $11^h\ 7^m\ 44^s = 17^d.4637$. Deducting August $9^d.5243$ from August $17^d.4637$ we have the interval $t = 7^d.9394$.

$t'+t'' = 48.2910$		$t' = 21.8903$	log 1.340252
$t \;=\; 7.9394$	log 0.899788	$t'' = 26.4007$	log 1.421616
$t'+t''-t = 40.3516$	log 1.605861	$t' \times t''$, log	2.761868

$t \times (t'+t''-t)$, or num. log 2.505649

$t' \times t''$ or denominator, log 2.761868

$\frac{t}{t' \times t''} \times (t'+t''-t)$, log 9.743781, or factor of b.

$t = 7.9394$	log 0.899788	$t'' = 26.4007$	log 1.421616
$t' = 21.8903$		$t' = 21.8903$	
$t'-t = 13.9509$	log 1.144602	$t''-t' = 4.5104$	log 0.654215
numerator	log 2.044390	$t''.(t''-t')$ denom. log	2.075831
denominator	log 2.075831		

$\frac{t\,(t-t')}{t''\,(t''-t')}$ log 9.968559, or factor of c which is negative, because t is less than t'.

		Diff.
August 9th, 12^h 35^m chronometer slow	51^m 57^s.35	. . . . . a
31st, 9 57	54 10 .33	. . 132^s.98 b
Sept. 4th, 22 12	54 39.16	. 28.83 c

What is the difference, August 17th, 11^h 8^m.

$b = 132^s.98$ log 5.123782 $c = 28^s.83$ log 1.459845

factor b . log 9.743781 factor c log 9.968559

$(f)\ b = +73^s.72$ log 1.867563 $(f)\ c = -26^s.82$ log 1.428404

$(f)\ c = -26\ .82$

cor. $= +$ 46 .90 to be applied to the error of the chronometer at the time a.

August 9th, 12^h 35^m chronometer slow for M. T.	51^m 57^s.35
correction . .	+ 46 .90
Chronometer slow for mean time . .	52 44.25

On August 17th, at 11^h 7^m 44^s.*

Table I. Decimal Fractions of a Day of 24h.				Table II. Decimal Parts of an Hour.		Table III. To convert Decimals of Time into Degrees at 15° to an Hour.	
T.	Decimal.	T.	Decimal.	T.	Decimal.	T.	Arc.
1^h	0^d.041667	10^m	0^d.006944	10^m	0^h.166667	0^s.1	1″.5
2	0.083333	20	0.013889	20	0.333333	0.2	3.0
3	0.125000	30	0.020833	30	0.500000	0.3	4.5
4	0.166667	40	0.027778	40	0.666667	0.4	6.0
5	0.208333	50	0.034722	50	0.833333	0.5	7.5
6	0.250000	1	0.000694	1	0.016667	0.6	9.0
7	0.291667	2	0.001389	2	0.033333	0.7	10.5
8	0.333333	3	0.002083	3	0.050000	0.8	12.0
9	0.375000	4	0.002778	4	0.066667	0.9	13.5
10	0.416667	5	0.003472	5	0.083333	0.01	0.15
11	0.458333	6	0.004167	6	0.100000	0.02	0.30
12	0.500000	7	0.004861	7	0.116667	0.03	0.45
13	0.541667	8	0.005556	8	0.133333	0.04	0.60
14	0.583333	9	0.006250	9	0.150000	0.05	0.75
15	0.625000					0.06	0.90
16	0.666667	10^s	0.000116	10^s	0.002778	0.07	1.05
17	0.708333	20	0.000232	20	0.005556	0.08	1.20
18	0.750000	30	0.000347	30	0.008333	0.09	1.35
19	0.791667	40	0.000463	40	0.011111	0.001	0.015
20	0.833333	50	0.000579	50	0.013889	0.002	0.030
21	0.875000	1	0.000012	1	0.000278	0.003	0.045
22	0.916667	2	0.000023	2	0.000556	0.004	0.060
23	0.958333	3	0.000035	3	0.000833	0.005	0.075
24	1.000000	4	0.000046	4	0.001111	0.006	0.090
For 12^h double that for 24^h.		5	0.000058	5	0.001389	0.007	0.105
		6	0.000069	6	0.001667	0.008	0.120
		7	0.000081	7	0.001944	0.009	0.135
		8	0.000093	8	0.002222		
		9	0.000104	9	0.002500		

* For Rossel's method of correcting the error in rate of a chronometer, see Biot's Astronomie, vol. III., or Myer's translation of this, page 95, and also this work in the explanation of the tables.

Explanation.

Table I. contains the decimal fraction of a day of 24^h. It is useful for finding what part of a day any number of hours, minutes, and seconds are, and consequently may be conveniently employed in many calculations where daily differences are necessarily involved, such as the daily rate of a clock, the change of which, in any given number of hours, &c., may be thereby readily obtained. It is also very useful in the preceding method of comparing chronometers, and other purposes.

Table II. serves the same purpose when an hour is taken for unit, and is useful in several astronomical operations.

Table III. is supplementary to the general Table V., which serves to convert time into degrees if less than 6^h or 90°. But as 6^h answers, to 90°, 12^h to 180°, and 18^h to 270°, this table will easily be applied to 24^h or 360°, the whole circle to every four seconds of time, and by the proportional parts at the bottom to every single second. Whence it is only necessary to convert the decimal part of the time into degrees by this table to complete the whole.

III. BY OCCULTATIONS AND ECLIPSES.

The moon in her periodical revolution frequently passes between the earth and a fixed star, of which she intercepts the spectator's view, thus producing what is called an *occultation*.

Since the instant of disappearance and reappearance of the star can be ascertained without the use of any instrument liable to error, the longitude may be determined more accurately by an observation of this phenomenon, than by a lunar distance. An observer possessed of an ordinary telescope, a chronometer, and an instrument to determine its error and rate,* can readily make the observations; and the necessary calculations are far from difficult. Several rules have been proposed for this purpose independent of the method of determining the parallaxes by the nonagesimal, and comparatively much more simple. Of these, Dr Inman's (of Portsmouth), which we shall in the mean time adopt with some alterations, appears to us the most convenient.†

At the instant of the disappearance or reappearance of the star, the apparent right ascension and declination of the point of the moon's limb in contact with the star is the same as the right ascension and declination of the star, which can be obtained with great facility and accuracy from tables. The apparent right ascension and declination of this point being corrected for parallax, its true right ascension and declination will be determined. Now since the distance of this point from the moon's centre, which is equal to her semidiameter, and the declination of the centre for the estimated time at Greenwich, may be found by the Nautical Almanac, the true right ascension of the moon's centre is easily computed. Should there be an uncertainty in the estimated Greenwich time amounting to about one minute, the operation must be repeated till the estimated and computed Greenwich time be very nearly the same.

* If the observations are made at sea, an allowance must be made for the rate of the chronometer between the disappearance and reappearance of the star and the run of the ship, as in lunars.

† Almost all of these methods are founded upon a paper by Lagrange. See Connaissance des Tems for 1819.

Rule.

By applying the estimated longitude in time to the observer's apparent time, the reduced Greenwich time to the nearest minute will be obtained.

To this time take from the Nautical Almanac the sun's R. A., the moon's R. A., and their declinations corrected for second differences, together with the variation of declination for 10', for the purpose of repeating the operation when supposed necessary; and the moon's semidiameter, and the horizontal parallax corrected for the spheroidal figure of the earth.

Take also the moon's R. A. for 3^h after the first estimated time corrected as formerly.

Find from the Nautical Almanac, or from other tables, the apparent R. A. and D. of the observed fixed star, and reduce the given latitude for the spheroidal figure of the earth.

To the apparent time add the sun's R. A., and from the sum, increased if necessary by 24^h, subtract the star's R A.; the remainder, if less than 12^h, will be the hour-angle; if greater than 12^h, its complement to 24^h will be the hour-angle.

Now write down the *proportional logarithm* of the reduced horizontal parallax under the numbers (1), (2), and (3). Under (1) and (2) put the *secant* of the reduced latitude; under (3) the *cosecant* of the same; under (1) the cosecant of the hour-angle (a), and take the sum of these.

Below the sum of the three logarithms under (1) put the *constant logarithm* 1.17609, and the *cosine* of the star's declination; at the same time under (2) put the *cosecant*, and under (3) the *secant* of the same; the sum of these three logarithms under (1) will be the *proportional logarithm* of *arc first*, or the parallax in R. A. in time, nearly; one half of which (b) is to be subtracted from the hour angle (a), giving (a—b), the corrected hour-angle.

Under (2) put the *secant* of the hour-angle thus corrected. The sum of the logarithms under (3) will be the *proportional logarithm* of the *first* part of the parallax in declination, and that under (2) the *second*. The first part must be applied with such a sign as to diminish the star's distance from the elevated pole: the second must be applied with the same sign as the first, if the hour-angle and polar distance are the one greater, and the other less than 90° or 6^h; otherwise with a contrary sign. The result will be the true declination of the observed point of the moon's limb. Take the difference between this true declination of the observed point and the declination of the moon's centre, found from the Nautical Almanac, under which put the moon's horizontal semidiameter properly corrected for the given time, and take the sum and difference. Add together the *proportional logarithm* of this sum and difference; and take half the sum, to which add the *cosine* of the mean of the two declinations just found, the sum will be the *proportional logarithm* of the moon's semidiameter in R. A. nearly.

Under (4) put the constant logarithm 1.17609, the first sum under (1), and the *cosine* of the declination of the observed point, the sum will be the *proportional logarithm* of the exact parallax of R. A. in time. This being *added* to the star's R. A. when *west* of the meridian, but *subtracted* if *east*, will give the true R. A. of the point observed. To the true R. A. thus obtained, add the moon's semidia-

meter in R. A., or subtract it therefrom, according as the reappearance or disappearance of the star has been observed, and the result will be the true R. A. of the moon's centre deduced from observation.

Under this put the moon's (1) R. A. taken from the Nautical Almanac for the Greenwich time, and then the moon's R. A. three hours after, or the (2) R. A. Take the difference between the first and second, and the difference between the second and third. Then from the *proportional logarithm* of the first difference subtract that of the second, the remainder will be the *proportional logarithm* of a portion of time which must be *added* to the Greenwich time when the first R. A. is greater than the second, otherwise *subtracted;* and the result will be the Greenwich apparent time. The difference between this and the apparent time of the observer will be the longitude in time.

Ex. 1.—On the 3d of March, 1823, at Bahia, in latitude 12° 57′ 17″ S., and longitude by estimation 38° 30′ W., the reappearance of Antares from the dark limb of the moon was observed at $15^h\ 30^m\ 0^s.3$; required the true longitude?

Bahia, March 3d,	$15^h\ 30^m\ 0^s.3$	Moon's 1st R. A.	244° 27′ 29″.75
Lon. in time	2 34	2d R. A.	246 6 16 .82
		Dec.	25 55 15 .6 S.
G. est. time	18 4	var. for 10^s +	0 .63 S.
To this time.		hor. S.D.	14 50
Sun's R. A.	$22^h\ 56^m\ 58^s.64$	eq. par.	54 26 .5
Antares R. A.	16 18 35 .8	red. to 13°	— 0 .5
Dec.	26 1 50 .1		
		red. par.	54 26 .0
App. time	15 30 0 .3		
Sun's R. A.	22 56 58 .64	latitude	12° 57′ 17″ S.
		red. to 13° S.	— 5 0
Sum	38 26 58 .94		
Antares R. A.	16 18 35 .82	red. lat.	12 52 17
Diff.	22 8 23 .12		
	24		
Hour-angle	1 51 36 .88		

As it is convenient that the work should follow from beginning to end in regular order, that of the foregoing example has been transferred to the two following pages, and to avoid unnecessary waste of room, the remainder of this has been filled with the following example for exercise:—

Ex. 3.—On the 26th of May, 1822, at San Blas, in latitude 21° 32′ 25″ N., and longitude by estimation 105¼° W., at $9^h\ 22^m\ 41^s.3$, A. T. the immersion of A Leonis was observed by Lieutenant H. Foster, then Master's Mate of his Majesty's ship Conway; what was the true longitude?

Ans.—105° 18′ 27″ W.

			(1)			(2)		(3)		(4)
Moon's h. p.	0° 54′ 26′	P. L.	0.51941		P. L.	0.51941	P. L.	0.51941	C. L.	1.17609
Reduced lat.	12 52 17	secant	0.01104			0.01104	cosec.	0.65216		
Hour-angle	$1^h\ 51^m\ 36^s.9$ (*a*)	cosec	0.32976	*'s dec. 26° 1′ 50 .1 S. cosec.		0.35768	sec.	0.04645		
Sum		P. L.	0.86021							0.86021
*'s declina.	26° 1′ 50″.1	cosine	9.95345	(2) 10′ 53″.7 S.			P. L.	1.21802		
		C. L.	1.17609	26° 12′ 43 .8 S.						
Arc (1)	$0^h\ 1^m\ 50^s.6$	P. L.	1.98975							
Half	0 0 55 .3 (*b*)									
Difference	1 50 41 .6 (*a*—*b*)	secant				0.05276				
				(3) 0 20 37 .5 N.		0.94089	P. L.			
Moon's true declination				25 52 6 .3		cosine				9.95415
Moon's reduced declination				25 55 15 .6			(4)	$0^h\ 1^m\ 50^s.4$	P. L.	1.99045
			Difference	3 9 .3			*'s R. A.	16 18 35 .8		
			☽'s semidia.	14 50 .0			T. R. A.	16 16 45 .4 =		244° 11′ 21″
			Sum	17 59 .3						

Moon's T. dec.	25° 52′ 6″.3
Moon's R. dec.	25 55 15 .6
Sum .	51 47 22 .0
Half, or mean,	25 53 41 .0

Sum . .	17′ 59″.3	P. L.	1.00028
Difference .	11 49 .7	P. L.	1.18790
Sum			2.18818
Half			1.09409
Cosine			9.95405
Arc (5) .	16 6 .7	P. L	1.04814

	T. R. A.	244° 11 21″
	arc (5)	+ 16 .7
	C. R. A. P.	244 27 28
	1st R. A.	244 27 30
	2d R. A.	246 6 16
3.73239 P. L.	1st diff.	0° 0′ 2″
0.26066 P. S.	2d diff.	1 38 46
3.47173 P. L.	P. P.	0h 0m 3s.7
	Est. time	18 4 0.0
	Green. T.	18 3 56 .3
	Bahia T.	15 30 0 .3
	L. in T.	2 33 56 .0
	in deg.	38° 29′ 0″ W.

Ex. 2.—On the 20th of July, 1823, at Rio Janeiro, in latitude 22° 54′ 10″ S., an estimated longitude 43° 15′ W., the disappearance of λ Sagittarii behind the moon's dark limb was observed at 6^h 49^m $9^s.2$; what was the true longitude?

Rio Janeiro, July 20,	$6^h\ 49^m$	Moon's 1st R.A.	273° 16′ 59″		
Lon. in time	2 53	2d R.A.	274 54 48		
		var. in 10^s	+	5 .44	
Est. Greenwich time	9 42	dec.	25 39 21		S.
To this time.		var. in 10^s	—	0 .38.	
Sun's R.A.	$7^h\ 57^m\ 23^s.7$	hor. S.D.	14′ 52″		
Star's R.A.	18 17 7 .3	eq. par.	53 58		
Dec.	25° 30 31″.0	red. to 23°	— 2		
App. time	$6^h\ 49^m\ 9^s.2$				
Sun's R.A.	7 57 23 .7	red. par.	53 56		
		latitude	22° 54′ 10″		S.
Sum	14 46 32 .9	reduc.	— 8 12		
Star's R.A.	18 17 7 .3				
		red. lat.	22 45 58		S.
Diff.	20 29 25 .6				
	24				
Hour-angle	3 30 34 .4				

It is hardly necessary to give the variation of the sun's R. A. and D. in 10^s, as it is very small, and as the true time must differ but a few seconds from the estimated, on repetition the longitude cannot vary much on this account.

Ex. 4.—On the 3d of January, 1825, at Port Bowen, in latitude 73° 13′ 40″ N., and longitude by estimation 5^h 56^m W., the immersion of η Geminorum of the 4th magnitude was observed at 6^h 14^m $23^s.26$ M. T., and the emersion at 7^h 11^m $12^s.17$ M. T., by Lieutenant Henry Foster, R. N.; what was the true longitude?

Ans.—By immersion the longitude is 15^h 55^m 40^s, and by emersion it is 5^h 55^m 35^s W.

It was intended, if room would have permitted, to put the whole of the calculation on one page, and, though not done here, may readily enough be so placed by the calculator. This little attention ought not to be slighted, as a neat form, like a convenient formula, will be found of some service in accurate computations.

			(1)		(2)		(3)		(4)
☽'s R. hor. par.	0° 53′ 56″	P. L.	0.52342	P. L.	0.52342	P. L.	0.52342	C. L.	1.17609
Red. latitude	22 45 58	secant	0.03522	. . .	0.03522	cosec.	0.41231		
Hour-angle	$3^h 30^m 34.4$ (*a*)	cosec	0.09970	*'s dec. S. 25° 30′ 31″ cosecant	0.36588	secant	0.04454		
		P. L.	0.65834	. . .	. . .	. . .	. . .	. . .	0.65834
		C. L.	1.17609	(2) 0° 18′ 50″ S.		P. L.	0.98027		
Moon's dec.	25 30 31	cos.	9.95545	25 49 21 S.					
Arc (1)	2 55 .2	P. L.	0.78988						
Half . . .	1 27 .6 (*b*)								
	2 29 6 .8 (*a*—*b*)		secant . . .	. . .	0.21337				
				(3) 13 7 N.	1.13789				
Moon's true dec.	25° 36′ 14″ . . .	. . .	. . .	25 36 14 S.	cosine	. . .	. . .	. . .	9.95511
Moon's est. dec.	25 39 21 . . .	. . .	. . .	25 39 21		arc (4)	$0^h 2^m 55^s.3$	P. L.	1.78954
Sum . .	51 15 35	Diff. . .		3 7		*'s E. R. A.	18 17 7.3		
		Moon's S D. .		14 42					
Half . .	25 37 47 .5	Sum . . .		17 49		*'s T. R. A.	18 14 12.0 = 273° 33′ 0″		

Half	25 37 47.5 ...	☽'s S. D.	14 42			18 14 12.0 =		273° 33′ 0″
		Sum	17 49	P. L.	1.00445		arc (5)	— 15 36
		Diff.	11 35	P. L.	1.19144		C. R. A. P.	73 17 4
		Sum			2.19589		1st R. A.	273 16 59
		Half			1.09794		2d R. A.	274 54 48
		Cosine			9.95502	3.34445 =	1st diff. P. L.	+ 5
		Arc (5) — 15′ 56″			1.05296	0.26486 =	2d diff. P. L.	1 37 49
						3.06959	P. L. cor.	+ 9ˢ
							Est. time	9ʰ 42ᵐ 0
							G. T. July 20	9 42 9
							Rio. Jan. T.	6 49 9
							Long. in T.	2 53 0
							Or in degrees	43 15 0 W.

As the variation in declination for 9ˢ would only be about 0″.3, it is almost unnecessary to repeat the final part of the work, unless the operations were carried to fractions of a second.

BY AN ECLIPSE OF THE SUN.

An eclipse of the sun depends upon the same cause as an occultation, his light being intercepted by the body of the moon passing between him and the spectator. The beginning and end of a solar eclipse is easily observed by a telescope of moderate power properly prepared, if the point of contact of the limbs be nearly known, and the rule for computing the longitude is similar to that now given for an occultation. If the semidiameter of the moon passing through that point of the sun and moon, apparently in contact, be supposed to be produced to the centre of the sun, as seen from the observer, and conceiving this centre to be at the distance of the fixed stars, so as to have no sensible parallax, then it is manifest, that the rule for an occultation must apply by substituting for the moon's semidiameter the sum of the sun and moon's semidiameters, considering the sun to be at the same distance as the moon when seen from the earth's centre,—that is, subtracting the augmentation for the sun's semidiameter as if it were the moon's from it, as found in the Nautical Almanac. In the supposition just made, the sun's centre was supposed to have no parallax; but, as it has a horizontal parallax of about 8″.7, in finding the apparent place we cannot proceed exactly as for a fixed star. The sun's right ascension and declination, as seen from the centre, must be taken from the Nautical Almanac, which, corrected for parallax, will give the apparent right ascension and declination, thus reducing the case of a solar eclipse to a similarity with that of an occultation. The apparent right ascension and declination of the sun's centre must now be corrected, using the horizontal parallax of the moon in the computation. This would evidently give the same true place as if, taking the right ascension and declination of the sun's centre from the Nautical Almanac, we considered these elements as apparent, and corrected them for parallax, instead of the moon's horizontal parallax employing the difference between the horizontal parallaxes of the sun and moon.

Whence the true right ascension of the point answering to the sun's centre is obtained, and consequently, as formerly, the true right ascension of the moon's centre, from which the Greenwich apparent time is determined. The apparent time of the observer is found by means of a chronometer, whose error and rate have been determined by double altitudes if possible; if not, by altitudes both to the east and west of the meridian.

Rule.

By applying the estimated longitude in time to the observer's apparent time expressed astronomically, the Greenwich time will be obtained to the nearest minute. For this time take from the Nautical Almanac the sun's right ascension and declination, the sun's semidiameter diminished by the augmentation, the moon's right ascension and declination, semidiameter, and horizontal parallax, corrected for the spheroidal figure of the earth, and diminished by the sun's horizontal parallax. Take also the moon's R. A. for 3 hours after the first R. A., or estimated Greenwich time.

Find the hour-angle, which, in the afternoon, is the observer's apparent time, and in the morning its complement to 24 hours.

Employing the moon's diminished horizontal parallax, correct the sun's right ascension and declination, as if for some point on the moon, extended, proceeding as formerly, only putting the sum of the sun's semidiameter, diminished by augmentation and the moon's semidiameter, instead of the moon's semidiameter alone. If the resulting Greenwich time differ from the estimated, the sun's R. A. and declination must be corrected for the difference, repeating the operation as often as necessary, till the Greenwich time by computation and estimation agree.

Ex.—On the 7th of September, 1820, at the Royal Naval College, Portsmouth, in latitude 50° 48′ 3″ N., and longitude by estimation 1° W., the end of a solar eclipse was observed at $3^h\ 12^m\ 55^s$; required the true longitude?

Ports. Sept. 7.	$3^h\ 13^m$	Moon's (1) R. A.	166° 54′ 47″
Lon. in time	4	var. in 10^s	4 . 4
	——	(2) R. A.	168 13 26
Est. G. T.	3 7	dec.	6 21 4 N.
To this time.		var. in 10^s	1 .84 S.
Sun's R. A. $11^h\ 4^m\ 13^s$. 60		hor. S. D.	14 42 . 7
Var. in 10^s	0^s.025	equa. par.	53 56 . 0
Dec.	5° 58′ 22″ N.	red. to lat.	— 6 . 3
Var. in 10^s	0 .16		————
Semidiameter	11 54 . 8	red. par.	53 49 . 7
Hor. par.	8 . 6	sun's hor. par.	— 8 . 6
Alt. 30° nearly			————
App. time $3^h\ 12^m\ 55^s$=H.A.		difference	53 41 . 1
		latitude	50 48 3 N.
		Reduction	— 11 15
			————
		Red. lat.	50 36 48
		Sun's S. D.	15 54 . 8
		Aug. to 30°	— 7 . 8
			————
		Red. S. D.	15 47 . 0
		Moon's S. D.	14 42 . 7
			————
		Sum	30 29 . 7

			(1)		(2)		(3)		(4)
Diff. of par.	53′ 41″	P. L.	0.52543	P. L	0.52543	P. L.	0.52543	C. L.	1.17609
Red. lat.	50° 36′ 48	secant	0.19751		0.19751	cosec.	0.11188		
Hour-angle	$3^h\ 12^m\ 55^s$ (*a*)	cosec	0.12737	☉'s dec. N. 5° 58′ 22″	0.98273	secant	0.00236		
Sum .	. .	P. L.	0.85031	cosec.	.	.	.	.	0.85031
		C. L.	1.17609	(2) 41″ 16′ N.	.	P. L.	0.63967		
Sun's dec.	5° 58′ 22″	cos.	9.99763	6° 39 38 N.					
Arc l.	$1^m\ 42^s.2$	P. L.	2.02403						
Half .	0 51.1 (*b*)								
Diff. .	3 12 3.9 (*a*—*b*)	secant	.	.	0.17463				
				2′ 22″ S.	P.L. 1.88030				

		T. dec.	6° 37 16 N.	cosine .	.	. .	9.99709
Sun's S. D. .	15′ 47″	☽'s dec.	6 21 4 N.	. .	$0^h\ 1^m\ 42^s.2$	P. L.	2.02349
Moon's S. D.	14 43	Diff.	16 12	sun's R. A.	11 4 13.64		
Sum .	30 30	. . .	30 30		11 5 55.84 =	. .	166° 28′ 58″
		Sum	46 42	P. L. 0.58596		arc. (5)	+ 26 0
True dec.	6° 37′ 16″	Diff.	14 18	P. L. 1.09994		red. R. A.	166 54 58

Moon's dec.	6 21 4			sum	1.68590			1st R. A.	166 54 47
Sum	12 58 20			half	0.84295			2d R. A.	168 13 26
Half	6 29 10	cosine			9.99721		2.99203	P. L.	11
		Arc (5)	26′ 0″	P. L.	0.84011		0.35957	P. L.	1 18 39
		Repetition					2.63246	P. L. to est. time	0h 0m 25s 3 17 0
Moon's dec.	6° 21′ 4″			R. A.	166° 28′ 58″				
Var. in 25s	— 5			arc (5)	25 57			G. T.	3 17 25
Cor. dec. ☽	6 20 59				166 54 55			app. T.	3 12 55
True dec.	6 37 16			(1) R. A. cor.	166 54 58			lon. in time in deg.	4 30 1° 7′ 30″
Diff. Sum S. D.	16 17 30 30			diff.	— 3 1 18 39	P. L. P. L.	3.55630 0.35957		
Sum Diff.	46 47 14 13	P. L. P. L.	0.58518 1.10247	est. T.	— 7s 3h 17m 25	P. L.	3.19673	(1) R. A. var. in 23s	166° 54′ 47″ + 11
								cor.	166 54 58
		Sum	1.68765	G. T.	3 17 18				
		Half	0.84382	Ap. T.	3 12 55				
Half sum	6° 29′ 10″	cos.	9.99721	lon. in T.	4 23	= 1° 5′ 45″ W.			
Arc (5)	25 57 .4	P. L.	0.84103						

On the 7th of July, 1823, at Dunglass House, the seat of Sir James Hall, Bart., in latitude 55° 56′ 32″ N., and longitude by estimation $9^m\ 30^s$ W., Captain Basil Hall, R. N., observed the end of a solar eclipse at $7^h\ 55^m\ 34^s.1$ mean time; required the true longitude of Dunglass?

July 7th,	$17^h\ 55^m\ 34^s$	July 7th, Mean Time,	$17^h\ 55^m\ 34^s.1$
Est. lon. in T.	+ 9 30	equ. of time to $18^h\ 1^m$	— 4 28.7
Eq. of T. at noon	— 4 21		
		apparent time at D.	17 51 5.4
Approx. G. T.	18 0 43		24
Or	18 1 nearly.		
		hour-angle,	6 8 54.6

To this time.

Sun's R. A.	$7^h\ 6^m\ 2^s.55$	Moon's (1) R. A.	106° 14′ 5″.8
Var. in 10^s,	0.03	(2) R. A.	108 16 16 .4
Sun's dec.	22° 35′ 45″.0	var. in 10^s,	+ 6 .8
Var. in 10^s,	— 0 .05	dec.	23 48 55 .7
Sun's S. D.	15′ 45″ .5	var. in 10^s	— 1 .3
Aug. to 20° alt.	— 5 .5	S. D.	16 43
		equat. par.	61 20 .5
Cor. S. D.	15 40 .0	red. to 56°	— 8 .1
Hor. par.	8 .54		
Latitude,	55° 56′ 32″	red. par.	61 12 .4
Reduction,	— 10 40	sun's par.	— 8 .4
Red. lat.	55 45 52	diff. par.	61 4 .0

			(1)		(2)		(3)		(4)
Diff. par.	1° 1′ 4″	P. L.	0.46947	P. L.	0.46947	P. L.	0.46947	C. L.	1.17609
Red. lat.	55 45 52	secant	0.24980	secant	0.24980	cosec	0.08264		
Hour-angle	$6^h\ 8^m\ 54^s.6$ (a)	cosec	0.00033	☉'s dec. N.					
				21° 35′ 45″	0.41541	secant	0.03469		
		P. L.	0.71960	cosec					0.71960
		C. L.	1.17609	(2) 46 36½		P. L.	0.58680		
Sun's dec.	22° 35′ 45″	cosine	9.96531	23 22 21½					
Arc 1	$2^m\ 28^s.8$	P. L.	1.86100						
Half	1 14.4 (b)								
	6^h 7 40.2 (a—b)	secant			1.47492				
				(3) $26\frac{1}{2}^s$ P. L.	2.60960				
		T. dec.		23 22 48		cosine			9.96279
Sun's S. D.	15′ 40″	moon's dec.		23 48 56		arc (4)	— $2^m\ 29^s.6$	P. L.	$\bar{1}$.85848
Moon's S. D.	16 43	diff.		26 · 8		sun's R. A.	7 6 2.6		
Sum	32 23			32 23			7 3 33.0		=105° 53′ 15″.0
		Sum		58 31	P. L. 0.48799	arc (5)	=		+ 20 52 .0
Sun's T. dec.	23° 22′ 48″	diff.		6 15	P. L. 1.45939	T. R. A.			106 14 7 .0
Moon's	23 48 56				sum 1.94738	(1) R. A.			106 14 5 .8
Sum	47 11 44				half 0.97369	(2) R. A.			108 16 16 .4

Half	23 35 52 cosine				9.96208	3.97321	P. L. (1) diff.	+		1.2
		arc (5)	20′ 52″	P. L.	0.93577	0.16829	P. L. (2) diff.	2	2	10.6
						4.14150	P. L. arc (6)	+		0.4
							est. G. T.	18h	1m	0.0
							T. A. G. T	18	0	0.4
							A. Dungl. T.	17	51	5.4
							Lon. in T.		9	55.0 W.
							in deg.	2°	28′	45″ W.

From angles taken by Captain Hall at Dunglass and at North Berwick Law, with the Isle of May light, it appears, on comparison with the latitudes and longitudes of the two latter places, that the longitude of Dunglass tower is 2° 21′ 42″ W. As this result is conceived to be correct, the errors of the lunar tables, and the observations taken together, must amount to about 7′ of longitude.

IV. BY THE MOON'S TRANSIT.

Mr Bailey's Method.

The method of determining the difference of longitude between two given points on the surface of the earth, which is one of the most difficult problems in practical astronomy, has long engaged the attention of various astronomers and mathematicians; and has been executed with more or less accuracy according to the means employed for that purpose. If the distance between the two observatories be not very great, their difference of meridian may be determined with considerable accuracy, by means of chronometers conveyed from one observatory to the other; or by means of signals previously agreed on. These methods have been practised very successfully on many recent occasions. But, where this is impracticable, we must have recourse to the observation of certain celestial phænomena for the solution of the problem: and for this purpose, five several and distinct methods have been proposed: 1° the eclipses of Jupiter's satellite's: 2° eclipses of the moon: 3° eclipses of the sun: 4° occultations of the fixed stars: 5° the meridional transits of the moon, compared with certain stars previously agreed on.

The results deduced from the observations of the eclipses of Jupiter's satellites are, for obvious reasons, very unsatisfactory. The phænomena will, in fact, appear to take place at different moments of time, with different instruments and to different observers. Moreover, they are visible only in certain positions of the planet in its orbit,—a circumstance which very much circumscribes the utility of the method.

The eclipses of the moon afford a still more unsatisfactory result: they occur but seldom in the course of a year, and the phænomena attending them cannot (on account of the indistinctness of the border of the earth's shadow) be observed with that degree of accuracy which the present state of astronomy requires for such purposes.

Eclipses of the sun are more certain in their deductions; but they so rarely occur, and are at the same time so limited in extent, that they can seldom be brought in aid of the general solution of the problem.

There remain, therefore, only the two other methods, on which the practical astronomer can safely and constantly depend. Of these, I am aware that occultations of the fixed stars by the moon have been long considered as affording the best means of determining the difference of longitude between two places: and, assuredly, the results deduced from such observations, made under favourable circumstances, have agreed with each other to a greater degree of accuracy, than those deduced by any of the preceding methods.

There are, however, many circumstances, attending the practical solution of the problem by this method, which tend to diminish the confidence which is reposed in the correctness of the theory. In the first place, it is necessary to know the apparent right ascension and declination of the star very exactly, on the day of observation; which, if the star is of inferior magnitude (and such being the most numerous, are the most likely to be occulted), may not be readily determined; for, we may not be able to find it in any catalogue; and, when found, we have to compute its precession, aberration, and nutation expressly for this purpose. In the second place, we have to calculate the parallax of the moon for the given moment of observa-

tion: and in this computation we must assume a given quantity for the compression of the earth; respecting which astronomers are by no means agreed, and which will consequently give rise to various results, according to the view which each astronomer may take of the subject. Thirdly, this method is dependent on the accuracy of the lunar tables, not only as to the position of the moon and her horary motion, but also to her horizontal parallax and semidiameter. Fourthly, the method is, in a great measure, dependent on a correct knowledge of the longitude and latitude of the place of observation. And, lastly, the apparent border of the moon is so uneven (consisting of projecting mountains and hollow valleys) that we cannot always depend on the immersion or emersion having taken place at the exact distance from the moon's centre, as computed from the lunar tables.

The *meridional transits* of the moon, agreebly to the method about to be described, are free from all these objections: the observations are made with the greatest facility; the opportunities are of frequent occurrence; the absolute time is of no material consequence; the computations are by no means intricate or troublesome; and the results are (I believe) more to be relied on than by any of the preceding methods.

This method consists in merely observing, with a transit instrument, the *differences* of right ascension between the *border* of the moon, and certain fixed stars *previously agreed on*;* which stars are so selected that they shall differ very little from the moon in declination. It is evident that this method is quite independent of the errors of the lunar tables, except as far as the horary motion of the moon (in right ascension) is concerned, and which, in the present case, may be depended on with sufficient confidence: that it does not involve any question as to the compression of the earth: that a knowledge of the correct position of the star is not at all required: and, finally, that an error in the state of the clock is of no consequence. Consequently, a vast mass of troublesome and unsatisfactory computation is avoided. Moreover, it is the only method that is *universal*, or that may be adopted, at one and the same time, by persons in every habitable part of the globe: for it is applicable to situations distant 180° in longitude from each other; and even *beyond* that if required.

It might indeed, at first sight, appear that the same results would be obtained, if we merely observed the correct *time* of the moon's transit, without any reference to the contiguous stars: but a moment's reflection will convince us that, by referring the moon's border to the adjacent stars, we obviate all errors not only of the clock, but also in the position of the transit instrument.

For the solution of this problem, let us make

t = the difference (in sidereal time) of the transit of the moon's *limb*, and of the star previously agreed on, at the observatory situated most *westerly*; which will be *positive* when the star precedes the moon, or when the Æ of the moon exceeds that of the star; but, on the contrary, *negative*.

τ = the similar difference at the observatory situated most *easterly*.

---

* Lists of such stars, called *moon culminating* stars, are now annually published.

$(t-\tau)=$ the true observed difference in the Æ of the moon's *limb*, for the time elapsed between the two observations.*

$c=$ the *apparent* time (as shown at Greenwich†) of the culmination of the moon, at the *western* observatory.

$\varkappa=$ the *apparent* time (as shown at Greenwich) of the culmination of the moon at the *eastern* observatory.

$a=$ ☽'s right ascension in *space*
$d=$ ☽'s true declination‡ } computed for the time c.
$r=$ ☽'s true radius, or semidiam.‡

$\alpha=$ ☽'s right ascension, in *space*
$\delta=$ ☽'s true declination‡ } computed for the time $\varkappa$.
$\varrho=$ ☽'s true radius, or semidiam.‡

$s=$ the length of the true solar day, expressed in seconds of time.

$m=$ the moon's motion in Æ, in *half* that interval, expressed in seconds of space. See page 168.

$\chi=$ the *assumed* difference of longitudes in time: *plus* when west, and *minus* when east.

$(\chi+e)=$ the *correct* difference of longitudes.

Find the apparent times, c and $\varkappa$, of the moon's culmination, *to the nearest minute*,§ in order to compute d, r and δ, ϱ, for those approximate times respectively:‖ and then make

$$\Delta=(t-\tau)\pm\frac{r}{15\,.\cos d}\pm\frac{\varrho}{15\,.\cos\delta}$$

which is the true observed difference in the Æ of the moon's *centre*,

---

* If more than one star has been observed at both observatories on any given night, t and τ must be taken equal to the *mean* of *all* the corresponding comparisons made at each observatory respectively.

† Or as shown at Paris, Berlin, Milan, or any other place for which the ephemeris is calculated, from which the computations are made. And this must always be understood when Greenwich is alluded to in this manner.

‡ The *true* declination and semidiameter of the moon are such as they are supposed to be if *seen from the centre of the earth*; in opposition to the *apparent* declination and semidiameter, which some persons have erroneously imagined ought to be adopted.

It may be sufficient to observe here once for all, that (with a view to prevent confusion) the quantities connected with the *eastern* observatory are denoted by *Greek* letters; and that the similar quantities connected with the *western* observatory are denoted by *Roman* letters.

§ The apparent time of the moon's culmination at Greenwich to the nearest minute may be seen in the Nautical Almanac; and the apparent time (at Greenwich) of its culmination on any other meridian may thence be easily deduced. Or, if the sidereal time is known, we may determine the Greenwich apparent time very nearly, by subtracting therefrom the sun's right ascension at Greenwich at the preceding noon; and diminishing the interval by the acceleration of the fixed stars. Or, we shall have, in all cases sufficiently near for this purpose, the required interval $c-\varkappa=[\chi+(t-\tau)]\times\cdot 99727$; where it should be observed that $[\chi+(t-\tau)]\times\cdot 99727$ is equal to the time $\chi+(t-\tau)$ diminished by the acceleration of the fixed stars for that interval.

‖ It may be useful here to remark, that it is not necessary to determine with strict accuracy the *absolute* value of the moon's semidiameter at *both* observatories, in order to find the value of Δ; for the values may be estimated (in most cases by inspection) in whole seconds only, for one observatory, and the correct *differences*, in the given interval, being added thereto, will give the proper values for the other observatory. With respect to the declination, it may be taken to the nearest ten or twenty seconds only.

for the time elapsed between the two observations: where the *upper* sign is to be taken when the first (or western) border of the moon is observed; and the *lower* sign when the second (or eastern) border is observed.* Then, by assuming χ equal to the presumed difference of longitude, and knowing the apparent time (at Greenwich) at one of the observatories to the nearest minute, we may determine the required apparent time (at Greenwich) at the other observatory, by the following equation:

$$c = \varkappa + (\chi + \Delta)\frac{86400}{s}$$

Compute a and α for the respective times c and $\varkappa$†; correcting the moon's motion for third differences, if required. And the formula for the correction of the assumed difference of longitude will be

$$e = \left[15\,\Delta - (a - \alpha)\right]\frac{s}{2m}$$

which, being added to χ, will give $(\chi + e)$ for the true difference of meridians required.

It is evident that, if $15\,\Delta - (a - \alpha) = 0$, the value of χ has been assumed sufficiently accurate, and does not require correction. In fact, the difference will in general be very small: and, when this is not the case, we may justly suspect some error in the steps of the process.

Example. On December 5, 1824, Lieut. Foster observed the differences in the culmination of the moon and of the two stars 62 and 95 *Tauri*, at Port Bowen; the station where the expedition (for

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* These expressions are the same as those which are used in my Paper on this subject, inserted in the 2d volume of the *Memoirs of the Astronomical Society*; where this subject is treated more at length.

† It may here also be useful to remark, that it is not necessary to determine with strict accuracy the *precise moment* of the apparent time of the transit of the moon at *both* observatories, for the purpose of determining a and α; for it will be sufficient to know the apparent time of the transit of the moon to the nearest minute only, for one observatory, and to find the correct *difference* of the apparent times, by means of the expression $(\chi + \Delta)\frac{86400}{s}$.

In fact, nothing more is required than to compute the true increase of the moon's Æ during this given *interval*; and for this purpose Dr Brinkley has suggested a very convenient rule, which is given in the first number of the *Dublin Philosophical Journal*. This distinguished astronomer has there shown that $(a - \alpha)$, as far as *first differences* only are concerned, may be expressed by $(\chi + \Delta)\,.\,\Delta''.\frac{2}{s}$; leaving the equation of *second* differences (and of the *third* differences, if required) to be applied in the usual manner.

Under this point of view, the problem admits of two cases: one where both the observations are made on the same side of noon or midnight; and the other where are made on different sides. In the former case, the expression $(\chi + \Delta).\Delta''.\frac{2}{s}$, as far as *first differences* are concerned, will lead us to the correct solution, and will save much time and labour: but, in the latter case, it must be divided into two parts, viz.

$$(12^{h} - \varkappa)\,.\,\Delta''\frac{2}{s} \text{ and } \left[\chi + \Delta - (12^{h} - \varkappa)\right].\,\Delta'''.\,\frac{2}{s};$$

and consequently becomes more intricate. In these expressions Δ'' and Δ''' denote the successive *first differences* in the moon's motion in Æ; or the same quantities that are alluded to in the explanation of tables of equations of second, third, &c. differences.

the discovery of a North-West passage, under the command of Capt. Parry) passed the winter of 1824-25. Similar differences were observed also at Greenwich. These differences, in sidereal time, were respectively as follow:

	at Greenwich	at Port Bowen
62 *Tauri*	$\tau = +\ 9^{m}\ 45^{s},58$	$t = +\ 24^{m}\ 53^{s},98$
95 ——	$\tau = -\ 9\ \ 25,98$	$t = +\ \ 5\ \ 42,90$

what was the longitude of the place?

For the solution of this question we must first assume an approximate longitude. Now it appears from some occultations of fixed stars, observed by Lieut. Foster, that the longitude might be considered as $5^{h}.55^{m}.39^{s},5$ west from Greenwich; but, for the sake of round numbers, I shall assume it equal to $5^{h}.55^{m}.40^{s}$. The operation therefore will be as follows.

The mean of the two observations gives $t = +\ 15^{m}.18^{s},44$ and $\tau = +\ 0^{m}.9^{s},80$: consequently we have $(t-\tau) = +\ 15^{m}.8^{s},64$; which being added to $5^{h}.55^{m}.40^{s}$. and the sum diminished by the acceleration of the fixed stars during that interval, will give in round numbers $6^{h}.10^{m}$ as approximate value of the apparent time elapsed between the two culminations.

By the Nautical Almanac it appears that the moon's *centre* passed the meridian at Greenwich at $11^{h}.35^{m}$: consequently the moon's *first limb* passed at $11^{h}.34^{m}$, at Greenwich; and at $17^{h}.44^{m}$ (Greenwich time) at Port Bowen. With these approximate values we find the declination and semidiameter of the moon, at those respective periods, as follow:

at $11^{h}\ 34^{m}$	at $17^{h}\ 44^{m}$
$\varrho = \ 0^{\circ}\ 15'\ 42''$	$r = \ 0^{\circ}\ 15'\ 44'',39$
$\delta = 23\ \ 39\ \ 20$	$d = 23\ \ 53\ \ 30$

whence we find

$$\Delta = +15^{m}.8^{s},64 + \tfrac{1}{15}(17'.12'',88 - 17'.8'',41) = 15^{m}.8^{s},938$$

and the correct value of c, for the subsequent computations, will be $c = 11^{h}.34^{m} + (5^{h}.55^{m}.40^{s} + 15^{m}.8^{s},938)\frac{86400}{86662} = 17^{h}.43^{m}.41^{s},67$. The moon's true right ascension must now be calculated for the apparent time $\varkappa = 11^{h}.34^{m}$ and $c = 17^{h}.43^{m}.41^{s},67$:* whence we have

$$\begin{aligned} a &= 69^{\circ}\ 53'\ 49'',21 \\ \alpha &= 66\ \ \ 6\ \ 29\ ,93 \\ \hline (a-\alpha) &= \ \ 3\ \ 47\ \ 19\ ,28 \\ 15\,\Delta &= \ \ 3\ \ 47\ \ 14\ ,07 \\ \hline 15\,\Delta - (a-\alpha) &= \qquad -5\ ,21 \end{aligned}$$

This remainder, being multiplied by $\frac{s}{2m}\left(=\frac{24^{h}.4^{m}.22^{s}}{2\times 7^{\circ}.24'.42''}\right) = 1.62$, will give $e = -8^{s},44$, and the correct longitude will be $(\chi + e) = 5^{h}.55^{m}.31^{s},56$.†

Should the value of $15\,\Delta - (a-\alpha)$ be considerable, it will show

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* Or, at once, for the given *interval;* agreeably to the method proposed by Dr Brinkley, as stated in the preceding note.

† From a mean of 21 eclipses of Jupiter's satellites, the longitude was found to be $5^{h}.55^{m}.29^{s}$.

either that there is some error in the computation, or that the value of χ has not been assumed sufficiently near. In the latter case, we must diminish e by the acceleration of the fixed stars during that interval, and apply the result to c as a new value for the computation of a. Thus $-(8^s,44-0^s,02)=-8^s,42$ being added to c, will give $17^h.43^m.33^s,25$ as the correct time for which a should have been computed, and the result would then have been*

$$\begin{aligned} a &= 69^\circ\ 53'\ 44'',00 \\ \varkappa &= 66\ \ \ 6\ 29\ ,93 \\ \hline (a-\varkappa) &= \ \ 3\ 47\ 14\ ,07 \\ 15\,\Delta &= \ \ 3\ 47\ 14\ ,07 \end{aligned}$$

For the convenience of those persons who make use of this method of solution, I have computed Table I. of the value of $\frac{s}{2m}$; the argument of which is m = the moon's motion in Æ in 12 true solar hours; or the quantity which is actually employed, as the *first difference*, in computing the moon's place, for c or $\varkappa$, as the case may be. The value of s is, in this table, assumed to be equal to $24^h\ 4^m$.

TABLE I.						TABLE II.		
Argument.	$\frac{s}{2m}$	Diff. to 10′	Argument.	$\frac{s}{2m}$	Diff. to 10′	Argument.	Log. of $\frac{86400}{s}$	Diff. for Seconds.
5° 0′	2.4067	—	6° 30′	1.8513	—	3m 20s	9.998996	—
5 10	2.3290	777	6 40	1.8050	463	3 30	9.998946	1s = 5
5 20	2.2562	728	6 50	1.7610	440	3 40	9.998896	2 = 10
5 30	2.1879	683	7 0	1.7190	420	3 50	9.998845	3 = 15
5 40	2.1235	644	7 10	1.6791	399	4 0	9.998795	4 = 20
5 50	2.0629	606	7 20	1.6409	382	4 10	9.998745	5 = 25
6 0	2.0055	574	7 30	1.6044	365	4 20	9.998695	6 = 30
6 10	1.9514	541	7 40	1.5696	348	4 30	9.998645	7 = 35
6 20	1.9000	514	7 50	1.5362	334	4 40	9.998595	8 = 40
6 30	1.8513	487	8 0	1.5042	320	4 50	9.998545	9 = 45

Table II. will be convenient for determining the logarithm of the value of $\frac{86400}{s}$. The argument is the increase in the sun's Æ in 24^h; which is found from an ephemeris, by taking the difference in the Æ of the sun for two successive days.

OF THE TRANSIT INSTRUMENT.

A transit instrument is a telescope properly placed in the meridian for the purpose of observing the times at which the celestial bodies pass this circle. If the clock or chronometer by which the time is marked be adjusted to show sidereal time, then their right ascensions will be found. This is perhaps the best method of determining the rates of chronometers.

* All these values of a and $\varkappa$ have been corrected for *third* differences, which diminish the value of $a-\varkappa$ about $0''.5$.

The telescope is fitted to an axis, of which the ends tapered into points turn in notches, from their shapes called *Vs* or *Ys*. This axis is made hollow, opposite one of the ends of which is placed a lamp for illuminating the wires in night observations.

These wires, generally five in number, are placed in the telescope equidistant from each other, and perpendicular to the horizon, having also a horizontal wire bisecting them, near or upon which the transits are observed.

When properly adjusted, the middle vertical wire coincides with the meridian, and the instant that the centre of any heavenly body passes this wire, is called its transit. The other parallel wires are intended to correct or verify the observation, by taking a mean between the transits over the *first* and *last*, the SECOND and FOURTH, and comparing it with the third or meridian wire; or, what is more correct, a mean of the whole called the reduction of the wires.

There are five principal adjustments necessary in placing a transit instrument, three relative to the telescope, and two to the axis.

1. *The wires should be set perfectly vertical.*—This is verified by observing that any distant object cut by a wire does not change its position relative to that wire, on moving the instrument up and down. If it does, the wires must be all turned till the object is kept upon them, when moved through their whole extent, and the adjustment is then complete.

2. *The telescope should have no parallax.*—When any distant object is bisected by the horizontal wire, if, on moving the eye up and down a little, the object should appear to separate from the wire, the instrument is said to have a parallax. This must be corrected by placing the object and eye-glasses at such a distance from each other, that their foci may meet in the point of intersection of the wires. When the object-glass has been properly fixed by the instrument-maker, the observer has only to adjust the eye-glass.

3. *The line of collimation should be correct.**—This is known by bisecting any object by the meridian wire, and if, on reversing the axis, the object still remains bisected as before, the line of collimation is correct. If not, it must be adjusted by means of the small screws in the sides of the telescope. This is effected by easing the one screw and tightening the other till the error appears one half diminished, when the axis is again reversed, and the operation is repeated till the adjustment is properly effected.

4. *To level the axis.*—This is performed by means of a screw placed under one of the *Ys* or notches, which raises or depresses that end of the axis at pleasure, while the true horizontal position is ascertained by a spirit-level.

5. *To bring the telescope to the meridian.*—This is accomplished by means of a horizontal screw acting on one end of the axis, by which it is moved backward or forward till its proper position is obtained.

As the problem of bringing a transit instrument into the meridian is one of considerable difficulty, it is proposed to treat it at some length.

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* The line of collimation is an imaginary straight line supposed to join the centre of refractions of the object-glass, and the intersection of the meridian and horizontal wire in the centre of the telescope.

To take a Transit.

With the latitude of the place and the declination of the object, compute its meridian altitude.

When it is known to approach the meridian, elevate the telescope to the given altitude by the circle attached to the end of the axis, or in some instruments, by one of the circles attached to the eye-end of the telescope. Now, because the telescope inverts objects, the object will appear to come into the field of view from the west, and move towards the east.

Mark the time of transit over each wire, using a dark glass to save the eye when the sun is observed.

FROM THE GREENWICH OBSERVATIONS.

1816. Nov.	Wires. I.	II.	III.	IV.	V.	Reduc. of Wires.	Star.
	1^s.4	20^s.0	21^h 55^m 38^s.4	56^s.5	15^s.2	38^s.30	α Aquarii.
3d	22.6	55.2	0 29 27.5	0.0	32.5	27.56	α Cassiop.
4th	0.4	18.4	21 55 37.2	55.7	14.1	37.16	α Aquarii.
	51^m 29^s.4	51^m 48^s.6	14^h 52^m 7^s.5	52^m 26^s.7	52^m 46^s.7		Sun's 1 L.
8th	53 45.0	54 4.3	14 54 23.4	54 42.5	55 1.6		Sun's 2 L.

By taking the means as directed.

That of the 3d will be . . .		21^h 55^m 38^s.30
4th		0 29 27.16
8th, both limbs . . .		14 53 15.50
By the Nautical Almanac the sun's right ascension that day was . . .		14 54 4.70
The error of the clock on the 8th is slow, or .	—	49.20

Suppose the observation had been made with one wire, as the middle one only, then

To sidereal time shown by clock . .		14^h 52^m 7^s.6
Add semidiameter, Table XV. . .	+	1 7.6
Red. 1^m 7^s.6 mean solar to sidereal time . .	+	0.2
Transit of centre		14 53 15.4
Mean of the whole		14 53 15.5
Difference only	—	0.1

The error of the clock may readily be determined from the stars, if one of those whose true places are given in the Nautical Almanac is observed. Otherwise the corrections must be applied from appropriate tables.

Observed the transit on 3d . . .		21^h 55^m 38^s.30
α Aquarii R. A. by tables		21 56 24.35
Error of clock by the star slow, on the 3d .	—	46.05
on the 8th .	—	49.20
Loss in 4.71 sidereal days		3.15
Or the daily loss is		0.67

TO BRING A TRANSIT INSTRUMENT INTO THE MERIDIAN.

To perform this problem, the time should be accurately determined by an altitude near the prime vertical, or still better by equal altitudes, as already explained. Bring the telescope to any celestial object when nearly passing the meridian, and, by turning the horizontal screw, make the middle wire bisect the object at the instant of its transit, then is the instrument in the meridian.

Should the object be the sun, as it cannot be accurately bisected, either limb must be observed when on the meridian, and by allowing for the time his semidiameter takes to pass the meridian, that of the centre becomes known. This is found most accurately in the Nautical Almanac, or, if it is not at hand, from Table XV.

To find the Time that any Star takes to pass from one Wire to another in a Transit Instrument, that of the Equinoctial being known.

Rule.—To the cosine of the star's declination add the proportional logarithm of the time at the equinoctial, the sum is the proportional logarithm of the time by the given star.

Ex.—On the 10th of April, 1826, by a transit telescope which gave 25ˢ.4 for the passage of a star of the equinoctial from wire to wire; what would be the time by Antares, having 26° 2′ S. declination?

Declination . .	26° 2′	cosine . .	9.95354
Time . .	25ˢ.4	P. L. . .	2.62867
Reduced time .	28 .26	P. L. . .	2.58221

Or this would be more readily performed by considering the seconds minutes, and converting the decimals into thirds to be estimated seconds, then the answer will come out in minutes and seconds to be estimated seconds and thirds.

Declination . .	26° 2′	cosine . .	9.95354
Time .	25ˢ. 4, or 25ᵐ 24ˢ	P. L. . .	8.05044
	28.27, or 28 16	P. L. . .	0.80398

Hence the star's expected time of approach to the other wires becomes known after its contact with the first is observed.

One of the most convenient methods of fixing the transit telescope in the meridian in mean northern latitudes is by means of Polaris.

It is required to set a transit instrument by Polaris, on the 1st of March, 1826, at Edinburgh, in latitude 55° 57′ 21″ N. By a reference to the Nautical Almanac, its altitude at its superior transit will be 57° 34′, and at its inferior 54° 21′; and its R. A. is $0^h\ 58^m\ 12^s.20$. It must therefore pass the meridian about $2^h\ 8^m$, and $14^h\ 8^m$ at the altitudes stated above, which serve as a guide to advertise the observer to be prepared.

Now let the clock be regulated to sidereal time, and when it shows $0^h\ 58^m\ 12^s.2$, make the middle wire bisect Polaris, then will the instrument be in the meridian. If, however, the first time assumed was not known with sufficient accuracy, the error of the clock can now be found very nearly by the transit of the sun or a star. By repeatedly observing Polaris, and correcting in this manner, the instrument will at last be truly in the meridian. This may be verified

in several ways. One of the most general methods is by observing that the semirevolutions of circumpolar stars are equal, supposing the rate of the clock to be uniform.

Rule.—Let a circumpolar star be observed by the transit above and below the pole. If the difference of these times is exactly half a revolution of the earth round its axis, found from observations of the fixed stars by the clock, the transit is truly in the plane of the meridian. If the difference between these times is not equal to half a revolution, take the difference between the interval of the times of the two passages, and half a revolution, and to the constant logarithm 0.30103, add the proportional logarithm of this difference, the log cotangent of the star's polar distance, and the log cosine of the latitude of the place, the sum, rejecting tens in the index, will be the prop. log of the angle which the transit makes with the true meridian. Then if the star, when above the pole, comes later to the meridian wires of the transit than half a revolution after it passed it when below the pole, the transit lies to the east of the true south meridian.

Again, to the prop. log of the error in azimuth just found, add the log secant of the star's altitude, and the log sine of its polar distance, the sum, rejecting tens in the index, is the prop. log of the error of the transit in time from the true meridian at that altitude.

Ex.—At Cambridge, in latitude 52° 12′½ N., longitude 30ˢ E., by a clock that loses $3^m\ 10^s$ a day on mean solar time, Capella was observed to pass the meridian wire below the pole, at $6^h\ 0^m\ 0^s$ in the evening, and the next morning to pass it at $5^h\ 56^m\ 21^s$.

Since the clock loses $3^m\ 10^s$ daily, then $\dfrac{24^h - 3^m\ 10^s}{2} =$ $\dfrac{23^h\ 56^m\ 50}{2} = 11^h\ 58^m\ 25^s =$ half a revolution by the clock.

To time of passage S. P.	$6^h\ 0^m\ 0^s$
Add half a revolution, ...	11 58 25
Sum, rejecting 12^h	5 58 25

The time it would have passed if the transit had been exactly in the meridian.

But it passed it at	5 56 21
Difference	2 4

Constant logarithm	0.30103
Difference $2^m\ 4^s$ P. L.	1.94000
Polar dist. 44° 15′ cotangent	0.01137
Latitude 52 12½ cosine	9.78731
Deviation $1^m\ 38^s.56$ W. P. Log.	2.03971

To find the error in the transit below the pole.

Deviation in azimuth P. L.	2.03971
*'s Altitude 7° 57′½ secant	0.00420
Pol. Dist. 44 15 sine	9.84373
Error of first observation —$2^m\ 20^s$ P. L.	1.88764

Error in first obs.	—0^h 2^m 20^s		
Observed time	6 0 0		
Time of transit	5 57 40 of the north meridian.		

To find the error in the transit above the pole.

Error in azimuth		P. Log.	2.03971
Altitude	96° 27′½	secant	0.94892
Pol. dist.	44 15	sine	9.84373
Error	— 0^m 16^s	P. L.	2.83236
Obs. time of tr.	5 56 21		
Time of tr.	5 56 5 of the south meridian.'		

The difference of these times is 11^h 58^m 25^s, being exactly half a revolution of the fixed star, as shown by the clock. If it is only necessary to find the error of the transit at the altitude when the star passes the meridian either above or below the pole, add together the constant logarithm 0.30103, the proportional logarithm of the difference between the interval of the two observations and half a revolution, the log cosine of the star's polar distance, the log cosine of the latitude, and the log secant of the star's altitude, the sum, rejecting tens in the index, will be the prop. log of the error of observation for the time of the true passage over the meridian.

Constant logarithm			0.30103
Difference	2^m 4^s	P. L.	1.94000
Pol. dist.	44° 15′	cosine	9.85510
Latitude	52 12½	cosine	9.78731
Altitude	7 57½	secant	0.00420
Deviation	—0^h 2^m 20^s	P. L.	1.88764
Obs. time	6 0 0		
True time	5 57 40 as before.		

Should the observer not choose to trust to the uniformity of the going of the clock, he may select two circumpolar stars whose right ascensions differ nearly 12^h, as it requires in this case only a few minutes' perfect regularity in the clock. Take the *difference* between the transits of circumpolar stars by the clock, which are nearly in the same azimuth, the one above, the other below the pole; repeat the operation 12 hours after successively, when the stars have reversed their positions, and if there be a variation in their *differences*, it shows a deviation in the instrument, which may be corrected by substituting half the difference for the error, and repeating the trial by approximation till the adjustment is complete.

If some of those stars whose apparent places are given in the Nautical Almanac be selected, the operation will be comparatively easy. These in pairs are; 1, α Cassiopeiæ and δ Ursæ Majoris; 2, Polaris and ζ Ursæ Majoris; 3, Polaris or α Arietis and α Draconis; 4, Capella and α Herculis; 5, β Tauri and β Draconis; 6, β Aurigæ and γ Draconis; 7, Pollux and γ Aquilæ. No doubt some of these can only be so observed in very high northern latitudes; and, there-

fore, recourse must be had in some instances to other tables, such as those of Dr Pearson.*

It sometimes happens that an observer has not a command of the whole meridian, especially if he has not an observatory properly adapted to the purpose, yet may find it necessary to take transits for the regulations of clocks or chronometers. In this case recourse must be had to the sun, and to pairs of high and low stars having nearly the same right ascension. Having, by the sun and a good watch or chronometer, placed the instrument nearly in the meridian, observe the transits of two stars having nearly the same right ascension, but differing at least 30° or 40° of declination. Now if the interval between their passing the meridian in sidereal time, be exactly equal to their difference of right ascension, the instrument is truly placed; if not, it wants correction.

If, when the latitude is N. and the stars S. of the zenith, the highest star come first to the meridian, and the interval between the transits be too great, it deviates towards the west; if too small, towards the east.

But if the lowest star come first to the meridian, and the interval between the transits be too great, it deviates towards the east; if too small, towards the west. In either case there is required a correction, which may be computed in the following manner:—

Rule.—To the secant of the star's declination add the sine of the *difference* of the latitude and declination, if they are of the *same name*, or the sine of their *sum*, if they are of *different names;* of the sum of which find the natural numbers. To the logarithm of the sum of these add the arithmetical complement of the logarithm of their difference, and the logarithm of the difference between the excess of the right ascension of one star above that of the other, and the observed interval of time between the transits, the sum will be the logarithm of an *arc* in time.

Half the sum of the excess of the right ascension of the one star above the other and the forgoing arc will be the deviation at the lowest star, and half the difference between these will be the deviation at the highest.

The deviation in time at each star being now known, the instrument may be easily rectified by either or both of them on the following night, or still more readily by a third star on the same evening; or, if the telescope is sufficiently powerful to show stars in the day, all the corrections may be performed at any time in a few successive hours. For the deviation of one star being known, that at another may be computed by the following

Rule.—To the logarithm of the given deviation add the cosine of the corresponding star's declination, the secant of the declination of the third star, the cosecant of the *sum* of the latitude and declination of the first star if they are of *different* names, or of their difference if they are of the same name, and the sine of the *sum* of the latitude and declination of the third star if they are of *different* names, or of their difference if they are of the same name; the sum of these will

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* Perhaps the catalogue in the Nautical Almanac might be extended, and the selection more judicious. For example, the places of some of the smaller stars in Orion might be properly exchanged for either circumpolar or high and low stars. An accurate catalogue of the principal stars is given among the tables in this work.

be the logarithm of the deviation in seconds of time at the third star.

Ex.—On the 1st of March, 1826, at the observatory of Edinburgh, in latitude 55° 57′ 20″ N., I observed the transits of Capella and Rigel, on the same evening, about a quarter past 6, and found the interval between the two transits 2ˢ.5 *less* than the difference between their true apparent right ascensions, as given in the Nautical Almanac; required the deviation of the instrument at either star, and also at a third, as Sirius?

Latitude	55° 57′ N.			55° 57′ N.		
Dec. of Capella	45 48 N.	sec. 0.156664	Rig.	8 25 S.	sec.	0.004703
Difference	10 9	sin. 9.246069	sum	64 22	sin.	9.955005
1. Nat. number	0.2528	9.402733				...9.959708
2. Nat. number	0.9114	75				
Sum	1.1642	log. 0.065953				
Difference	0.6586	ar. co. 1.181378				
Diff. of R. A. and Obs. interval	2ˢ.5	log 0.397940				
Arc in time	2.42	log 1.645346				
Sum	6.92	half = 3ˢ.46 = the deviation at Rigel.				
Difference	1.92	half = 0.96 = the deviation at Capella.				

Now since the highest star comes first to the meridian, and the interval between the transits is too short, the deviations are *easterly*.

If the stars had been between the zenith and the north pole, the deviations would have been *westerly*.

Since it has been found necessary to fix the instrument as soon as possible, we shall proceed to compute the deviation at the third star, which can be easily done, as we have an hour and three quarters nearly to perform the calculations and complete the arrangements; thus:

Declination of Rigel (a)	8° 25′ S.	cosine	9.995297
Latitude (b)	55 57 N.		
Declination of Sirius (c)	16 29 S.	secant	0.018226
First sum, or, $(a+b)$	$=$ 64 22	cosecant	0.044995
Second sum, or $(b+c)$	$=$ 72 26	sine	9.979260
Deviation at Rigel	$=$ 3ˢ.46	log	0.539076
Deviation at Sirius	3.77	log	0.576854

After having corrected the instrument by means of Sirius, I observed the transits of Castor and Procyon, and again those of Procyon and Pollux, and found the interval of time to agree with their difference in right ascension, from which I concluded, that in the space of about three hours I had placed my transit instrument exactly in the meridian.

As it is rather a difficult operation to fix a transit instrument ac-

curately in the meridian, these operations should be repeated a considerable number of times to ensure the utmost possible accuracy. After the observations prove satisfactory, a meridian-mark may be put up in a horizontal direction at a considerable distance, with which the central wire may be frequently examined and rectified previous to any very nice observation. This mark may be of various constructions, such as a copper-plate with a hole in it, so as a small segment of light may be seen on each side of the vertical middle wire, or a small notch in a building, or even a post at some distance. A thin slip of brass or copper painted black, with white lines or divisions at every inch, and numbered throughout, will also be found very convenient, and by knowing its distance, the deviation upon it may be computed.*

The transit instrument being now properly rectified, it will be found the most accurate of all for determining the error and rate of a clock or chronometer, by taking the transit of the sun or stars daily, and marking the difference regularly in a column prepared for that purpose. If a star be observed, sidereal time must be reduced to mean solar time by Table XXXI. when necessary.

Ex. 1.—The observed times of the sun's passing the meridian of the observatory were as follows:—What was the original error on the last day of observation and the daily rate?

Bar.	Ther.	1826.	Obs. Time Sun's Transit.	Mean Time. App. Noon.	Chronometer too fast.	Daily Rate.
29.87	52°	Mar. 1	$0^h\ 25^m\ 27^s.1$	$0^h\ 12^m\ 40^s.7$	$0^h\ 12^m\ 46^s.4$	
29.90	53	2	0 25 16.6	0 12 28.6	0 12 48.0	+ $1^s.6$
29.86	51	3	0 25 5.4	0 12 16.0	0 12 49.4	+ 1.4
29.88	50	4	0 24 54.0	0 12 3.0	0 12 51.0	+ 1.6
29.85	52	5	0 24 42.0	0 11 49.5	0 12 52.5	+ 1.5
29.84	54	6	0 24 29.8	0 11. 35.6	0 12 54.2	+ 1.7
29.87	52					5|7.8

Mean daily rate is therefore +1.56
And the original error at noon, on the 6th of March, 1826, is $0^h\ 12^m\ 54^s.2$ fast.

Hence its error, supposing the rate to remain uniform, may, at any moderately distant future time, be determined.

Ex. 2.—On the same evenings the star Rigel passed the meridian as follows:—Required the daily rate and the original error on the sixth at the time of observation, about 6 o'clock in the evening?

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* Hor. deviation = sec. alt. × cos. dec. × obs. diff. of time × 15, to radius 1.

On Captain Kater's plan, by contracting the diameter of the object-glass by some contrivance for that purpose, the meridian-mark may be only a few feet distant.—See his paper on the Floating Collimator.

1826.	Obs. Time. Star's Transit.	Daily Diff. of Star's Transit.	Diff. of Mean and Sidereal Time.	Daily Rate.	Bar.	Ther.
March 1	6h 42m 53s.6				29.87	48°
2	6 38 59.3	3m 54s.3	3m 55s.9	+ 1.6	29.88	47
3	6 35 4.8	3 54.5	3 55.9	+ 1.4	29.85	46
4	6 31 10.4	3 54.4	3 55.9	+ 1.5	29.86	47
5	6 27 16.2	3 54.2	3 55.9	+ 1.7	29.87	45
6	6 23 22.0	3 54.2	3 55.9	+ 1.7	29.85	49
				5)7.9	29.86	47

Rate by the star . . .	+1.58
By the sun . .	+1.56
	2)3.14
Mean rate by both . .	+ 1.57
Sun's R. A. at noon, on the 6th,	23h 6m 21s.4
Prop. part of daily var., to 5h 59m .	+ 55.2
Reduced R. A. . .	23 7 16.6
Star's R. A. by Nautical Almanac	5 6 12.4
Apparent time of transit .	5 58 55.8
Equation of time at 5h 59m .	+ 11 32.0
Mean time of transit of star .	6 10 27.8
Time of transit by chronometer on the 6th	6 23 25.0
Error of chronometer, fast by star,	0 12 54.2
Allowing for change of rate in 6h, by sun,	0 12 54.6
Mean error at 6h fast : .	0 12 54.4
With a daily rate of . . .	+ 1.57

The barometer being at 29.86, and thermometer, 49.5° Fah.

As opportunities may not occur daily for celestial observations, it is in that case necessary to compare a chronometer with a good clock, the rate of which can be depended on, and is occasionally ascertained by the heavenly bodies.

Ex. 3.—Given the daily difference between a chronometer and a clock, the rate of the clock being occasionally determined by celestial observations; to find the error and rate of the chronometer?

Bar.	Ther.	1826.	Clock before Mean Time.	Chron. differs from Clock.	Chron. before Mean Time.	Daily Rate.
29.74	64°	May 1	+ 8s.5 *	+ 2s.5	+ 11s.0	
29.78	65	2	+ 8.9	+ 3.8	+ 12.7	+ 1s.7
29.80	65	3	+ 9.4	+ 5.2	+ 14.6	+ 1.9
29.76	66	4	+ 9.8 *	+ 6.5	+ 16.3	+ 1.7
29.74	63	5	+10.1	+ 7.9	+ 18.0	+ 1.7
29.75	65	6	+10.5 *	+ 9.3	+ 19.8	+ 1.8
29.76	64.7				5	+ 8.8

Mean daily rate	+ 1.76
And on the 6th at noon, the original error was fast	19s.8

Hence the error of the chronometer may be found at any moderate distance of time, so far as its steady rate can be depended on.

The clock was examined by celestial observation, only where the asterisks are placed, or on the 1st, 4th, and 6th, and these are sufficient to ascertain, with the requisite precision, the rate of the chronometer when the clock is good. It is in a somewhat similar manner that the prize-chronometers are tried at Greenwich.

Table of the variations of the sun's R. A. and dec. in 1ˢ for every month in the year.

Month.	Var. in R. A. for 1 Second.	Var. in Dec. for 1 Second.
January	0ˢ.0029	0″.008 N.
February	0 .0027	0 .014 N.
March	0 .0025	0 .016 N.
April	0 .0026	0 .014 N.
May	0 .0028	0 .009 N.
June	0 .0029	0 .000
July	0 .0028	0 .006 S.
August	0 .0026	0 .013 S.
September	0 .0025	0 .016 S.
October	0 .0026	0 .015 S.
November	0 .0028	0 .010 S.
December	0 .0031	0 .002 S.

This table will be useful when the change of the sun's R. A. or D. for a few seconds only is wanted.

Summary of Directions for making a Series of Observations with the Transit.

1. Place the transit instrument as nearly in the meridian as possible, and select or place substantial meridian-marks at a considerable distance both to the north and south.
2. The clock must be set to sidereal time, and its daily rate ascertained.
3. Observations of pairs of high and low Greenwich stars must be made each evening, along with others whose right ascensions are required.
4. The apparent right ascensions of the Greenwich stars must be computed for the time of observation, or taken from the Nautical Almanac.
5. The azimuthal error must be found by several of these pairs of stars.
6. The error of the clock must be deduced at the time of transit of one of the Greenwich stars.
7. This error must be reckoned constant to every observation made during the same night.
8. The azimuthal error must be considered to have a contrary sign between the zenith and the pole to that on the opposite side of the zenith.

9. A proportional part of the daily rate of the clock must be applied to every observation from the first.

10. The error of each star from the true meridian must be computed from tables prepared for the purpose.

11. To the time of transit of each star add the error of the clock (6), the proportional part of the daily rate (9), and the error from the meridian (10), the respective sums will give the true apparent right ascension required.

12. Compute the sum of the corrections for precession, aberration, lunar and solar nutation, for every star at the time of culmination or observation, and apply each sum with a contrary algebraic sign to each true apparent right ascension, the result will give the mean right ascension for the beginning of the year.

13. Let series of these for each star be registered, and the mean of each series may be expected to give the mean right ascension at the beginning of the year with considerable accuracy.

PART III.

MENSURATION, SURVEYING, &c.

Section I.

Mensuration of Surfaces.

Mensuration is the application of Arithmetic to Geometry, by which the values of geometrical magnitudes are obtained in numbers.

In this case some determinate magnitude of the same kind with that to be measured is assumed as unit, and the number of times this unit is contained in the given magnitude is the measure of that magnitude.

See Leslie's Geometry, Book V. Prop. XXV.

1. To find the area of a *parallelogram,* multiply the length by the perpendicular breadth.

2. *Triangle.*—Multiply the base by the perpendicular altitude; half the product is the area. Or take half the product of the two sides, and the natural sine of the contained angle. Or when the three sides are given, multiply half the sum of the three sides, and the differences between that half sum and the three sides together, the square root of this product will be the area. This may be performed readily by logarithms.

3. *Trapezium.*—Multiply the base into half the sum of the perpendiculars.

4. *Trapezoid.*—Multiply half the sum of the parallel sides by the perpendicular distance between them.

5. *Irregular Polygon.*—Divide it into triangles, find their areas, the sum of these will be the area.

6.—*Regular Polygon.*—Multiply the square of the side given into the proper *multiplier for areas* from the table, page 183, for that purpose, and the product will be the area. Or divide the polygon into triangles; find the area of one of them by some of the foregoing rules. Multiply this by the number in the whole polygon, the product is the area.

7. *Circle.*—The *diameter* is to the *circumference* as 1 to 3.1415926536, or 1 to 3.141593 nearly.

The *circumference* is to the *diameter* as 1 to 0.318309.

The *area* is equivalent to the square of the diameter multiplied into 0.785398, or the square of the circumference by 0.0795575.

The *area* is equivalent to half the diameter multiplied into half the circumference.

8. *Circular Arc.*—The *length* of a circular arc is equivalent to the

radius of the circle multiplied by 0.0174533, and by the number of degrees in the arc.

Or from eight times the chord of half the arc subtract the chord of the whole arc, one third of this remainder is the length of the arc nearly.

9. *Circular Sector.*—The area is equivalent to the radius multiplied into half the length of the arc.

10. *Circular Segment.*—Multiply the square of the radius by either half the *difference* of the arc of the segment and its sine, or by half their SUM, according as the segment is *less* or GREATER than a semicircle, and the product will be the area.

11. *Parabola.*—The area is equivalent to two-thirds of the product of its base and altitude.

12. *Ellipse.*—The area is equivalent to the product of the transverse axis into the conjugate axis multiplied by 0.785398. *Periphery.*—Multiply the square root of half the sum of the squares of the two axes by 3.141593, the product will be the periphery nearly.

I. Imperial Land Measure.

Marked.			*Square In.*
sq. in.	144 square inches	= 1 square foot =	144
sq. ft.	9 square feet	= 1 square yard =	1296
sq. yd.	$30\frac{1}{4}$ square yards	= 1 square pole =	39204
po.	40 square poles	= 1 rood =	1568160
ro.	4 roods	= 1 acre, ac. =	6272640

43560 sq. feet = 4840 sq. yards = 1 acre, and 640 acres = 1 sq. mile.

II. Solid Measure.

Marked.			*Cubic or Solid In.*
sq. in.	1728 solid inches	= 1 solid foot =	1728
sq. ft.	28 solid feet	= 1 solid yard =	46656

It may be remarked too, that the Scottish land-surveying chain, derived from the standard ell, has been at different times estimated at different lengths.

The late Dr Mackay of Aberdeen, in his collection of Mathematical Tables, published in 1804, Table LXX., estimated it at 74.4 feet. Dr Hamilton estimated it at 74 feet exactly, and this length had been much employed in land-measuring previous to 1811, when Mr Jardine, civil-engineer, found the standard ell to be 37.069 inches. This gives 74.138 feet at the temperature at which the comparison was made. In 1826 the length of the standard ell is again stated at 37.0598, which gives 74.1196 feet for the Scottish land-surveying chain, and the ratio of the Scottish acre to the imperial as 1.26118345 to 1. Hence it is obvious that those estates measured previous to 1811 are likely to have been measured with a chain of 74, or 74.4 feet. After 1811 a few were likely measured with a chain of 74.138 feet, and those after 1826 with one of 74.1196; and the reduction to imperial acres ought to be made accordingly.

Under circumstances of such uncertainty, the only unexceptionable method of converting estates from the old to the new standard is to employ an experienced surveyor with the imperial chain.

Examples for Exercise.

1. Required the area of a square of which the side is 5 feet 9 inches? *Ans.*—33.0625 feet.

2. Required the area of a rectangle, if the length is 1375 links, and the breadth 950? *Ans.*—13ac. 0r. 10p.

3. Required the area of a rhombus, of which the length of the side is 12.24 feet, and height 9.16 feet?
Ans.—112.1184 square feet.

4. Required the area of a rhomboid, of which the length is 7 feet 9 inches, and height 3 feet 6 inches?
Ans.—27ft. 1in. 6pa.

5. Required the area of a rhomboid, of which the adjacent sides are 2535 and 1040 links and the contained angle 30°?
Ans.—13ac. 0r. 29p.

6. Required the area of a triangle, of which the base is 1225 links, and altitude 850?
Ans.—5ac. 0r. 33p.

7. Required the area of a triangle, of which two of the sides are 30 and 40, and the contained angle 28° 57′ 18″?
Ans.—290.47356.

8. Required the area of a triangle, of which the three sides are 20, 30, and 40 feet? *Ans.*—290.4737 square feet.

9. How many acres are there in a triangle, of which the three sides are 380, 420, and 765 yards?
Ans.—9ac. 0r. 38p.

10. A ladder, 50 feet long, being placed in a street, reached a window 28 feet from the ground on one side; and, by turning it over, without removing the foot, it reached another window 36 feet high on the other side; required the breadth of the street?
Ans.—76.1233 feet.

11. How many acres are there in the trapezium, of which the diagonal is 4750 links, and the two perpendiculars falling upon it on opposite sides, 225 and 360 links respectively?
Ans.—13ac. 3r. 23p.

12. Required the area of a regular hexagon, one of whose equal sides is 14.6 feet, and the perpendicular from the centre 12.64 feet?
Ans.—553.632 feet.

13. If the diameter of a circle be 17, what is the circumference?
Ans.—53.4072.

14. If the circumference of the earth be 24850 miles, what is the diameter? *Ans.*—7910.

15. If the chord of an arc be 30, the height or versed sine 8, what is the length of the arc?
Ans.—35½.

16. Required the length of an arc of 57° 17′ 44″.8; the diameter of the circle being 25 feet?
Ans.—12.5, which is equal to the radius.

17. Required the area of a circle, of which the semi-diameter is 15½ feet?
Ans.—81.1798yds.

18. Required the radius of a circle in yards, of which the area is an acre?
Ans.—39¼yds.

19. The diameters of two circles are 16 and 10; what is the area of the ring formed between these two circles, the centre being common to both? *Ans.*—122.5224.

20. Required the area of the sector whose height or versed sine is 4, and the diameter of the circle 16?
Ans.—67.02.

21. Required the area of the segment of a circle, of which the chord is 16, and the diameter of the circle $16\frac{2}{3}$? *Ans.*—70.7083.

22. Let ABCD be a four-sided field, and from the side AB to the points C, D, let fall the perpendiculars PC and QD. Now the measure of AP is 110 links, PC is 352 links; AQ is 745 links, QD is 595, and AB is 1110 links; required the area of the field?

Ans.—$4^{ac.}$ $1^{ro.}$ $5.8^{po.}$

TO FIND THE AREAS OF CIRCULAR SEGMENTS.

Rule.—Divide the height of the segment by the diameter, and find the quotient in the column of heights in the following table: Take out the corresponding area in the next column on the right hand; and multiply it by the square of a circle's diameter, for the area of the segment.

TABLE OF THE AREAS OF CIRCULAR SEGMENTS.

Height.	Area of the Segment.	Height.	Area of the Segment.	Height.	Area of the Segment.	Height.	Area of the Segment.	Height.	Area of the Segment.
0.01	0.00133	0.11	0.04701	0.21	0.11990	0.31	0.20738	0.41	0.30319
0.02	0.00375	0.12	0.05339	0.22	0.12811	0.32	0.21667	0.42	0.31304
0.03	0.00687	0.13	0.06000	0.23	0.13646	0.33	0.22603	0.43	0.32293
0.04	0.01054	0.14	0.06683	0.24	0.14494	0.34	0.23547	0.44	0.33284
0.05	0.01468	0.15	0.07387	0.25	0.15354	0.35	0.24498	0.45	0.34278
0.06	0.01924	0.16	0.08111	0.26	0.16226	0.36	0.25455	0.46	0.35274
0.07	0.02417	0.17	0.08853	0.27	0.17109	0.37	0.26418	0.47	0.36272
0.08	0.02944	0.18	0.09613	0.28	0.18002	0.38	0.27386	0.48	0.37270
0.09	0.03502	0.19	0.10390	0.29	0.18905	0.39	0.28359	0.49	0.38270
0.10	0.04088	0.20	0.11182	0.30	0.19817	0.40	0.29357	0.50	0.39270

Ex. 1.—Taking as an example the chord 12, and the radius 10, or diameter 20.

And having found the perpendicular from the centre upon the chord $= 8$; then $10 - 8 = 2$. Hence, by the rule, $= 2 \div 20 = \cdot 1$ the tabular height. This being sought in the first column of the table, the corresponding tabular area is found $= \cdot 04088$. Then $\cdot 04048 \times 20^2 = \cdot 04088 \times 400 = 16.352$, the area.

The use of the following tables will be readily understood, from considering that the *areas* of similar figures are as the *squares* of their like dimensions, and their SOLIDITIES as the CUBES.

TABLE OF POLYGONS.

No of Sides.	Names.	Multipliers for areas.	Radius of circum. circle.	Factors for sides.
3	Trigon	0.4330127	0.5773503	1.732051
4	Tetragon, or Square	1.0000000	0.7071068	1.414214
5	Pentagon	1.7204774	0.8506508	1.175570
6	Hexagon	2.5980762	1.0000000	1.000000
7	Heptagon	3.6339124	1.1523824	0.867767
8	Octagon	4.8284271	1.3065628	0.765367
9	Nonagon	6.1818242	1.4619022	0.684040
10	Decagon	7.6942088	1.6180340	0.618034
11	Undecagon	9.3656399	1.7747324	0.563465
12	Dodecagon	11.1961524	1.9318517	0.517638

Section II.

Mensuration of Solids.

1. *Prism.* (1.) *Surface.* Multiply the perimeter of one end by the length or height, the product will be the surface of the sides. To this add the areas of the two ends, and the sum will be the whole surface.

(2.) *Solidity* or *Capacity.* Multiply the area of the base by the height, the product will be the solid content. The same rules determine the surface and capacity of a cylinder.

2. *Pyramid* or *Cone.* (1.) *Surface.* Multiply half the perimeter of the base by the slant height. To this add the surface of the base, the sum is the whole surface.

(2.) *Capacity.* Multiply the area of the base by one-third the perpendicular height.

3. *Frustum* of a *Pyramid.* (1.) Multiply half the sum of the perimeters of the two ends by the slant height. To this add the areas of the two ends, the sum will be the whole surface.

(2.) *Capacity.* Add a diameter or side of the greater base to one of the less; from the square of the sum subtract the product of these two sides or diameters; multiply the remainder by a third of the height, and this last product by the proper number for the circle, .785398, or polygon, the last product will be the content.

4. *Sphere.* (1.) *Surface.* Multiply the square of the diameter by 3.141593, the product is the surface.

(2.) *Capacity.* Multiply the cube of the diameter by 0.5236, or the cube of the circumference by 0.016887.

5. *Spheric Segment.* (1.) *Surface.* Multiply the circumference of the sphere by the height of the segment.

(2.) *Capacity,* or $c = 0.5236\, h^2\, (3\, d - 2\, h)$, in which d is the diameter of the sphere, and h the height; or $c = 0.5236\, h\, (3\, r^2 + h^2)$; in which r is the radius of the base of the segment, and h its height.

6. *Paraboloid,* or solid formed by the rotation of a parabola about its axis.

Capacity. Multiply the base by its height, half the product is the content.

7. *Spheroid,* or solid formed by the revolution of an ellipse about one of its axes.

Capacity. Multiply the square of the revolving axis by the fixed axis, and the product by 0.5236, the result will be the content.

8. *Regular,* or Platonic bodies, as they are sometimes called, are contained under like, equal, and regular plane figures, of which the solid angles are all equal. The names and descriptions of these bodies, together with their multipliers, the side of each being unity, are contained in the following tables:—

Surfaces and Solidities of Regular Bodies, the Side being Unity, or 1.

No. of Sides.	Name.	Surface.	Solidity.
4	Tetraedron	1.7320508	0.1178513
6	Hexaedron	6.0000000	1.0000000
8	Octaedron	3.4641016	0.4714045
12	Dodecaedron	20.6457288	7.6631189
20	Icosaedron	8.6602540	2.1816950

The diam. of a sphere being 1; the side of a	That may be inscribed in the sphere, is	That may be circumscribed about the square, is	That is equal to the sphere, is
Tetraedron	0.816497	2.44948	1.64417
Hexaedron	0.577350	1.00000	0.88610
Octaedron	0.707107	1.22474	1.03576
Dodecaedron	0.525731	0.66158	0.62153
Icosaedron	0.356822	0.44903	0.40883

Examples for Exercise.

1. Required the solidity of a cube, of which the side is 5 feet 3 inches? *Ans.* 144.7 feet.

2. What is the solidity of a block of marble, of which the length is 10 feet, breadth $5\frac{3}{4}$ feet, and depth $3\frac{1}{2}$ feet? *Ans.* $201\frac{1}{4}$ feet.

3. Required the solidity of a prism, of which the base is a hexagon, each of the equal sides being 1 foot 4 inches, and the length of the prism 15 feet? *Ans.* 69.282 feet.

4. Required the convex surface of a cylinder, of which the circumference is 8 feet 4 inches, and length 14 feet? *Ans.* $116\frac{2}{3}$ feet.

5. What is the solidity of a cylinder, of which the length is 5 feet, and diameter of its base 2 feet? *Ans.* 15.708 feet.

6. The diameter of the base of a right cone is $4\frac{1}{2}$ feet, and the slant height 20 feet; required the convex surface? *Ans.* 141.372 feet.

7. Required the convex surface of a frustum of a right cone, the circumference of the greater end being 30 feet, that of the less 10 feet, and the slant height 20 feet? *Ans.* 400 feet.

8. What is the solidity of a triangular pyramid, of which the height is 30, and each side of its base 3? *Ans.* 38.97.

9. What is the solidity of a cone, of which the circumference of the base is 40 feet, and its height 50 feet? *Ans.* 2122 feet.

10. What is the solidity of the frustum of a cone, of which the diameter of the greater end is 5 feet, that of the less 3 feet, and the perpendicular height 9 feet? *Ans.* 115.454 cubic feet.

11. What is the solidity of a frustum of a square pyramid, one side of the greater end being 18 inches, that of the less 15 inches, and the height 5 feet? *Ans.* 16380 cubic inches.

12. Required the convex superficies of a sphere, of which the diameter is 17 inches? *Ans.* 907.92 square inches.

13. Required the solidity of the same? *Ans.* 1.48868 cubic feet.

14. Required the solidity of the earth, considering it as a perfect sphere, of which the diameter is 7910 miles? *Ans.* 259136798136 cubic miles.

15. What is the solidity of the segment of a sphere, of which the

diameter of the base is 20 feet, and its height 9 feet? *Ans.* 1795.4244 cubic feet.

To find the area of a curvilinear figure by the method of equidistant ordinates.

RULE I.

Supposing the number of ordinates to be odd.

1. Collect all the even ordinates into one sum, and multiply this sum by 4.

2. Take the sum of the odd ordinates and multiply it by 2.

3. To the sum of these two products, add the sum of the first and last ordinates, this result multiplied by one third of the common interval will be nearly the area required; or

$$A = \frac{i}{3}\left(a + l + 4e + 2o\right) \quad . \quad . \quad (1.)$$

in which A is the area, i the interval, a the first ordinate, l the last, e the sum of the even ordinates, and o that of the odd.

RULE II.

1. Collect into one column every third ordinate, beginning with the fourth, such as the 4th, 7th, 10th, &c. increasing by the common difference 3, except the *first* and *last*, and multiply this sum by 2, and call the result $2m$.

2. In another column write down also all the other ordinates, excepting the *first* and *last*, and multiply the sum by 3.

3. To the first and last ordinates add these two products, and the result multiplied by three-eighths of the common interval will be the approximate area required; or

$$A = \frac{3i}{8}\left(a + l + 2m + 3n\right) \quad . \quad . \quad (2.)$$

in which A is the area, i the common interval, m the sum of $1 +$ a multiple of 3 ordinates, n the sum of the remaining ordinates, a the first ordinate, and l the last.

SECTION III.

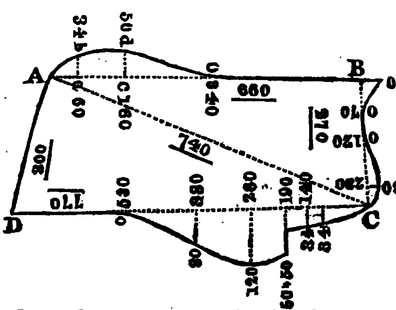
Surveying.

In land-surveying, the instruments commonly employed for the ordinary purposes are—

1. Gunter's chain, and ten iron pins.
2. Cross-staff, and signal-staves.
3. Field-book, or paper.
4. Case of mathematical instruments.
5. Plotting scales.
6. Parallel ruler, and beam compasses.
7. A small quadrant, if a theodolite is not at hand, to reduce the hypotenusal lines to their horizontal measure.

It would exceed our present limits to decribe all these, as well as some others, which may, however, appear perhaps in a work proposed with that view.

An Example of laying off a Field.



Having set up poles at A, B, C, and D, so as with the different dotted lines to reduce the body of the field to a quadrilateral form, and drawn a sketch of it, into which the measures when taken may be inserted; begin at any point A, measuring the successive distances A a, A c, &c., on the chain-line A B, and the corresponding offsets ab, cd, &c., and marking them as in the figure till a complete circuit A B C D A of the field and the diagonal A C are measured; these afford data for planning it, and computing the area. For the various portions may be considered either as trapezoids or triangles, whose contents may be ascertained by the rules given for that purpose.* The area computed in this manner will be 2.4295 acres, or 2 ac. 1 ro. 28.72 po., though it is better in general to retain it in acres and decimals. It is necessary to take an account of the roads, dikes, ponds, &c., of which the contents must all be stated distinctly by themselves when a whole estate is surveyed. In the case of the sale of crops, that in tillage only must be measured.

Required the plan and area of the field from the following field-book, in which the angles were measured with the pocket-box sextant, and the distances with the chain, beginning the operations at the gate near the south-east corner?

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* Many land-surveyors first construct an accurate plan, from which, by scale and compass, the area is obtained with sufficient precision; and this is at least a good method of checking the result by computation. The imperial surveying-chain is 66 standard feet in length, divided into 100 links, each 7.92 inches; the Scotch surveying-chain in use previous to 1811 was generally 74 English feet, though that derived from the standard ell should be 74.1196. In both cases, 10 chains, or 100,000 square links, make an acre. The ratio of the acre by the Scotch surveying-chain of 74 feet is to the imperial acre as 1.25711662 to 1, that of 74.4 feet as 1.270744 to 1, and that of 74.1196 feet as 1.26118345 to 1, and the ratio of an acre measured by a chain of 74.1196 feet to an acre, measured by a chain of 74 feet is 1.003235 to 1. In such circumstances a judicious surveyor will uniformly state the length of his chain upon his plan as I have frequently seen done, one of which of 74 feet exactly occurred so late as 1819, and I believe this chain is still used in some parts of the country.

Field-Book.

Left-hand Offsets, &c.	Links.	Stations, Distances, and Angles.	Right-hand Offsets, &c.
Hedge.	0	0	*Remark.* The chain-line bears nearly west along the north side of Bitterick Syke.
	38	143	
		⊙ 1st 99° 45′ 30″ W.	
	73	200	
Deadriggs* or	83	240	
Crosshall lands	70	300	
on the south or left	34	400	
hand	0	480	
	0	510	
	44	650	
	42	726	
	100	810	
		⊙ 2d 85° 43′ 30″ N.	
Boundary.	0	0	The chain-line bears nearly north.
	2	200	
	5	400	
	3	600	
	0	800	
	10	860	
	0	866	
		⊙ 3d 73° 8′ 0″ E.	
Hedge.	0	0	
	50	100	
Hardacres land	66	200	The chain-line bears nearly east.
on the north	83	264	
or left hand	30	350	
	66	456	
	5	544	
	130	700	
	0	755	
		⊙ 4th 101° 23′0″ S.	The chain-line bears almost south along the road from Greenlaw to Eccles. The diagonal from ⊙ 1st to ⊙ 3d, measuring 1053 links, was also taken, that the area might by the three sides of the triangles be a check upon that determined from using the angles.
	0	100	
	12	200	
	38	300	
	65	400	
	108	500	
To ⊙ 1st, or	143	600	
		Area = 6.14537 ac. or 6 ac. 0 ro. 23 po.	

If there are dikes, ditches, or fences of any kind, they must be measured during the survey, and their amount stated. Also plantations, roads, commons, lakes, ponds, &c. must be all surveyed and classed separately from the arable land. For these we cannot here enter into detail.

* This place is mentioned in Sir Walter Scott's Minstrelsy of the Scottish Border.

Levelling.

It is often necessary to ascertain the difference of elevation of one point above another, for the purpose of conveying a stream of water to drive machinery. This may be performed in several ways, but the readiest and most accurate is by means of a spirit-level of the best construction. It must be accompanied by a pole, or rod divided into feet, and at least hundredths of a foot. On this rod a sliding vane is fitted, capable of moving easily up and down, and having a dark strong line or other well-defined mark upon it, by which the telescope, or in common levels the sight, may be directed. The slider must be moved upwards or downwards on the rod, till the mark coincide with the intersections of the cross hairs in the focus of the telescope. When this is accomplished, and the level being properly adjusted, the height in feet and hundreth parts is to be carefully read off and marked in a book for the purpose. Now, by means of a chain or measuring tape,* let the pole-bearer place it at equal distances, alternately on each side of the level, such as about one or two hundred yards, if convenient, if a level with a good telescope be used. If an ordinary level with a plain sight be used, the distance must be reduced to as many feet. The heights taken with the telescope turned towards the place whence the observer set out, are called the back-observations; and those taken towards the place where he means to finish, are called fore-observations, for the sake of distinction. Since the pole is always placed at equal distances from the level, no allowance need be made for the curvature of the earth.†

EXAMPLE.

Back.		Fore.	
Dist.	Height on Pole.	Height on Pole.	Dist.
Links.	Feet.	Feet.	Links.
100	2.92	4.68	100
100	1.56	3.79	100
200	0.48	5.63	200
200	1.35	4.86	200
150	1.27	3.74	150
150	1.34	2.56	150
100	2.36	3.94	100
50	3.28	4.36	50
1050	14.56	33.56	
2		14.56	
2100		19.00	

* A chain of one hundred feet is very convenient, and generally used for this purpose. If the rod consist of several slides which pull out, having feet and tenths very distinctly marked upon them, which are visible through the telescope, the observer is enabled to read the different heights without either trusting to the reading of his assistant, or submitting to the trouble of calling him up each observation, and the work is thus carried on more expeditiously.

† The difference of level is about 8 inches in a mile, which increases as the square of the distance. The difference of level in *feet*, allowing for refraction, is $\frac{4}{7}$ of the square of the distance in English miles.

Hence the difference of level on a sloping height of 2100 links of Gunter's surveying chain, or $2100 \times 0.66 = 1386$ feet, is 19 feet. When a spirit-level exactly adapted to this purpose is not at hand, if there is a theodolite to be had, it will perform the operation, though it is not quite so convenient.

In case of levelling for canals, the process is not different, only the canal is carried on an exact level, by judiciously choosing the situation winding round rising grounds, conveying it across ravines by aqueduct bridges, and allowing it to descend at particular points by means of locks. Roads ought to be carried along a level line as nearly as possible, and only having gentle acclivities and declivities. This may be readily obtained by following routes somewhat circuitous in uneven parts of the country, taking the advantage of ravines, water-courses, and the sides of lakes; for a greater distance on a road nearly level is productive of less expense of animal strength, than by passing over considerable elevations. All very quick turns in the road, particularly when entering upon a bridge, ought to be avoided, as the danger from centrifugal force, which may be easily estimated by the formula, Part III., Sec. IV., is considerable. The justice of these remarks may be readily appreciated by considering many parts in most of our public roads which have hitherto been constructed upon the very worst principles, having been intrusted to what are called practical men, who are frequently the mere slaves of custom.

Section IV.

Rules and Formulæ.

When two angles of a plane triangle are known, the third may be found, consequently, for general purposes, it is unnecessary to measure the third angle. But when great accuracy is required, or when the sides on the surface of the earth are large, they become spherical arcs, and then the third angle should always be measured as a check upon the results. If the sum of the three angles of any triangle actually measured amount to 180° very nearly, it is probable that they have been taken with great accuracy, and the excess or defect must be distributed among the three angles according to the judgment of the observer, by applying such a part of the error to each as appears most likely upon a due consideration of all the circumstances of the case to be the most probable quantity. If all the angles be taken with equal care, one-third of the error must be applied to each angle with its proper sign, so as to reduce their sum to $180° + e$ exactly, e being the spherical excess to be afterwards explained.

Let the spherical excess, or $e = 0''.49$, and

$A' =$	$40° \; 36' \; 56''.81$	Hence $A =$	$40° \; 36' \; 56''.836$
$B' =$	$75 \; 39 \; 29 .81$	$B =$	$75 \; 39 \; 29 .836$
$C' =$	$63 \; 43 \; 33 .79$	$C =$	$63 \; 43 \; 33 .816$
	$180 \; 0 \; 0 .41$		$180 \; 0 \; 0 .488$
$180° + e =$	$180 \; 0 \; 0 .49$		To each of which $\frac{E}{3}$ has been
$\frac{E}{3} =$	$\frac{0 .08}{3} = -0''.026$		applied with a contrary sign.

Again, let $e = 0''.15$ and

$$\begin{array}{rrrrr} A' = & 42^\circ & 2' & 32''.0 \\ B' = & 67 & 55 & 39\ .0 \\ C' = & 70 & 1 & 48\ .0 \\ \hline & 179 & 59 & 59\ .0 \\ 180^\circ + e = & 180 & 0 & 0\ .15 \\ \hline \end{array}$$

$$\frac{E}{3} = \frac{1\ .15}{3} = -0''.38$$

$$\begin{array}{rrrrr} \text{Hence } A = & 42^\circ & 2' & 32''.38 \\ B = & 67 & 55 & 39\ .38 \\ C = & 70 & 1 & 48\ .38 \\ \hline & 180 & 0 & 0\ .14 \end{array}$$

To each of which $\frac{E}{3}$ has been applied with a contrary sign.

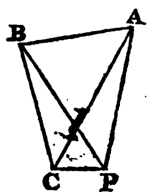
When the ordinary instruments are employed where the error of reading may be 30″ or so, then it will be sufficient to apply one-third of the excess above, or defect from 180° to each of the angles, so as to reduce their sum to 180° exactly. In conducting geodetical operations, the triangle should be so chosen, if possible, as to produce the most accurate conclusions. To diminish the probability of error, the following rules should be observed:—

I. When one side only of a triangle is to be determined, the measured base should be nearly equal to the required side.

II. When two sides of a triangle are to be determined, the triangle should, if possible, be equilateral.

III. When the base cannot be equal to one or to both the required sides, it should be as long as possible, and the two angles at the base equal, and not less than twenty or thirty degrees.*

IV. When the centre of the instrument cannot be placed in the vertical line occupied by the axis of a signal, the observed angles must be reduced to it by an appropriate formula. Let C be the centre of the station, such as a tower, P the place of the centre of the instrument, by which the angle subtended by A B at P is to be measured. Let the angle A P B be observed, and the distance C P be measured, it is required to find C, the measure of the angle A C B? Suppose A P B = P, B P C = p, C P = d, A C = D and B C = D′.



Since the exterior angle of the triangle A P I is equal to the sum of the two interior and opposite angles, AIB = P + I A P, and of the triangle B I C, the exterior angle A I B = C + C B P. Making these two values of A I B equal, by transposition, we have C − P = I A P − C B P. But the triangles C A P, C B P give sin C A P = sin I A P = $\frac{\text{C P}}{\text{A C}}$ sin A P C = $\frac{d \sin (\text{P}+p)}{\text{D}}$; sin C B P = $\frac{\text{C P}}{\text{B C}}$ sin B P C = $\frac{d \sin p}{\text{D}'}$. And since the angles C A P, C B P, are, by hypothesis, always very small, their sines may be substituted for their arcs, hence, $\text{C}-\text{P} = \frac{d \sin (\text{P}+p)}{\text{D}} - \frac{d \text{ sine } p}{\text{D}'}$, which in seconds becomes $\frac{d}{\sin 1''}\left\{\frac{\sin (\text{P}+p)}{\text{D}} - \frac{\sin p}{\text{D}'}\right\}$; or R″ being the length of an arc in seconds equal to the radius, or 206264″.8, then

* For a demonstration of these properties, see vol. III. of Hutton's Course of Mathematics.

$C-P = R'' d \times \left\{ \frac{\sin(P+p)}{D} - \frac{\sin p}{D'} \right\}$. If this be developed, we have $C-P = R'' d \sin P, \sin(A-p)$.

Cor. 1.—If c fall upon the line P B, then $C-P = \frac{R'' \ d \sin P.}{D}$

Cor. 2.—In operations where the angles are measured by a common theodolite or pocket-sextant, R′ may be used instead of R″.

Cor. 3.—When the theodolite cannot conveniently be placed at the same height as the top of the signal observed,

$$\delta \Delta = \frac{R'' \delta h \sin \Delta}{D}$$

In which Δ is the observed zenith distance, R″ the length of an arc equal to the radius in seconds, δh the difference between the height of the centre of the circle and the top of the signal, and D the distance.

The use of this formula cannot be embarrassing, provided the signs of sin p, and sin $(P+p)$ be properly attended to, as is illustrated by the following example:—Let the observed angle P be 43° 52′ 49″.44, $p = 264°\ 41'\ 24''$, $d = 10.706$ feet, $D = 57508$ feet and $D' = 66750$ feet, required the reduction?

(1.)				(2.)
Log R″	5.314425			
log d	1.032860			
+	6.347285			—6.347285
Sin (P+p) 308 34′ 14″	—9.893118	sine p 264° 41′ 24″		—9.998132
arc cor log D+	5.240272	arc cor log D′		+ 5.175549
(1.) —30″ 246	—1.480675	(2.) + 33″.187		+ 1.520966
		(1.) — 30 .246		
C—P =		+ 2 .941		
A		43° 52′ 49 .440		
C		43 52 52 .381		

Scholium.—The error in height or distance arising from an error of one minute in measuring the horizontal or vertical angle when the altitude is small, or the height small, in comparison of the distance.

Supposing the error of the measured angle to be 1′.

Dist. in Feet.	Error in Height.
1000	0.291
10000	2.909
100000	29.089

Error arising from the eccentricity of the telescope of a theodolite to the vertical circle additive to elevations, but subtractive from depressions.

Eccentricity.	Distance.	Error.	Distance.	Error.
1 inch.	1000 feet.	17″.2	10000	1″.72
2 inches	1000	34 .4	10000	3 .44

The ordinary 5-inch theodolites have an eccentricity of about 1.86 inch, and the error at 1000 feet is about 32″.0. These may enable observers to allow for such small errors when thought necessary.

When signals are circular or polygonal towers, various methods may be employed to find the true angle, from a due consideration of the nature of the case, which, to any one possessing a knowledge of the elements of geometry, will readily occur.

V. The angles measured in an inclined plane should be reduced to the horizontal plane.

In this case the altitudes must be also observed, and then there is formed a spherical triangle, of which the three sides are given to compute the angle at the zenith, which may be performed by the rules of spherical trigonometry.

VI. A spherical triangle being proposed, of which the three sides are very small compared with the radius of the sphere; if from each of its angles, one-third of the excess of the sum of its three angles, above two right angles be subtracted, the angles so diminished may be taken for the angles of a rectilineal triangle, whose sides are equal in length to those of the proposed triangle.

To find the spherical excess when the three sides are given in feet.

1. *Rule.*—To the constant logarithm 1.349380, add the logarithm of half the sum of the three sides, the logarithms of the three differences between these sides and that half sum, half the sum of these five logarithms will be the logarithm of the spherical excess in seconds.

2. To the logarithm of the area of the triangle taken as a plane one in feet, add the constant logarithm 0.674690; the sum is the logarithm of the excess above 180° in seconds.

3. If the base and perpendicular of a triangle be given. To the logarithm of the base in feet, add the logarithm of the perpendicular, and the constant logarithm 0.373660; the sum will be the logarithm of the spherical excess in seconds.

The spherical excess amounts to one second for an area of 76 English square miles, whence, if the area in square miles be known, the spherical excess may be readily obtained by dividing it by 76.

VII. To reduce a base on an elevated level to that at the surface of the sea.

Let r represent the radius of the earth, corresponding to the base b at the level of the sea, and $r+a$ the radius referred to the level of the measured base B; then it is obvious that $r+a : r :: \mathrm{B} : b = \mathrm{B}\times\frac{r}{r+a}$. Hence, $\mathrm{B} - b = \mathrm{B} - \mathrm{B}\frac{r}{r+a} = \mathrm{B}\times\frac{a}{r+a} = \mathrm{B}\times\left(\frac{a}{r} - \frac{a^2}{r^2} +, \&c.\right)$. But the radius of the earth being very great in comparison of the difference of level a, we have the correction δ sufficiently accurate, by retaining the first term. Hence, $\delta = \mathrm{B}\times\frac{a}{r}$.

Rule.—By logarithms. To the logarithm of the measured base in feet, add the logarithm of its height above the sea, and the constant logarithm 2.680110; the sum will be the logarithm of a number of feet which, taken from the measured base, will be that at the level of the sea required.

VIII. To determine the horizontal refraction from observation.

Rule.—From the measure of the intercepted terrestrial arc subtract the sum of the two depressions at its extremities; half the remainder is the refraction. If by reason of the smallness of the contained arc, one of the objects has an elevation instead of a depression, then the depression must be taken from the sum of the contained arc and elevation; half the remainder is the refraction.

FORMULÆ.

$$R=\frac{c-d-d'}{2}=\frac{c-(d+d')}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

If $-d'$ becomes an elevation, it changes its sign, and becomes $+e$, and in that case $R=\frac{c+e-d}{2}$ . . . . . . (2.)

The exact quantity of terrestrial refraction is very variable. It is estimated by Dr Maskelyne at one-tenth of the intercepted arc, by Delambre at one-eleventh, by General Mudge at one-twelfth, and by Legendre at one-fourteenth at a mean state of the atmosphere. In peculiar circumstances it varies very considerably from this, as from one-sixth to one-eighteenth of the contained arc. If one-twelfth of the contained arc be allowed, then, since the length of a second is about 100 feet in moderate distances, $\frac{1}{1200}$ of the distance in feet will be the refraction in seconds.

IX. To find the angle made by a given line with the meridian.

With a good instrument measure the greatest and least angular distance of the pole star from the vertical plane in which the given line is situated; half the sum of these two measures will be the angle required, or if the exact place of the pole star be computed, the true azimuth may be found at any time in the same manner as the latitude. See page 112.

This may also be done, though less accurately, by computing the azimuth of the sun, or a star, when on the line, from an altitude taken for that purpose.

X. In addition to what has already been said relative to finding the latitude of the place, we may add here, that the same thing may be very accurately obtained, by observing the greatest and least altitude or zenith distance of a circumpolar star, and correcting them for the effects of refraction; half the sum of the altitudes, thus corrected, will be the latitude, or half the sum of the zenith distances will be the colatitude.

XI. To determine the ratio of the earth's axes, and their actual magnitude from the measure of a degree of the meridian in two given distant latitudes, supposing the earth a spheroid generated by the rotation of an ellipse about its minor axis.

Let d and d' be the measures of two degrees, d being the least, or that nearest the equator, l and l' the latitudes of their middle points, t the semitransverse axis of the meridian or radius of the equator, c the semiconjugate or semipolar axis, e the excess of the equatorial radius above the polar semiaxis, and $r^\circ = 57^\circ.2957795$, the number of degrees in an arc are equal to the radius.

$$\text{Then, } e=\frac{r^\circ\,(d'-d)}{3\sin(l'+l)\times\sin(l'-l)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

$$\text{And } \frac{e}{t}=\iota=\frac{d'-d}{3\,d\sin(l'+l)\times\sin(\,'-l)} \quad . \quad . \quad . \quad . \quad (2.)$$

If $\frac{e}{t} = \iota$, ellipticity or compression, $t = \frac{r^\circ d}{1 - \frac{1}{3} - \frac{2}{3}\iota \cos 2l}$ (3.)

When l is nothing, or when one of the degrees is at the equator from formula (1.) $e = \frac{r^\circ (d' - d)}{3 \sin^2 l}$ . . . . (4.)

Therefore, the excess of the degree in any latitude above this degree at the equator, when divided by the square of the sine of the latitude, should always give the same quotient; or the excess of the degrees of the meridian above the degree at the equator, should be as the squares of the sines of the latitudes.

Since $e = \frac{r^\circ (d' - d)}{3 \sin (l' + l) \sin (l' - l)}$, then $d' - d = \frac{3e}{r^\circ} \sin (l' + l) \times \sin (l' + l)$ . . . . (5.)

If d' and d are two contiguous degrees, so that $l' = l + 1^\circ$, then $d' - d = \frac{3e}{r^\circ} \sin (2l + 1^\circ) \sin 1^\circ$, and since the sine of one degree is 0.017453, $d' - d = \frac{3e \times 0.017453}{r^\circ} \sin (2l + 1^\circ)$ . . . . . (6.)

The contiguous degrees therefore differ by a quantity proportional to the sine of twice the middle latitude. The difference is a maximum when $2l + 1^\circ = 90^\circ$, or when the middle latitude is 45°.

Places.	Latitude.	Degree in Fathoms.
Peru	1° 31′ 0″ N.	60468
India	13 6 31	60493
France	44 51 3	60756
England	52 2 18	60824
Sweden	66 20 10	60955

From the foregoing five different measures combined so as to produce the most accurate result, I found $\iota = 0.003227 = \frac{1}{310}$ nearly, and the equation representing the degrees of the meridian setting out from 45°, will be

$D = 60759.06 - 294.58 \cos 2l$ . . . . (7.)

in fathoms, or,

$D = 69.0444 - 0.334 \cos 2l$ . . . . (8.)

in English miles.

	Fathoms.		Miles.
Hence, $e =$	11252.12	=	12.786
$t =$	3486863. 0	=	3962.344
$c =$	3475611. 0	=	3949.588

The radius of curvature for the parallel of $45^\circ = t - \frac{e}{2} = 3481197$ fath. = 3955.906 miles. The circumference of the meridian is therefore equal to the product of the mean degree at 45° by 360 = 24856 miles; and the circumference of the equator is 24896 miles, or about 40 miles more than the preceding.

A geographical mile is therefore 1012.6 fathoms, or 6075.6 feet.

The semidiameter or distance from the centre to the surface, at any latitude l, or $r = t\,(1 - \iota \sin^2 l + \frac{5}{2} \iota^2 \sin^2 l \cos^2 l.)$. (9.)

If d be a degree of the meridian at any point of which the latitude is l, and D a degree of the curve perpendicular to the meridian at the same point, then, $e = \frac{r^{o}}{2}(D-d)\sec^{2} l$. . . . (10.)

$$\iota = r^{o} D - \frac{r^{o}}{2}(D-d)\tan^{2} l. \quad (11.)$$

$$\frac{e}{t} \text{ or } \iota = \frac{D-d}{2\,D\cos^{2} l} = \frac{D-d}{2\,D} \times \sec^{2} l. \quad (12.)$$

Ex. 1.—In the Philosophical Magazine for 1828, D the degree perpendicular to the meridian is given equal to 60973 English fathoms; $d = 60819$, and $l = 50° 40'$ N. By formula . (12.) $\iota = \frac{154}{2 \times 60973} \times \sec^{2} 50° 40' = \frac{1}{312}$ nearly.

Ex. 2.—The length of a degree in latitude 52° 2′ 20″ N. is 57074 toises, that in 11° 0′ N. is 56755 toises; required the ellipticity by formula (2.)?

$l' = 52° 21' 20''$	const. log	. . . .	9.522879
$l = 11\ 0\ 0$			
$l'+l = 63\ 2\ 20$	cosecant,	. . . .	0.049969
$l'-l = 41\ 22\ 0$	cosecant,	. . . .	0.182718
$d' = 57074$			
$d = 56755$	ar. co. log	. . . .	5.245996
$d'-d$ 319	log	. . .	2.503791
$\iota = 0.003202$	log	. . . . .	7.505353
or $= \frac{32}{10000} = \frac{1}{312.5}$ nearly.			

If L = the length of a degree of longitude, then
$L = \frac{\cos l}{r^{o}}(1 + \iota \sin.^{2} l + \frac{3}{4}\iota^{2} \sin.^{4} l)$. . . (13.)*

If the value of the degree is wanted in toises, fathoms, or feet, the second member of this equation must be multiplied by the semitransverse axis in the same measure.

Ex. 3.—Required the length of a degree of latitude at Edinburgh, in 56° N.?

By formula (7), D = 60759.06 + 294.58 × sin. 22° = 60759.06 + 294.58 × 0.374607 = 60869.4 fathoms.

Ex. 4.—Required the length of a degree of longitude in latitude 56° N., the ellipticity being $\frac{1}{300}$?

By formula (13), $L = \frac{0.559193}{57.2957795}\left\{1 + \frac{1}{300} \times 0.68694\right\}$; or L= 0.009760 × 1.00229 × 20921200=204651 feet, or, taking in the second term mentioned in the note, it is 204664 feet. These formulæ are useful for fixing the latitude and longitude of a particular point when

* If great accuracy is not required, $\frac{3}{4}\iota^{2} \sin.^{4} l$. may be omitted in the quantity within the parenthesis. Indeed any small error in the ellipticity or radius of the equator will produce an equal or greater effect.

referred to some object whose situation has been well determined, such as many places in Britain are by the trigonometrical survey. In this case any amateur observer may verify the latitude and longitude of his observatory deduced from his own observations, by a comparison with some point well settled in that work, when properly connected by trigonometrical operations. Even by taking a few angles with great care, the situation of a particular point may be well determined by spherical trigonometry.

If p be the length of a degree perpendicular to the meridian, t the equatorial radius, c the semipolar axis, $t-c=d$ the difference of these, r° the length of an arc in degrees equal to radius, or $57^\circ.2957795$, and l the latitude, then $p=\dfrac{t+d\sin.^2 l}{r^\circ}$ nearly. . . . (14.)

Ex. 5.—If $t=3486850$ fathoms, $d=11160$ fathoms, and $l=56^\circ$, then $p=\dfrac{3486850+11160\times\cdot 68694}{57.29578}=60992$ fathoms.

If p be the measure of a degree of a great circle perpendicular to a meridian at a certain point, m that of the corresponding degree on the meridian itself, and o the length of a degree on an oblique arc, making an angle a with the meridian, then

$$o=\frac{p\,m}{p-(p-m)\sin^2 a}=\frac{m}{1-\dfrac{p-m}{p}\sin^2 a} \quad . \quad . \quad (15)$$

Ex. 6.—If $p=60973$ fathoms, $m=60819$ fathoms, and $a=81^\circ\,56'\,53''$, therefore

$$o=\frac{60819}{1-\dfrac{154}{60973}\times 0.98038}=\frac{60819}{1-0.00253}=\frac{60819}{0.99847}=60912,$$ the length of the oblique degree in fathoms.

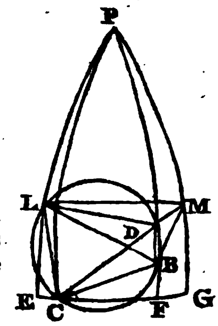
For an extension of this subject, see Mr Ivory on the properties of a line of the shortest distance traced on the surface of the oblate spheroid, in the sixty-seventh volume of the Philosophical Magazine, and vol. IV., new series, for a correction of the trigonometrical survey. The papers are rather too long and difficult to be inserted in this place.

Edinburgh, 4th July, 1827.

Ex. 8.—Determination of the latitude and longitude of the Observatory on the Calton-hill.

The observations were made with a repeating theodolite placed near the old entrance from the east into the grounds surrounding the Observatory.

The angle between North Berwick Law, in latitude 56° 3′ 8″ N., longitude 2° 42′ 11″ W., and the Isle of May Light, in latitude 56° 11′ 22″ N., longitude 2° 32′ 47″ W., was 13° 48′ 48″; and the angle between the Isle of May and the West Lomond, in latitude 56° 14′ 57″ N., longitude 3° 17′ 4″ W., was 68° 40′ 46″; what is the latitude and longitude of the Observatory?



Let P be the North Pole, C the Calton-hill, L the West Lomond, M the Isle of May Light, and B North Berwick Law; also ECFG a parallel of latitude passing through C, and PE, PC, PF, and PG meridians passing through L, C, B, and M.

Then, by the problem originally proposed by Townley, let the marginal figure be constructed, (page 47) and apply the principles of spherical trigonometry, since the lines employed are arcs on the surface of the earth, which in this case may be considered as a sphere.

Now by the trigonometrical survey we have

West Lomond latitude	56° 14′ 57″ N. longitude		3° 17′ 4″ W.
Isle of May Light .	56 11 22		2 32 47
North Berwick Law . .	56 3 8		2 42 11
Whence are obtained	PL = 33° 45′ 3″		LPB = 34′ 53″
	PB = 33 56 52		LPM = 44 17
	PM = 33 48 38		BPM = 9 24

1. In the triangle LPB there are given the sides PL, PB, and the angle LPB, to find LB = 22′ 44″.48 = 26.17 English miles.

2. In the triangle LPM are given PL, PM, and the angle LPM, to find LM = 24′ 52″.88 = 28.632 English miles.

3. In the triangle BPM are given PB, PM, and the angle BPM, to find BM = 9′ 45″.56 = 11.23 English miles.

4. In the triangle PLB are given all the sides to find the angles PLB and PBL = 121° 3′ 52″, and 58° 27′ 10″ respectively.

5. In the triangle LBM are given all the sides now found by computation to determine the angles LBM = 90° 51′ 40″, and BML = 66° 2′ 54″. Also LBM — LBD = 90° 51′ 40″ — 68° 40′ 46″ = DBM = 22° 10′ 54″; and 180° —(BLD+LBD) = 180° — 82° 29′ 34″ = 97° 30′ 26″ = LBD. BLM will also be found to be 23° 5′ 28″, whence BLM — DLB = 23° 5′ 28″ — 13° 48′ 48″ = 9° 16′ 40″.

Likewise LD will be found = 21′ 22″.08, LDM = 128° 28′ 28″, LMD = LMC = 42° 14′ 52″. But LMC + LCM = 110° 55′ 38″, and consequently CLM = 69° 4′ 22″, CLM — BLM = 69° 4′ 22″ — 23° 5′ 28″ = 45° 58′ 54″ = BLC. But BLC + PLB = PLC = 167° 2′ 46″.

6. It is now necessary to compute the sides LC, MC, and BC, which will be found to be LC = 17′ 57″.46, MC = 24′ 56″.84, and BC = 16′ 29″.70.

Hence LC = 20.66 English miles, MC = 28.71, and BC = 18.98.

7. In the triangle PLM are given the sides PL, PM, and LM, to find the angles PLM = 97° 58′ 24″, PML = 81° 24′ 48″. If Napier's analogies be applied to the triangle LPM, these will be found to be 97° 58′ 34″, and 81° 24′ 38″ respectively. Also PLM + BLM + CLB = PLC = 167° 2′ 46″, as before. In like manner PML + LMC = PMC = 123° 39′ 40″.

8. In the triangle LBC are given the sides to find the angle LBC = 51° 31′ 32″, and consequently LBC + PBL = 109° 58′ 42″ = PBC.

9. In the triangle PLC are given the two sides, PL, LC, and the contained angle PLC, to determine PC the colatitude = 34° 2′ 32″.6, and the latitude = 55° 57′ 27″.4 N. deduced from the West Lomond.

10. In the triangle PMC are given the sides PM, MC, and the contained angle PMC, to deduce PC the colatitude, and thence the latitude, $= 55° 57' 26''.8$ obtained from the Isle of May.

11. In the triangle PBC are given the sides PB, BC, and the contained angle PBC to find PC, the colatitude, and thence the latitude, $55° 57' 28''$, deduced from North Berwick Law.

The mean of these three is	55 57 27.4 N.
Error in the latitude of Greenwich from which these were deduced	— 1.6
Correct latitude	55 57 25.8 N.

12. In the triangle PLC there are given the three sides to find the angle PLC, the difference of longitude between the West Lomond and the Calton-hill, $= 7' 16''.5$ E.

Hence $3° 17' 4'' - 7' 16''.5 = 3° 9' 47''.5$ W. by West Lomond:

13. Again in the triangle PMC there are given all the sides to find the angle MPC, the difference of longitude between the Isle of May and the Calton-hill, $= 37' 5''.6$ W.

Hence $2° 32' 47'' + 37' 5''.6 = 3° 9' 52''.6$ W., the longitude of the Calton-hill deduced from the Isle of May.

14. Lastly, in the triangle PCB, the sides are given to find the angle CPB, the difference of longitude between North Berwick Law and the Calton-hill, $= 27' 42''.4$ W.

Whence $2° 42' 11'' + 27' 42''.4 = 3° 9' 52''.6$ W.

The mean of these three gives $3° 9' 51''.2 = 12^m 39^s.5$ W.

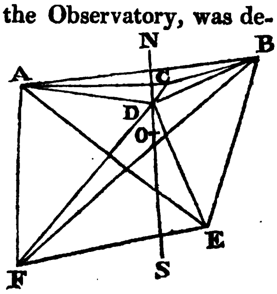
Ex. 9.—Given the latitude of the staff on North Berwick Law, 56° 3′ 8″ N., longitude 2° 42′ 11″ W., and the latitude of the Isle of May Light 56° 11′ 22″ N., longitude 2° 32′ 47″ W.; the angle at North Berwick Law, between the Isle of May and Dunglass Tower, was observed by Captain Hall, R. N., to be 87° 41′ 1″, that at Dunglass, between the Isle of May and North Berwick Law, being 37° 20′ 13″; required the latitude and longitude of Dunglass Tower?

Ans.—Latitude 55° 56′ 31″.7 N., longitude 2° 21′ 42″ W.

Ex. 10.—Determination of the latitude and longitude of Makerston Observatory by comparison with the trigonometrical survey communicated by Lieut. General Sir Thomas Makdougal Brisbane, K. C. B.

By means of angles taken at the various stations, A, B, C, D, &c. and the latitudes, longitudes, and distances, obtained from the trigonometrical survey, the point D, from which the other stations could be seen, was accurately fixed, and the distance DO correctly ascertained, whence O, the position of the Observatory, was determined.

Let A denote Eildon middle hill
B Mordington Law
C Hume Castle, south-east turret
E Cheviot
F Wisp
D Point whence these places could be properly observed
O the Observatory, and
N S its meridian.



Distances.	Angles with the Meridian.
AB = 156876 feet.	NDC = N 22° 14′ 34″ E.
AC = 59464	NDB = N 50 16 4 E.
CB = 97438	SDE = S 60 26 56 E.
AD = 41842	SDF = S 39 13 47 W.
DF = 147961	SDA = S 81 33 15 W.
FB = 266432	A Lat. 55 35 6 N. Long. 2° 42′ 27″ W.
FE = 185024	B 55 48 39 2 3 45
EB = 121278	C 55 40 5 2 37 38
DO = 7266	E 55 28 52 2 8 12
	F 55 17 13 2 57 22

1. Mordington 119696 ft.	N. 50° 16′ 4″E.	Lat. 55° 36′ 8″N.	Long. 2° 30′ 30″W.
Cheviot . 88968	S. 60 26 56 E.	55 36 6.5	2 30 31
Wisp . . 147961	S. 39 13 47 W.	55 36 5.5	2 30 32
Means of first series	. . .	55 36 6.7	2 30 31
2. Hume Castle 26164 ft.	N. 22 14 34 E.	Lat. 55 36 6.3	Long. 2 30 30.3
Cheviot . 88851	S. 60 26 56 E.	55 36 6.0	2 30 28 7
Mordington 119758	N. 50 16 4 E.	55 36 7.4	2 30 31.2
Means of second series	. . .	55 36 6.6	2 30 30.0
3. Eildon-hill 41842 ft.	S. 81 33 15 W.	Lat. 55 36 7.0	Long. 2 30 29.0
Hume Castle 25985	N. 22 14 34 E.	55 36 8.0	2 30 29.0
Mordington 119607	N. 50 16 4 E.	55 36 8.4	2 30 29.0
Means of third series	. . .	55 36 7.8	2 30 29.0
General Means (1)		55 36 6.7	2 30 31
Ditto ditto (2)		55 36 6.6	2 30 30
Ditto ditto (3)		55 36 7.8	2 30 29
Means of the whole		55 36 7.0	2 30 30
Centre of Observatory South of Meridian mark, 7266 feet		= — 1 11.6	
Latitude and Longitude of Observatory		55 34 55.4N	Long. 2 30 30 W.

SECTION V.

Rules and Formulæ.

SPECIFIC GRAVITY.

The difference between the absolute weight of a body and its weight when entirely immersed in a fluid is the same with the weight of a quantity of the fluid equal in bulk to the body.

If W be the weight of a body in *vacuo*, (which is nearly the same as that in air,) and W′ its weight in water, then W—W′ is the weight of a quantity of water equal in bulk to the body; and since the weight of any body divided by an equal bulk of water measures the specific gravity, S, of the body, then $S = \frac{W}{W-W'}$ . . (1)

The specific gravities of bodies are determined by the hydrostatic balance, the hydrometer, &c., described in books on natural philosophy.

To compute the specific gravity of air under given circumstances.

It is shown in Playfair's Outlines, vol. I. § 333, that if the elasticity or tension at the freezing point be denoted by unity, and x, any number of degrees above that point, then the elastic force f at

that point, will be $f = (1.375)^{\frac{x}{180}}$ of Fahrenheit's scale, or

$$\log f = \frac{x}{180} \times \log (1.375) = \frac{x}{180} \times 0.138303 = 0.00076835x^{*} \qquad (2)$$

This also gives the bulk of gas in like circumstances. But the specific gravity is reciprocally as the bulk, therefore the reciprocal of the bulk or the natural number answering to the arithmetical complement of the $\log f$, will be the specific gravity of permanently elastic fluids. Thus let the bulk and specific gravity of air at 32° F. = 1, then at 52° F. they will be 1.036, and 0.9652 respectively.

From the experiments of Gay Lussac, it may be shown that 0.4545 will be the specific gravity of aqueous vapour, when compared with atmospheric air, at 32° F. Now, when the temperature is given, the specific gravity of aqueous vapour is directly as its temperature, and the tension being given, the specific gravity is reciprocally as its bulk, the specific gravity s of aqueous vapour, (that of water being 1), in saturated air at any temperature t, and elastic force f, (from table IV., page 64,) will be obtained from the following formula, the barometer being at 30 inches.

$$s = 0.4545 \times \frac{f}{30} \times \frac{660}{448+t} = \frac{10f}{448+t} \qquad (3)$$

If it be not saturated, and t' being the dew-point

$$s = \frac{10f}{448+t} \times \frac{448+t}{448+t'} = \frac{10f}{448+t} \times \left(1 + \frac{t-t'}{448+t'}\right) \qquad (4)$$

The quantities in this expression are all known except f, which is to be taken from any good table, such as Dalton's or Ure's. See Table IV., page 64.

If, therefore, s' be the specific gravity of air fully saturated with moisture; a the specific gravity of dry air obtained from formula (2), and s the specific gravity of aqueous vapour in saturated air, derived from formulæ (3), then from the law of expansion discovered by Dalton and Gay Lussac, that $v = \frac{p}{p-f}$,† p being the barometric pressure, f the elastic force, and v the volume,

$$s' = a + \left(0.4545 \times \frac{660}{448+t} - a\right) \times \frac{f}{30}, \text{ or by simplification,}$$

$$s' = a + s - \frac{af}{30} \qquad (5)$$

If t' be the dew-point, and s'' be the specific gravity, according to the actual state of the atmosphere,

$$s'' = \left(a + s - \frac{af}{30}\right)\left(1 + \frac{t-t'}{448+t'}\right) \qquad (6)$$

in which a and s are got from the following table, page 213, and f from Dalton's.

Ex.—Required the specific gravity of air saturated with moisture, at 92° F.?

By formula (2), $\frac{60}{180} \times 0.138303 = \frac{1}{3} \times 0.138303 = 0.046101$, ar. co. of which is 9.953899. To this the natural number is $0.89929 = a$.

* This depends upon the accuracy of the hypothesis of Dalton, of which, in his late publication, he seems to have some doubt. See page 56.

† The elasticity in the example was not taken from Dalton. It is difficult to obtain correct formulæ for these researches.

But by formula (3), $s=\frac{10f}{448+t}=0.02782$, and $\frac{af}{30}=0.04502$. Now, $s'=a+s-\frac{af}{30}$ by formula (5); therefore, $s'=0.89929+0.02782-0.04502=0.88209$ the specific gravity of air saturated with moisture, at 92° F. If the air is not saturated. Suppose 87° F. the dew point represented by t', then the factor $1+\frac{t-t'}{448+t'}$, in formula (6), becomes $1+\frac{92-87}{448+92}=1+\frac{1}{108}$, therefore, $0.88209+\frac{0.88209}{108}=0.88209+0.00817=0.89026$, the specific gravity of air in the given circumstances, that of dry air at 32° F. being unity.

It is shown in Playfair's Outlines, vol. I., art. 256, that if the specific gravity of air be called m, that of water being 1; if W be the weight of any body in air, and W′ its weight in water, then $W+m\,(W-W')$ is its weight in vacuo very nearly. In a mean state of the atmosphere at 30 inches of the barometer and 60° F. $m=0.00122$ nearly, which may be reduced to any other temperature by the foregoing formula (4), and to any other pressure by multiplying $\frac{p}{30}$.

If s be the specific gravity of a body ascertained by weighing it in air and water, and m the specific gravity of the air at the time when the experiment was made; the correct specific gravity s', or that which would have been found if the body had been weighed in a vacuum instead of air, or

$$s'=s+m\,(1-s). \qquad (7.)$$

Where the body is heavier than water, this correction is subtractive; when lighter it is additive.

Ex.—The weight of Captain Kater's experimental pendulum was carefully determined in air, by Barton's balance from the Mint, and found to be 66904 grains. The trough, which had been previously placed under the pendulum, was then filled with distilled water, and the weight of the water displaced, was 9066 grains. The small portion of iron wire which was immersed in the water was carefully noted; the weight of the wire by which the pendulum was suspended was 56 grains, and the weight of water equal in bulk to that part of the wire which was immersed was 2.5 grains. The temperature of the water was 68° F., that of the atmosphere 62° F., and the barometer 29.9 inches. Now since $s=\frac{w}{w-w'}$, w being the weight in air, and w' that in water, then

$s=\frac{66848}{9063.5}=7.37552$ at 29.9 bar. and 62° F., and $s'=7.37552+0.00120678\,(1-7.37552)=7.36783$ at 68° Fahrenheit.

But the specific gravity of water σ at 68° is ·99936, that at 62° being 1; and, therefore,

$$\frac{1}{\sigma}\times s'=\frac{1}{0.99936}\times 7.36783=7.37254 \text{ at } 62° \text{ F.}$$

Biot's experiments give at 30 inches bar., and 60° F., the specific gravity of air 0.00122, or $\frac{1}{820}$, water being 1.

Mr S. Rice, from Sir G. Shuckburgh's experiments, deduces

0.0012085, not differing much from Biot's, and generally supposed the more correct. According to Gay Lussac, the expansions of fluids from 32° to 212° F. is 0.375, whence $\frac{\cdot 375}{180} = \frac{1}{480}$ for 1° F.

Now suppose $c =$ the first correction of the length of the pendulum, c' the second, l the measured length of the pendulum, p the barometric pressure, the standard being 30 inches; and δt the difference of temperature from the standard, then

$$c = \frac{30 \times 820}{p} = \frac{24600}{p} \quad . \quad . \quad . \quad (8.)$$

$$c' = \frac{c\,(t' - t)}{480} \quad . \quad . \quad . \quad (9.)$$

If $l' =$ the corrected length of the experimental pendulum l'', from a mean of Captain Kater's experiments at London in air, then

$$l' = l'' + \frac{l''}{s\,(c + c')} \quad . \quad . \quad . \quad (10.)$$

s being the specific gravity of the pendulum.

Whence $c = \frac{24600}{29.786} = 826$, and $\delta t = 69°.62 - 62° = 7°.62$, hence $c' = \frac{826 \times 7.62}{480} = 13$, therefore $c + c' = 839$.

Hence by formula (10) $l' = l'' + 39.13284 \times \frac{1}{839} \times \frac{1}{7.37254} = l'' + 0.00633$.

It is now only necessary to correct for the height above the sea, which is 92.5 feet.

The correction for this height found by the formula, (13), which will presently be given, is 0.00023.

Hence the true length $l = 39.13284 + 0.00633 + 0.00023 = 39.13940$. In this case no allowance is made for the hygrometer. Now if the air were supposed half saturated with moisture, since Captain Kater does not give the state of the hygrometer, and the mean between Biot's and Rice's specific gravity of air taken, the true length would come out 39.13938, which differs from Captain Kater's result by 0.00009 in excess.

It is shown by writers on mechanics, that when the semiarc described by a pendulum on each side of the perpendicular is 1°, the time lost by oscillating in a circular, instead of a cycloidal or infinitely small arc, is $\frac{1}{52524}$ in each second, and that in different small arcs of the same circle, the time lost varies nearly as the square of the arc; hence if a pendulum makes v vibrations in 24^h, when vibrating in very small circular arcs, of which the mean at the commencement and termination of each experiment is d degrees, it would, in the same time, make $v + \frac{d^2 v}{52524}$ *infinitely small vibrations*. Hence to correct the oscillations of a pendulum for the arcs of vibration, multiply the square of the *mean arc* when it makes

Daily	86000 oscillations by	1.637	(A)
	86100	1.639	
	86200	1.641	
	86300	1.643	
	86400	1.645	
	86500	1.647	
	86600	1.649	

Since the force of gravity varies directly as the length of the pendulum, or inversely as the squares of the number of vibrations, and the diminution of the force of gravity, arising from the buoyancy of the atmosphere, is $\frac{1}{m}$ part; therefore if v be the number of vibration in air, and V those in a vacuum, then

$$V=\left\{v^2\left(1+\frac{1}{m}\right)\right\}^{\frac{1}{2}}=v\left\{1+\frac{1}{2m}-\frac{1}{8m^2}+\&c.\right\} \quad . \quad (10.)$$

$V=v+c$, and hence $c=\frac{v}{2m}$ nearly.

In Captain Kater's experiments at Unst, the specific gravity of the pendulum to that of air was as 7099 to 1, hence $\frac{1}{m}=\frac{1}{7099}$, and therefore $\frac{v}{2m}=\frac{86090.77}{14198}=6.07$ nearly.

If n' be the number of oscillations performed in 24[h] by the experimental pendulum, n the true number, e the expansion for a change of one degree Fahrenheit, t the standard temperature, and t' the observed, then

$$n=n'+\tfrac{1}{2}n'\,e\,(t'-t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.)$$

In Captain Kater's pendulum $e=0.00001$ of an inch nearly, whence $n=n'+\frac{1}{2}n'\times 0.00001\,(t'-t)$.

Hence if $v=86058.82$, $t'=71^\circ.6$ and $t=62^\circ$, the number of vibrations at the latter temperature are $n=86058.72+\frac{1}{2}\times 86058.72\times 0.00001\times 9.6=86082.77$.

To reduce the length of the pendulum from any height to the level of the sea, the true length being denoted by l, the observed by l', the height above the sea by a, and the radius of the earth by r, then

$$l=l'+\frac{2\,a\,l'}{r} \quad . \quad . \quad . \quad . \quad (12)$$

Some allow one-third for the effect of the dense strata immediately under the pendulum, in which case $l=l'+\frac{4\,a\,l'}{3\,r}$* . (13.)

In a similar manner $v=v'+\frac{2\,v'\,a}{3\,r}$† . . . . (14.)

At Unst $\frac{2\,v'\,a}{3\,r}=0.06$, therefore

$86090.77+6.07+0.06=86096.90=$ the number of oscillations of the pendulum in a mean solar day at the level of the sea in vacuo.

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* To the const. log $\bar{6}.396$ add the log of the height of the pendulum above the sea in feet, the sum will be the log of the correction in decimals of an inch.

† To the const. log $\bar{3}.440$ add the log of the height above the sea in feet, the sum will be the log of the correction in vibrations nearly.

These formulæ are sufficient for most purposes. Biot has, however, demonstrated, that if c be the correction in seconds for the mean arc of vibration, n the number of oscillations, M the logarithmic modulus, a the arc of vibration at the commencement of the interval, and b that at the end, then

$$c = \frac{n' \sin(a+b) \sin(a-b)}{32 \text{ M} \log\left(\frac{a}{b}\right)} \qquad (15.)$$

These arcs being small, their lengths will not differ sensibly from their sines, whence if a and b are given in degrees, the lengths of these arcs will be $0.0174533\ a$ and $0.0174533\ b$, and $\text{M} = 2.302585$, these values being substituted for a, b, and M, equation (15) will become $c = \frac{n'(a+b)(a-b)}{241886 \log(\log a - \log b)}$, and adopting logarithms, we finally have $\log c = \{\log n' + \log(a+b) + \log(a-b)\} - \{\text{C. L. } 5.383611 + \log(\log a - \log b)\}$ . . . (16.)

To apply this to practice let us assume Kater's 5th experiment marked E, and we have $a = 1°.21$ and $b = 1°.09$, whence

$a + b = 2.30$ log			0.361728	
$a - b = 0.12$ log			9.079181	
$n' = 86056.47$ log . . .			4.934785	
Sum			4.375694	(A)
Constant logarithm .		5.383611		
Log $a = 1°.21$.	0.082785			
$b = 1\ .09$.	0.037426			
Diff. .	0.045359 log	8.656663		
Sum (B) . . .		4.040274	4.040274	(B)
$(\text{A} - \text{B}) = \log c = 2''.165$ . . .			0.335420	

Hence $n = n' + c = 86056.47 + 2.165 = 86058.635$. Captain Kater thinking this an unnecessary refinement in practice, multiplies the square of the mean arc by 1.638 Table (A); thus $1.15 \times 1.15 \times 1.638 = 2''.166$ nearly the same as before; and, by selecting the proper number, this is sufficiently correct for almost any purpose, and much more simple, though when great nicety is required the first is without doubt the more accurate.

If the length of a pendulum oscillating seconds of mean time at one place or point on the earth's surface be known, its length at another place, where the same invariable pendulum makes a different number of vibrations, may readily be found. For if l be the length at the first place, l' that at the second, v the number of vibrations at the first place in 24 hours, and v' that at the second, then, as is shown by writers on mechanics,* $l : l' :: v^2 : v'^2$. (17.) consequently if three of these be known the fourth may be found.

As this is rather laborious, an approximate rule may be obtained sufficiently correct for most purposes where the difference of oscillations does not exceed 30 or 40, or in an arc of five or six degrees.

* See Gregory's Mechanics, vol. I., section II., for this and other formulæ and corrections more simple than those given here.

If ΔL represent a small variation of the length of the pendulum, and ΔN that in the number of oscillations, then $\Delta L = \frac{L \Delta N}{\frac{1}{2} N}$,

and $\Delta N = \frac{\frac{1}{2} N \Delta L}{L}$* . . . . . . (18.)

Let δL be the variation of L for one degree of Fahrenheit's thermometer, and n the number of degrees of change of temperature, for this then $\Delta L = n \delta L \times L$, and $\Delta N = \frac{1}{2} N n \delta L$. (19.)

Since the variation of brass from expansion is nearly 0.00001 inch for 1° Fah. $\Delta N = 0.432\, n$, and $\Delta L = \frac{n L}{100000}$. (20.)

Example I.

Captain Kater found the experimental pendulum made at London in latitude 51° 31′ 8″ N. 86061.52 oscillations at 62° Fah. in a mean solar day, while at Unst in latitude 60° 45′ 28″ N., it made 86096.90 oscillations in the same time; required the length of the pendulum at Unst, that at London being 39.13929 inches?

Here $86096.90 - 86061.52 = 35.38 = \Delta N$. Now $\Delta L = \frac{L \Delta N}{\frac{1}{2} N}$ formula (18) $= \frac{39.13929 \times 35.38}{43048.45} = 0.03217$, consequently $39.13929 + 0.03217 = 39.17146$ inches, the length at Unst.

Ex. 2.—Captain Hall found an experimental pendulum, making 86235.98 oscillations at London at 68° Fah. made 86098.75 oscillations at Galapagos at the temperature of 74°. Hence to the number of oscillations at London (since $74° - 68° = 6°$,) we must add (formula 20) $0.432 \times 6 = 2.59$ oscillations, which becomes 86101.34, at 68° Fahrenheit.

Now by formula (17), as the places are very distant, $v^2 : v'^2 :: l : l' :: 39.13929 : 39.01717$, the length of the pendulum at Galapagos.

Of late the figure of the earth has been determined with great accuracy by means of the pendulum. It is demonstrated by the theory of gravitation, that the length of the pendulum is augmented from the equator to the pole, proportionally to the square of the sine of the latitude†, in such a manner that if the length of the equatorial pendulum be represented by z, and its absolute variation from the equator to the pole by y, then l, its length in any other latitude L, will be represented by the following equation:—

$$l = z + y \sin^2 L. \qquad (1.)$$

If we have two equations of this form, in which l and L are determined by observation, we can obtain the values of z and y.

$$l = z + y \sin^2 L$$
$$l' = z + y \sin^2 L'$$

$$\text{hence } y = \frac{l' - l}{\sin (L' + L) \sin (L' - L)} \qquad (2.)$$

$$\text{And } z = l - y \sin^2 L \qquad (3.)$$

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* These formulæ will be very convenient for estimating small variations in either the length of the pendulum, or in the number of vibrations. See Captain Sabine's work, page 345.

† Strictly speaking this should be the reduced latitude.

Consequently $\frac{y}{z}$ represents the diminution of gravity from the pole to the equator.

Now by the doctrine of central forces, if f denote the centrifugal force; π the circumference of a circle to diameter unity; r the radius of the given circle in which a body revolves; t the time of revolution, and g the gravitating force, then $f = \frac{4\pi^2 r}{g t^2}$. But by the theory of the pendulum, if l is its length, $g = \pi^2 l$; hence

$$f = \frac{4r}{t^2 l} = \frac{r}{(\frac{t}{2})^2 l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

The ratio of the centrifugal force to gravity may be expressed by $\frac{f}{1+f}$, and the ellipticity of the meridian or flattening of the earth is from theory equal to $\frac{5}{2}$* of the ratio of the centrifugal force to gravity, diminished by the fraction obtained from dividing the difference of the lengths of the pendulum at the pole and equator by its length at the equator. Wherefore if ι denote the ellipticity.

$$\iota = \tfrac{5}{2} \times \frac{f}{1+f} - \frac{y}{z}$$

By substituting the value of f from equation (4.)

$$\iota = \tfrac{5}{2} \times \frac{r}{r+(\frac{t}{2})^2 l} - \frac{y}{z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

As t in these investigations denotes the time which the earth takes to perform a rotation about its axis, or $86164^s.0908$; $\frac{1}{2}t^2 = 1856062632$, r, the radius of the equator, is 20921200 feet, l, the length of the equatorial pendulum by numerous observations, is 39.01326 inches, or 3.25110 feet, and $y = 0.20686$ inch.

$$\text{Whence } \iota = 0.0086375 - \frac{y}{z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

By combining a great number of the best observations I have found $\iota = 0.00333 = \frac{1}{300}$ nearly.

From these we may get a formula to compute the length of the pendulum at any latitude.

Commencing at the equator $l = 39.01326 + 0.20686 \sin^2 L$. (A)

Setting out from 45°, $l = 39.11600 - 0.10343 \cos 2L$. (B)

Ex.—Required the length of the pendulum at Leith, in latitude 55° 58′ 40″ N.?

Ans.—39.1555 inches.

Since $g = \pi^2 l$, we have $g = 32.1716$ feet at 45° N.

Hence the length of the pendulum and force of gravity may be found at any latitude.

But the force of gravity may be found more readily by a particular formula for that purpose.

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* This fraction is obtained by approximation, and is not perfectly correct. By taking in the quantities of the second order, the ellipticity would vary about $\frac{1}{100}$ from the first approximation. It is difficult to solve the equations involving these. Still, however, no error should be allowed, if possible, to affect the final results, but what unavoidably belongs to the observations. Instead of $\frac{5}{2} = 2.5$, I have found that 2.491516 should be used, which makes the constant 0.00861 nearly, and the sine square of the reduced latitude should properly be employed, otherwise the computed pendulums are in a small degree too great about 45°.

Since g is equal to 32.1716 feet, or 9.8058 metres at 45°, then G at any other latitude will be

$$G = g\,(1 - 0.00268 \cos 2\,L) = 32.1716 - 0.08622 \cos 2\,L \qquad (7.)$$

Log 0.08622 is 8.935608.

Let L be the length of the sexagesimal pendulum and l that of the French decimal-metrical pendulum, then

$$L = 52.74079\,l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8.)$$

of Sir George Shuckburgh's scale,

$$\text{or } L = 52.740564\,l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9.)$$

of Bird's Parliamentary Standard of 1758.

Let V be the velocity of sound per second, t the temperature, f the elastic force of vapour for the dew-point, obtained by Daniell's hygrometer, or otherwise by formula, page 68, p the barometric pressure, λ the latitude of the place of observation, ω the velocity of the wind, and φ the angle which the direction of the wind makes with that of the sound, then

$$V = (102.4225 + 0.1103\,t)\left(1 + \frac{f}{5\frac{1}{4}\,p - 2f}\right)(10.2738 - 0.01378 \cos 2\,\lambda) + \omega \cos \varphi, \text{ in English feet.} \quad . \quad . \quad . \quad . \qquad (10.)$$

Ex. 1.—On the 19th of July, 1826, in mean latitude 56° N., longitude 3° 10′ W., several experiments were tried on the velocity of sound, when the guns on Edinburgh Castle were fired in honour of his Majesty's coronation. They were made on the coast of Fife at the distance of 42546 feet, the barometer standing at 29.96 inches, the thermometer at 72°, the dew point by Daniell's hygrometer, or by a thermometer, having its bulb moistened with tissue paper, (page 68,) at 66°, the velocity of the wind by an anemometer was 15 miles per hour, or 22 feet in a second, making an angle of 120° with that of the sound; required the true rate per second and the difference between theory and experiment, when the arithmetical mean of a number of experiments gives 37.36 seconds for the time elapsed between seeing the flash and hearing the report?*

$$V = \{102.4225 + 7.9416\}\left(1 + \frac{0.635}{158.52}\right)(10.2738 + 0.1136 - 22 \times 0.5 = 110.3641 \times 1.004 \times 10.3874 - 11 = \qquad 1140.0$$

$$\text{Experiment gives} \quad . \quad . \quad \frac{42546}{37.36} = \quad . \quad . \quad . \quad . \quad . \qquad 1138.8$$

$$\text{Difference} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad +1.$$

or excess of the formula.

Ex. 2.—Required the velocity of sound at Port Bowen in latitude 73° 14′ N., the barometer being at 30.398 inches, Fahrenheit's thermometer —39°.5, the air being very dry, and the wind almost insensible?

Ans.—By formula	.	1009.78 feet.
By experiment	.	1014.39
Difference		— 4.61

Ex. 3.—At the same place, what was the velocity of the sound

* If a series of experiments are made by a gun at each end of the measured base, the geometrical means of the times should be taken. See *Bulletin de Sciences* for 1826.

when the barometer was at 30.118 inches, and thermometer at $+33^{\circ}.5$ Fahrenheit?

Ans.—By formula . 1092.82 feet.
By experiment . 1098.32

Difference . — 5.50

These slight errors of Captain Foster's are perhaps owing to the effects of light airs of wind during the observations.

Also $V=(105.9518+0.19845t)\left(1+\frac{f}{5\frac{1}{3}p-2f}\right)(3.13143-0.0042\cos 2\lambda)+\omega\cos\varphi$, the velocity in French metres, the metrical barometer and centigrade thermometer being used. . (11.)

Ex. 4.—Dr Moll, near Utrecht, in Holland, in latitude 52° 13′ 33″ N. found the velocity of sound to be 340.06 metres per second, when the height of the metrical barometer was $0^{m}.74475=p$, the temperature $11^{\circ}.16$ centigrade, the mean tension of aqueous vapour by Daniell's hygrometer was $0^{m}.009253=f$, the effects of the wind being destroyed by firing guns at both ends of the base.

$$V=(105.9518+2.2147)\left(1+\frac{0.009253}{3.972-0.0185}\right)(3.13143+0.00105)$$

$=339^{m}.622$
Experiment gives 340 .060

Difference . —0 .438

Ex. 5.—Required the velocity of sound when the metrical barometer stands at $0^{m}.74815$, the temperature $11^{\circ}.215$ centigrade and $f=0^{m}.008405$, in the same latitude?

Ans.—Formula gives 339.582 metres.
Experiment . 339.340

Difference . + 0.242

In a river or open canal, let v be the velocity of the stream measured by the inches it moves over in a second of time; r a constant quantity, called the radius of the section, and obtained by dividing the area of the tranverse section of the stream expressed in square inches by the boundary or perimeter of that section, diminished by the superficial breadth of the stream expressed in linear inches. Also let λ be the length of an open canal or of a close pipe; δ the difference of the level of its extremities, d the diameter in the case of a pipe, h the height of the water in the reservoir above the upper orifice of the pipe, and h' the height above the lower orifice, at which the water stands in the cistern into which it is emptied.

Now let $\frac{\delta}{\lambda}=i$, or the sine of inclination and $\frac{h+\delta-h'}{\lambda}=k$.

The formula for the velocity of water in pipes, per second, will be $v=\{32806.6\,d\,k+0.023751\}^{\frac{1}{2}}-0.154113$ . . (12.)

Ex. 6.—Let $\delta=65$ feet, $d=19$ inches, $\lambda=18300$ feet, $\frac{\delta}{\lambda}=\frac{65}{18300}=0.00352=k$, therefore

$v=\{32806.6\,d\,k+0.023751\}^{\frac{1}{2}}-0.154113=46.9$ inches the velocity per second.

In rivers and open canals the formula is

$$v = \{32806.6\, r\, i + 0.023751\}^{\frac{1}{2}} - 0.154113 \quad . \quad . \qquad (13.)$$

These formulæ have been simplified, and are tolerably correct.

Suppose v, d, ∂, and λ, are all expressed in feet,

$v = 50 \left\{ \frac{d\,\partial}{\lambda} \right\}^{\frac{1}{2}}$ nearly the velocity in feet per second. . (14.)

Let D be the discharge per minute in cubic feet, then

$$D = 2356\, d^2 \left(\frac{d\,\partial}{\lambda} \right)^{\frac{1}{2}} \quad . \; . \; . \; . \; . \; . \; . \qquad (15.)$$

To find the fall in a river caused by obstruction, such as the piers of a bridge, &c.

Let v be the velocity of the stream in feet per second, b the whole breadth of the channel in feet, c the contracted breadth between the obstacles, and f the fall, then

$f = \left\{ \left(\frac{25\, b}{21\, c} \right)^2 - 1 \right\} \frac{v^2}{64} = \frac{1.42\, b^2 - c^2}{64\, c^2} \times v^2$ very nearly. (16.)

Let, as is nearly the case with the old London Bridge,

$v = 3\frac{1}{6}$, $b = 926$, $c = 200$,

Hence $f = \frac{1.42\, b^2 - c^2}{64\, c^2} \times v^2 = 0.46 \times 10\frac{1}{36} = 4.73$ feet, or 4 feet $8\frac{3}{4}$ inches by the formula, while that by experiment was 4 feet 9 inches. For a more extensive collection of these formulæ, see Dr Gregory's *Mathematics for Practical Men.*

TO FIND THE TONNAGE OF A SHIP BY LOGARITHMS, ACCORDING TO THE COMMON METHOD.

Rule.—If the vessel is a ship of war, let fall a perpendicular from the fore-side of the stem, at the height of the hause-holes; but if a merchantman, the perpendicular is to be let fall from that part of the fore-side of the stem which is at the same height above the keel as the wing-transom: also let fall another perpendicular from the back of the main post, at the height of the wing-transom. Find the distance between these two perpendiculars, from which subtract three-fifths of the extreme breadth; and also the product of the height of the wing-transom above the upper edge of the keel, by $2\frac{1}{2}$ inches, and the remainder is the length of the keel for tonnage. To the logarithm of which add the logarithm of the breadth, and that of the half-breadth, and the constant logarithm 8.02687;* the sum, rejecting 10 from the index, will be the logarithm of the tonnage required.

Ex.—Let the length between the perpendicular at the fore-part of the stem, and the back of the post, be 100 feet: the extreme breadth $27\frac{1}{2}$ feet, and the height of the wing-transom 15 feet. Required the tonnage?—*Ans.* 321 tons.

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* The arithmetical complement of the logarithm of 94 being the common divisor for finding the tonnage. This method is far from being correct. See papers on *Naval Architecture*, published by Morgan and Creuze. G. B. Whittaker, London, 1826.

TABLES OF SPECIFIC GRAVITY.

SOLIDS.

Platina	20.722	Sapphire, Oriental	3.994
Gold, pure, hammered	19.362	Ditto Brazilian	3.131
Guinea of George III.	17.629	Oriental Topaz	4.019
Tungsten	17.600	Oriental Beryl	3.549
Mercury, at 32° F.	13.598	Diamond from 3.501 to	3.531
Lead	11.352	English Flint Glass	3.329
Palladium	11.300	Tourmalin	3.155
Rhodium	11.000	Asbestus	2.996
Virgin Silver	10.744	Marble, green Campanian	2.742
Shilling of George III.	10.534	———, Parian	2.837
Bismuth, molten	9.822	———, Norwegian	2.728
Copper, wire-drawn	8.878	———, green Egyptian	2.668
Red Copper, molten	8.788	Emerald	2.775
Brass, wire-drawn	8.544	Pearl	2.752
Brass, common	7.824	Chalk, British	2.784
Molybdena	8.611	Jasper	2.710
Arsenic	8.308	Coral	2.680
Nickel, molten	8.279	Rock Crystal	2.653
Uranium	8.109	English Pebble	2.619
Steel from 7.769 to	7.816	Limpid Felspar	2.564
Cobalt, molten	7.812	Glass, green	2.642
Bar Iron	7.788	——, white	2.892
Pure Cornish Tin	7.291	——, bottle	2.733
Ditto hardened	7.299	Porcelain, China	2.385
Cast Iron	7.207	———, Limoges	2.341
Zinc	6.862	Native Sulphur	2.033
Antimony	6.712	Ivory	1.917
Tellurium	6.115	Alabaster	1.874
Chromium	5.900	Alum	1.720
Globe, M. S[t] G.	5.200	Copal, opaque	1.140
Granite (mean)	3.500	Sodium	973
Hornblende	3.000	Oak, heart of	950
Limestone	2.950	Ice	930
Basalt	2.860	Potassium	866
Felspar	2.650	Beech	852
Chalk	2.780	Ash	845
Quartz	2.640	Apple-Tree	793
Slate	2.670	Orange-Wood	705
Stone, common	2.500	Pear-Tree	661
Clay	2.160	Linden-Tree	604
Earth, common	2.000	Cypress	598
Sand	1.500	Cedar	561
Spar, heavy	4.430	Fir	550
Jargon of Ceylon	4.416	Poplar	383
Oriental Ruby	4.283	Cork	240

LIQUIDS.

Sulphuric Acid	1.841	Burgundy Wine	991
Nitrous Acid	1.550	Olive Oil	915
Water from the Dead Sea	1.240	Muriatic Ether	874
Nitric Acid	1.218	Oil of Turpentine	870
Sea-Water	1.026	Liquid Bitumen	848
Milk	1.030	Alcohol, absolute	792
Distilled Water	1.000	Sulphuric Ether	716
Wine of Bourdeaux	944	Air at the Earth's sur. about	1¼

1. Since a cubic foot of water, at the temperature of 40° Fahrenheit, weighs 1000 ounces avoirdupois, or 62½ pounds, the numbers in the preceding tables, omitting the decimal points, exhibit very

nearly the respective weights of a *cubic foot* of the several substances in avoirdupois ounces.

2. If the weight of a body be known in avoirdupois ounces, its weight in Troy ounces will be found in multiplying it into ·91145, And, if the weight be given in Troy ounces, it will be found in avoirdupois by multiplying it into 1.0971.

GASES.

Atmospheric air*	1·0000	Muriatic acid-gas	1·2474
Vapour of hydriotic ether	5·4749	Sulphuretted hydrogen	1·1912
——— oil of turpentine	5·0130	Oxygen-gas	1·1036
Hydriotic acid-gas	4·4430	Nitrous-gas	1·0288
Fluo-silicic acid-gas	3·5735	Olefiant-gas	0·9784
Vapour of sulph. of carbon	2·6447	Azote, or nitrogen-gas	0·9691
——— sulphuric ether	2·5860	Oxide of carbon	0·9569
Chlorine	2·4700	Hydro-cyanic vapour	0·9476
Fluo-boric gas	2·3709	Phosphoretted hydrogen	0·8700
Vapour of muriatic ether	2·2190	Steam of water	0·6235
Sulphurous acid-gas	2·1920	Ammoniacal-gas	0·5967
Cyanogen	1·8064	Carburetted hydrogen	0·5550
Vapour of absolute alcohol	1·6133	Arseniated hydrogen	0·5290
Nitrous oxide	1·5204	Hydrogen-gas	0·0732
Carbonic acid	1·5196		

* Air . . 0·00122 water being = 1, hence Gas S. G. × 0.00122 = S. G. Water = 1.

Specific Gravity of Distilled Water at different Temperatures, that at 62° being taken as Unity.

70°	0·99913	62°	1·00000	54°	1·00064	26°	46°	1·00102	34°
68	0·99936	60	1·00018	52	1·00076	28	44	1·00107	36
66	0·99958	58	1·00035	50	1·00087	30	42	1·00111	38
64	0·99980	56	1·00050	48	1·00095	32	40	1·00113	40

MISCELLANEOUS COMPUTATIONS AND EXPERIMENTS.

The pendulum vibrating seconds of mean solar time at London in a vacuum, and reduced to the level of the sea, is 39·1393 inches; consequently the descent of a heavy body from rest in one second of time, in a vacuum, will be 193·145 inches. The logarithm 2·2858835.

A platina metre at the temperature of 32°, supposed to be the ten millionth part of the quadrant of the meridian, 39·3708 inches. The ratio to the imperial measure of three feet, as 1·09363 to 1, the logarithm 0.0388717.

The following standards, accurately measured, give these results:—

General Lambton's scale, used in the Trigonometrical Survey of India, 35·99934 inches.

Sir G. Shuckburgh's scale (which, for all purposes, may be considered as identical with the imperial standard)	35·99998
Gen. Roy's scale	36·00088
Royal Society's standard	36·00135
Ramsden's bar	36·00249

Weight of a cubic inch of distilled water in a vacuum at the temp. 62° Fahrenheit, bar. 30 inches, as opposed to brass weights in a vacuum also, 252·766 grains	log 2.4027186
Consequently a cubic foot 62.3971 pounds avoirdupois in vacuo	log 1.7951644
Weight of a cubic inch of distilled water in air at 62° of temperature, height of the barometer 30 inches, 252.458 grains	log 2.4021892
Consequently a cubic foot 62.3211 pounds avoirdupois	log 1.7946351
And an ounce of water 1.732966 cubic inches	log 0.2387886
Cubic inches in the imperial gallon, 277.273844	log 2.4429092
Diameter of the cylinder containing a gallon at one inch high, 18.78926	log 1.2739096

SPECIFIC GRAVITY OF DRY AND SATURATED AIR.

That at 30 in Bar., and 32° Fahr. being 1.

Temp. Fahr.	Specific Grav. of Dry Air.	Specific Grav. of Saturat. Air.	Temp. Fahr.	Specific Grav. of Dry Air.	Specific Grav. of Saturat. Air.
32°	1.00000	0.99750	67°	0.93996	0.93164
33	0.99824	0.99568	68	0.93829	0.92968
34	0.99647	0.99385	69	0.93664	0.92772
35	0.99471	0.99203	70	0.93499	0.82576
36	0.99294	0.99021	71	0.93333	0.92380
37	0.99119	0.98839	72	0.93168	0.92184
38	0.98944	0.98654	73	0.93004	0.91988
39	0.98769	0.98470	74	0.92839	0.91792
40	0.98595	0.98286	75	0.92675	0.91596
41	0.98420	0.98101	76	0.92511	0.91400
42	0.98246	0.97917	77	0.92347	0.91203
43	0.98073	0.97731	78	0.92184	0.91005
44	0.97900	0.97545	79	0.92021	0.90811
45	0.97726	0.97358	80	0.91859	0.90609
46	0.97553	0.97172	81	0.91656	0.90411
47	0.97381	0.96986	82	0.91534	0.90213
48	0.97209	0.96798	83	0.91373	0.90013
49	0.97038	0.96610	84	0.91211	0.89814
50	0.96866	0.96421	85	0.91050	0.89615
51	0.96695	0.96233	86	0.90889	0.89415
52	0.96524	0.96045	87	0.90728	0.89216
53	0.96354	0.95855	88	0.90567	0.89014
54	0.96183	0.95665	89	0.90408	0.88813
55	0.96013	0.95475	90	0.90248	0.88611
56	0.95843	0.95285	91	0.90089	0.88410
57	0.95674	0.95095	92	0.89929	0.88208
58	0.95504	0.94902	93	0.89770	0.88006
59	0.95336	0.94710	94	0.89612	0.87803
60	0.95168	0.94518	95	0.89453	0.87602
61	0.94999	0.94326	96	0.89295	0.87401
62	0.94831	0.94134	97	0.89137	0.87199
63	0.94664	0.93940	98	0.88979	0.86995
64	0.94496	0.93746	99	0.88821	0.86790
65	0.94329	0.93552	100	0.88664	0.86585
66	0.94162	0.93338	110	0.87110	0.84329

On this subject see Biot's *Traité de Physique*, vol. I., ch. xix.

EXPANSIONS OF SOLIDS AND LIQUIDS AT DIFFERENT TEMPERATURES, FROM 32° TO 212° Fah.

	Means.
Glass tube, linear,	1.000822
Plate glass,	1.000878
Deal,	1.000808
Platina,	1.000911
Cast iron,	1.001110
Steel,	1.001213
Iron,	1.001249
Gold,	1.001458
Copper,	1.001796
Brass,	1.001873
Silver,	1.002002
Tin,	1.002372
Lead,	1.002858
Zinc,	1.002976
Mercury, volume,	1.018100
Water,	1.044660
Alcohol,	1.105000
Fixed Oils,	1.075000

TABLE FOR COMPUTING THE FLEXIBILITY AND STRENGTH OF TIMBER.*

Name of the kind of Wood.	Spec. Grav.	Value of U.	Value of E.	Value of S.	Value of S'.	Value of C.
Teak	745	818	9657802	2462	2488	15550
Poon	579	596	6759200	2221	2266	14787
Eng. Oak	969	598	3494730	1181	1205	9836
Do. Spec. 2.	934	435	5806200	1672	1736	10853
Canadian Oak	872	588	8595864	1766	1803	11428
Dantzic Oak	756	724	4765750	1457	1477	7386
Adriatic Oak	993	610	3885700	1583	1409	8808
Ash	760	395	6580750	2026	2124	17337
Beech	696	615	5417266	1556	1586	9912
Elm	553	509	2799347	1013	1042	5767
Pitch Pine	660	588	4900466	1632	1666	10415
Red Pine	657	605	7359700	1341	1368	10000
New Eng. Fir	553	757	5967400	1102	1116	9947
Riga Fir	753	588	5314570	1108	1131	10707
Do. Spec. 2.	738	—	3962800	1051	1081	—
Mar Forest Fir	696	588	2581400	1144	1168	9539
Do. Spec. 2.	693	403	3478328	1262	1310	10691
Larch	531	411	2465433	653	890	—
Do. Spec. 2.	522	518	3591133	832	850	—
Do. Spec. 3.	556	518	4210830	1127	1149	7655
Do. Spec. 4.	560	518	4210830	1149	1172	7352
Norway Spar	577	648	5832000	1474	1492	12180

* From Barlow on the Strength of Timber.

SOLUTION OF PRACTICAL PROBLEMS, FROM THE PRECEDING DATA.

Prob. I.—*To find the Strength of Direct Cohesion of a Piece of Timber of any given Dimensions.*

Rule.—Multiply the area of the transverse section, in inches, by the value of C, in the preceding table of data, and the product will be the strength required.

Note.—If the specific gravity be not the same as the mean tabular specific gravity; say, as the latter is to the former, so is the above product to the correct result.

Ex.—What weight will it require to tear asunder a piece of teak 3 inches square, the specific gravity being 745?—*Ans.* 139950 lbs.

Prob. II.—*To compute the Deflection of Beams fixed at one End and loaded at the other with any given Weight.*

Rule 1.—Multiply the tabular value of E by the breadth and cube of the depth of the given beam, both in inches.

2.—Multiply also the cube of the length in inches by the given weight, and that product again by 32.*

3.—Divide the latter product by the former, for the deflection sought.

Ex.—An ash batten, 3 inches square, is fixed in a wall, and projects from it 4 feet. If a weight of 200 lbs. be hung on its extremity, how much will it be deflected?—*Ans.* $1\frac{1}{3}$ inches.

Note.—The same rule will apply, when the weight is distributed throughout the length, by multiplying the second product by 12 instead of 32.

Prob. III.—*To compute the Deflection of Beams supported at each End, and loaded in the Middle with any given Weight.*

Rule 1.—Multiply the tabular value of E by the breadth and cube of the depth, both in inches.

2.—Multiply also the cube of the length, in inches, by the given weight in lbs.; then divide the latter product by the former for the deflection sought.

Ex.—A square beam of English oak, whose side is 6 inches, is supported on two walls, 20 feet distant, and is to be loaded at its middle point with 1000 lbs., what will it be deflected?—*Ans.* 1·8 inch.

Note.—If the beam be *fixed* at each end, the deflection will, with equal weights, be two-thirds of that found by the above rule.

Prob. IV.—*To compute the Deflection of Beams supported at each End, and loaded uniformly throughout their Length with a given Weight.*

Rule.—Compute the deflection the same as in the last problem. Multiply that result by 5, and divide the product by 8, and the quotient will be the answer.

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* Mr Bevan says this should be 16, and Mr Barlow that it is 32 in the usual method of fixing beams in ordinary erections.

Ex.—A uniform bar of Adriatic oak, 2 inches square, is rested upon two props, distant 24 feet, how much will it be deflected by its own weight, its specific gravity being 960, or 60 lbs. to the cubic foot?—*Ans.* $9\frac{1}{2}$ inches.

PROB. V.—*To compute the ultimate Deflection of Beams or Rods before their Rupture.*

Note.—The beams are supposed to be supported at each end.

Rule.—Multiply the tabular value of U, in the preceding table of data, by the depth of the beam in inches, and divide the square of the length, also in inches, by that product, for the ultimate deflection sought.

Ex.—A square inch rod of ash, 6 feet long, is broken by a weight applied to its centre: how much will it be deflected before it breaks? *Ans.* 13·1 inches.

PROB. VI.—*To find the ultimate transverse Strength of any rectangular Beam of Timber, fixed at one End and loaded at the other.*

Rule I.—Multiply the value of S, in the preceding table of data, by the breadth and square of the depth, both in inches, and divide that product by the length, also in inches, and the quotient will be the weight in lbs. This is approximative.

Rule II.—1. Take the ultimate deflection 8 times that of the last problem, and divide the deflection by the length, which will give the sine of the angle; whence by a table find the secant.

2. Multiply the secant by the breadth and square of the depth in inches, and the product again by the value of S′ in the table of data.

3. Divide this last product by the length in inches, and the quotient will be the answer in lbs.

Ex. 1.—What weight will it require to break a piece of Mar forest fir, fixed by one end in a wall, and loaded at the other; the breadth being 2 inches, depth 3 inches, and length 4 feet?—*Ans.* 518 lbs.

PROB. VII:—*To compute the ultimate transverse Strength of any rectangular Beam, when supported at both Ends and loaded in the Centre.*

Rule I.—Multiply the tabular value of S by 4 times the breadth and square of the depth in inches, and divide that product by the length, also in inches, for the weight.

Rule II.—1. Compute the ultimate deflection by Prob. V.; square that deflection, and divide it by the square of half the length of the beam, and add the quotient to 1, for the square of the secant of deflection; which multiply by the length in inches.

1. Multiply the tabular value of S′ by 4 times the breadth, and the square of the depth; and divide that product by the former answer in lbs.

Ex.—What weight will be necessary to break a piece of larch similar to the third specimen, the length being 8 feet 4 inches, the breadth 8 inches, and depth 10 inches; being supported at each end, and loaded in the middle?—*Ans.* 36676 lbs.

For farther information on this subject, the works of Barlow, Bevan, Tredgold, Farey, Young, Gregory, (Mathematics for Practical Men in particular), Playfair, Leslie, Robison, Hutton, Smeaton, Rennie, &c., may be consulted.

GUNNERY.

I.—*Theorems relative to Projectiles on the Horizontal Plane.*

Let s denote the sine, S the sine
c the cosine, v the versed sine of twice the angle of elevation
t the tangent of the angle of elevation
R the horizontal range
T the time of flight
H the greatest height of the projectile
$g = 32.2$ feet
a the altitude or impetus, as it is commonly called, due to V the velocity; then

1. $R = 2aS = 4asc = \frac{SV^2}{g} = \frac{scV^2}{\frac{1}{2}g} = \frac{\frac{1}{2}gcT^2}{s} = \frac{\frac{1}{2}gT^2}{t} = \frac{4H}{t}$

2. $V = \sqrt{2ag} = \sqrt{\frac{gR}{s}} = \sqrt{\frac{gR}{2sc}} = \frac{\frac{1}{2}gT}{s} = \frac{2}{s}\sqrt{\frac{1}{2}gH}$

3. $T = \frac{sV}{\frac{1}{2}g} = 2s\sqrt{\frac{a}{\frac{1}{2}g}} = \sqrt{\frac{tR}{\frac{1}{2}g}} = \frac{sR}{\frac{1}{2}gc} = 2\sqrt{\frac{H}{\frac{1}{2}g}}$

4. $H = as^2 = \frac{1}{2}av = \frac{1}{4}tR = \frac{sR}{4c} = \frac{s^2V^2}{2g} = \frac{vV^2}{4g} = \frac{1}{8}gT^2$

II.—*Theorems relative to Projectiles on Oblique Planes.*

Let c denote the cosine of the direction above the horizon
C the cosine of the inclination of the plane to the horizon
s the sine of the direction above the plane
R the range on the oblique plane
T the time of flight
V the projectile velocity
H the greatest height above the plane
a the impetus or altitude due to the velocity V
$g = 32.2$ feet; then

1. $R = \frac{cs}{C^2} \times 4a = \frac{2cs}{C^2g}V^2 = \frac{gc}{2s}T^2 = \frac{4c}{s}H$

2. $H = \frac{s^2}{C^2}a = \frac{s^2V^2}{2gC^2} = \frac{sR}{4c} = \frac{g}{8}T^2$

3. $V = \sqrt{2ag} = C\sqrt{\frac{gR}{2cs}} = \frac{gc}{2s}T = \frac{2c}{s}\sqrt{\frac{1}{2}gH}$

4. $T = \frac{2s}{C}\sqrt{\frac{a}{\frac{1}{2}g}} = \frac{sV}{\frac{1}{2}gC} = \sqrt{\frac{sR}{\frac{1}{2}gc}} = 2\sqrt{\frac{H}{\frac{1}{2}g}}$

III.—*To find the Velocity of any Shot or Shell.*

Let v denote the velocity, B the weight of the ball or shot, and C the weight of the charge of powder; then

1. $v = 1600\sqrt{\frac{2C}{B}} = 2260\sqrt{\frac{C}{B}}$ by Hutton's experiments.

2. $v = 1600 \sqrt{\frac{3C}{B}} = 2770 \sqrt{\frac{C}{B}}$ by Gregory's on better powder.

3. $v = 2500 \sqrt{\frac{C}{B}}$ = the mean of these two, which may be considered good in usual practice.

IV.—*Given the Range at one Elevation to find the Range at another Elevation.*

Rule.—As the sine of twice the first elevation is to the sine of twice the second elevation, so is the first range to the second range.

V.—*Given the Range for one Charge, to find the Range for another Charge.*

The ranges are directly as their charges, the elevation being the same; or as one range is to another range, so is the charge corresponding to the first range to the charge corresponding to the second range.

Example 1.—If a ball of 1 lb. be projected with a velocity of 1600 feet per second, when fired with a charge of 8 ounces of powder, with what velocity will each of the several kinds of shells be discharged by the full charges of powder?

Diam. of Shells.	Weight.	Charge.	Velocity in Feet.	
Inches.	Pounds.	Pounds.	III. 1.	III. 3.
13	196	9	485	536
10	90	4	477	527
8	48	2	462	501
$5\frac{1}{2}$	16	1	566	625
$4\frac{2}{5}$	8	$0\frac{1}{2}$	566	625

Ex. 2.—If a shell range 1000 yards when discharged at an elevation of 45°, what will be its range when the elevation is 30° 16′ with the same charge of powder?

Ans. 871 yards, or 2612 feet.

Ex. 3.—At an elevation of 45° the range of a shell was 3750 feet, to what angle must the piece be elevated to strike an object at the distance of 2810 feet?

Ans. 24° 16′, or 65° 44′, equally distant from 45°.

Ex. 4.—With what impetus, velocity, and charge of powder must a 13-inch shell be fired, at an elevation of 32° 12′, to strike an object at the distance of 3250 feet?

Ans. impetus 1802, velocity 340 feet, charge 4 lb. $7\frac{1}{2}$ oz.

Ex. 5.—A shell ranges 3500 feet, when discharged at an elevation of 25° 12′, how far will it range at an elevation of 36° 15′ with the same charge of powder? *Ans.* 4332 feet.

Ex. 6.—If, with a charge of 9 lb. of powder, a shell range 4000 feet, what charge will throw it 3000 feet at an elevation of 45° in both cases? *Ans.* $6\frac{3}{4}$ lb. of powder.

Ex. 7.—What will be the time of flight for the greatest range at the elevation of 45°?

Ans. The time in seconds is one-fourth of the square root of the range in feet.

Ex. 8.—In what time will a shell range 3250 feet at an elevation of 32°? *Ans*. $11\frac{1}{4}$ seconds nearly.

Ex. 9.—How far will a shot range on a plane which ascends 8° 15′, and on another which descends 8° 15′, the impetus being 3000 feet?

Ans. 4244 feet on the ascending plane,
and 6745 feet on the descending plane.

Ex. 10.—How much powder will throw a 13-inch shell 4244 feet on an inclined plane which ascends at an angle of 8° 15′, the elevation of the mortar being 32° 30′?

Ans. 7.38 lb., or 7 lb. 6 oz. nearly.

Ex. 11.—At what elevation must a 13-inch mortar be pointed to range 6745 feet on a plane descending at an angle of 8° 15′ with a charge of $7\frac{3}{8}$ lb. of powder? *Ans*. 32° $41\frac{1}{2}$′.

Ex. 12.—In what time will a 13-inch shell strike a plane which rises 8° 30′, at an elevation of 45°, and discharged with an impetus of 2304 feet? *Ans*. $14\frac{2}{5}$ seconds.

For more complete information on this subject, see Hutton's Course of Mathematics.

EXPLANATION OF THE TABLES.

TABLE I.—*The Miles and Parts of a Mile in a Degree of Longitude at every Degree of Latitude, supposing the Earth to be a Sphere.*

THE first column of this table contains degrees of latitude, the second the miles and hundredth parts of a mile in a corresponding degree of longitude,—of these the remaining columns are a continuation. If the given latitude consists of degrees and minutes, a proportional part of the difference between two contiguous degrees, the one greater and the other less than the given latitude, must be applied to the miles, &c. corresponding to either of the adjacent degrees, by addition or subtraction, according as it is greater or less than the given latitude.

Example 1.—Required the number of miles in a degree of longitude at the Isle of May, in latitude 56° 11′ 22″ N.?

Miles in a degree of longitude in latitude 56° = 33.55
in latitude 57 = 32.68

Difference 87

Then 60′ : 11′ 22″ : : ·87 : ·165, which, subtracted from 33.55, gives 33.385, the measure of a degree of longitude in latitude 56° 11′ 22″.

Ex. 2.—Suppose the error of a chronometer, allowing for rate, to be half a minute, after a voyage from Leith to the West Indies and back, how many geographical miles would that amount to at the mouth of the frith of Forth, near the Isle of May?

Since 1° of longitude is equal to four minutes of time, then half a minute will be the eighth part of a degree, and ⅛ of 33.385 = 4.178, or about 4¼ miles.

Ex. 3.—What is the distance in geographical and nautical miles between Stockholm in longitude about 18° E., and Petersburgh in longitude 30° E., the common latitude being 60° N. nearly?

$30° - 18° = 12°$, and $12 \times 30 = 360$ miles nearly, since at 60 one degree is 30 miles.

TABLE II.—*Logarithms of Numbers.*—Part I. contains the logarithms of all numbers from 1 to 100, inclusive, with their proper indices prefixed. Part II. contains the decimal part of the logarithms of all numbers from 100 to 10,000, without their indices. The indices are easily supplied by the computist, being always one unit less than the number of integers in the given natural number. The index of the logarithm of a number in which there are any integers is always positive; but, if the number be properly a fraction, the index is negative, usually marked by the sign — either

before, or more generally above the index. If the first effective figure of the decimal fraction be adjacent to the decimal point, the index is $\bar{1}$; if there be one cipher between them, the index is $\bar{2}$; if two ciphers, the index is $\bar{3}$; and, in general, the number denoting the place of the first significant figure from the decimal point will be the negative index. Instead of negative indices, their arithmetical complements are frequently used, especially by those unacquainted with the first principles of Algebra.

The decimal parts of the logarithms of numbers consisting of the same figures are the same whether the number be integral, fractional, or mixed, which may be illustrated as follows:—

Numbers	Logarithms
546800	5.737829
54680	4.737829
5468	3.737829
546.8	2.737829
54.68	1.737829
5.468	0.737829
0.5468	$\bar{1}$.737829, or 9.737829
0.05468	$\bar{2}$.737829, or 8.737829
0.005468	$\bar{3}$.737829, or 7.737829
0.0005468	$\bar{4}$.737829, or 6.737829

Problem I.—*To find the Logarithm of any given Number.*

Rule.—If the given number be under 100, its logarithm is found in the first page of the table immediately opposite to it.

If the number consist of three figures, find it in the first column of the following or second part of the table, opposite to which, and under or above 0, is its logarithm.

If the given number contains four figures, the three first are to be found, as before, in the side-column; and under the fourth at the top, or above it at the bottom, will be found the logarithm required. To this prefix the proper index, and the whole is completed.

If the given number exceeds four figures, find the difference between the logarithm answering to the first four figures of the given number, and the next immediately following; multiply this difference by the remaining figures in the given number, point off as many figures to the right hand as there are in the multiplier, and the remainder added to the logarithm, answering to the first four figures, will be the logarithm required nearly. The logarithm of a vulgar fraction is found by subtracting the logarithm of the denominator from that of the numerator; and that of a mixed quantity is found by reducing it to an improper fraction, and proceeding as before; or the vulgar fractions may be reduced to decimals, and the logarithms found as usual.

To find the logarithm of a number exceeding unity by a small fraction, formula (3), page 5th, must be employed. $\text{Log}(1+n)=\text{M}(n-\frac{n^2}{2}\ \&c.)=\text{M}n$ very nearly when n is a small fraction; for in that case $\frac{n^2}{2}$ becomes insensible.

Ex. 1.—What is the logarithm of 56?

In the first part of the table, opposite to 56, and under N, is 1.748188.

Ex. 2.—What is the logarithm of 366?

In the second part of the table, opposite to 366, and under 0, is 2.563481, supplying the index. The first two figures are understood to be supplied in the blank space, till the change takes place at 57; and this must be attended to throughout the whole of this table, as well as several others that follow.

Ex. 3.—Required the logarithm of 7854?

Opposite to 785, and under 4 is 3.895091.

Ex. 4.—Required the logarithm of 100176?

The log of 1001 is 000434
1002 is 000868

The difference is 434

Then 434 × 76 is 32984. From this cut off two figures, because the difference has been multiplied by two figures, 76, and it becomes 329.84. If the figure next the decimal point is less than 5, the whole may be rejected; but if greater, increase the figure before the point by unity, and consequently, in the present case, 329.84 would become 330. Whence to 000434

Add 330, and supply the index 330

And the log of 100176 will be 5.000764

In general the difference may be taken from the right-hand column, under D, unless the logarithms vary very rapidly, which happens only near the commencement of the table, as in the preceding example, where the difference under D is 432, the mean difference of the whole line, instead of 434 by actual subtraction. This would cause a difference of two units, in the last decimal place, less than that found above, or the logarithm would turn out to be 5.000762, instead of 5.000764.

To facilitate the method of obtaining proportional parts, there has been added to these tables an additional column on the left-hand side of the page, under P. P. In the column under N, the two first figures are omitted, and the third alone retained, by which means a regular series of the arithmetical digits, beginning with 1 and ending with 9, are obtained between each bar, or line across the page. Hence the proportional parts corresponding to the mean difference within the space marked out by each pair of cross bars, answering to any of the nine digits, can be placed opposite to each, which, in these tables, has been accordingly done. By this means the logarithm corresponding to any number extended to five or six places of figures, may be very readily obtained with sufficient accuracy, excepting, perhaps, when it falls in the second and third pages, where the differences vary rapidly.

Ex. 5.—Required the logarithm of 546876?

Log of 546800 is	5.737829	, or	5.737829
Prop. part for 70	56	, or	56
——— for 6	48	, or	5
Log of 546876 is	5.7378898	, or	5.737890

If the number consists of one figure more than four, or five figures altogether, the proportional part may be added at sight.*

* When there is wanted one figure only by prop. parts, they may be added at sight, by keeping the forefinger of the left hand at the log corresponding to the four first figures, and the little finger at the proportional parts for the fifth, and thus the table may be said to extend to 100,000.

Ex. 6.—Required the log of $\frac{15}{17}$?

Log of 15 is	1.176091
17 is	1.230449
Log of $\frac{15}{17}$ is therefore . . .	$\bar{1}.945642$ or 9.945642

Ex. 7.—Required the logarithm of $\frac{27600}{27599} = 1 + \frac{1}{27599} = 1 + \frac{1}{27600}$ nearly. Hence $M \times n = \frac{0.43429445}{27600} = \frac{1}{63551}$.

See Hutton's Course of Mathematics, vol. II., p. 262.

Required the log of $7\frac{5}{8}$, or $\frac{61}{8}$, or 7.625?

Log of 7.625 is 0.882240

Required the logarithms of 24, 56, 102, 546, ·7854, 78653, 54.4768, 97685.46, 0.001546, 0.176804, 0.00043689, $3\frac{1}{7}$, $\frac{555}{113}$, $768\frac{1}{4}$, $4857\frac{13}{13}$, $39766\frac{356}{748}$, $8546\frac{1}{16}$?

Problem II.—*To find the Number answering to any given Logarithm.*

Find the logarithm next less than that given in the column marked 0 at the top, and continue the sight along that horizontal line till a logarithm the same as that given, or as near as possible, be found; then the three first figures of the corresponding natural number will be found opposite to it in the side-column, and the fourth immediately above at the top or below at the bottom of the page. If the index of the given logarithm be 3, the *four* figures thus found are integers; if the index be 2, the three first figures are integers, and the fourth is a decimal, and so on, as may be easily understood by consulting Problem I. If the given logarithm cannot be exactly found in the table, and if more than four figures be wanted in the corresponding natural number, then find the difference between the given and the next less logarithm. To this annex on the right hand as many ciphers as there are figures required above four in the natural number. Divide the whole by the difference between the next less and next greater logarithm, and the quotient annexed to the four figures formerly found will be the natural number required. The same thing may be done by the table P. P., by subtracting a part corresponding to each unit from the difference between the given logarithm and the next less, and annexing these units successively in order to the number previously found.

Ex. 1.—Required the natural number corresponding to the logarithm 2.495544?

This logarithm is found opposite to 313 and under 0, and, as the index is 2, then 313 is the number required.

Ex. 2.—What is the number answering to the logarithm 3.828338?

The logarithm is found 673, and under 5, therefore, since the index is three, the natural number is 6735. If the index had been 2, then it would have been 673.5, or the natural number must always consist of one integer (if there are integers) more than the index expresses.

Ex. 3.—Required the natural number answering to the logarithm $\bar{2}.627980$?

The natural number corresponding to this is 4246; but the index being $\bar{2}$, one cipher must be prefixed, from what has been said in Prob. I., and it becomes 0.04246.

Ex. 4.—What is the number answering to the logarithm 5.687956?
The nearest less logarithm than this is 687886, corresponding to which will be found the number 4874. The difference between 687956 and 687886 is 70, to this annex two ciphers, and it becomes 7000, which being divided by 89, the difference of the columns found under D, gives 79. This being subjoined to 4874, gives 487479, the number required. Or the same may be performed thus:—

	Original log	5.687956
487400	corresponds to . . .	5.687886
	Diff. in P. P.	70
gives . . 70	for	63
	remainder as diff.	7
gives . . 8	for	72

72

or in all 487478, differing only one unit in the last place from the former number.

LOGARITHMIC ARITHMETIC.

Problem III.—*To perform Multiplication by Logarithms.*

Rule.—Add the logarithms of the factors, and the sum is the logarithm of the product.

If there are both negative and affirmative indices, their sum is taken according to the rules of algebra ; or the arithmetical complements of the negative indices may be used, rejecting the tens in their sum.

The arithmetical complement of the logarithm of any number is found by subtracting the given logarithm from 10, or by subtracting each of its figures, beginning at the left hand from 9, and the last effective figure from 10. When the arithmetical complement of the index alone is wanted, it is found by subtracting it from 10.

Ex. 1.—Multiply 6564 by 836.

Factors {	6564 logarithm	3.817169
	836 logarithm . . .	2.922206
	sum	6.739375
	5487000 corresponds to . .	6.739335
	diff. in P. P.	40
gives	500 for	40

or in all 5487500, which agrees as nearly with the real product 5487504, as tables extending to six places of decimals will give.

Ex. 2.—Multiply the numbers 43.68, 0.534, and 0.007685 together logarithmically.

Factors {	43.68	log 1.640283, or	1.640283
	0.534	log $\bar{1}$.727541 —	9.727541
	0.007685	log $\bar{3}$.885644 —	7.885644
Product	0.179254	$\bar{1}$.253468	9.253468

Problem IV.—*To perform Division by Logarithms.*

Rule.—From the logarithm of the dividend subtract the logarithm of the divisor, the remainder is the logarithm of the quotient.

Ex. 1.—Divide 5486 by 96.

Dividend	5486	log	3.739256
Divisor	96	log	1.982271
Quotient	57.146		1.756985
			40
			45

Ex. 2.—Divide 0.07856 by 0.003482.

Dividend 0.07856	. .	log $\bar{2}.895201$
Divisor 0.003482	. .	log $\bar{3}.541829$
Quotient 22.5617		1.353372
		39
		33
		19
		14

PROBLEM V.—*To perform Proportion by Logarithms.*

RULE.—From the sum of the logarithms of the second and third terms subtract the logarithm of the first term; the remainder will be the logarithm of the answer. Or, instead of subtracting the logarithm of the first term, its arithmetical complement may be added to the other two, which, in many cases, is more convenient.

The ar. co. is found by taking the log from 10,000000, or, beginning at the left hand, taking each figure mentally from 9, and the last significant figure from 10. In Table V. the ar. co. of the sine is the cosecant rejecting 10 in the index, the ar. co. of the tangent is the cotangent rejecting 10 in the index, or, generally, the ar. co. of any of these is found adjacent to it within the strong lines on each side of them; and by an expert calculator this method will always be preferred.

Ex.—A merchantman distant twenty miles, going at the rate of 5 knots or miles an hour, is pursued by a privateer, sailing at the rate of 7 miles; after three hours' chase the breeze freshened, the merchantman's rate was increased to 6 knots, and the privateer's to 10; in what time will the privateer come up with the merchantman?

As the privateer gained 2 miles an hour on the merchantman at the end of the first 3 hours, the distance between them is obviously 14 miles. During the remainder of the chase the hourly gain of the privateer was 4 knots. Hence,

As the hourly gain	4^m ar. co. log	9.397940
Is to the distance	14^m log	1.146128
So is	1^h log	0.000000
To the time required	$3^h.5$ or 3^h 30^m	0.544068

Consequently, from the time the breeze freshened, the privateer would come up with the merchantman in three hours and a half, or in six hours and a half from the commencement of the chase.

PROBLEM VI.—*To perform Involution by Logarithms.*

RULE.—Multiply the logarithm of the given number by the index of the power, and the product will be the logarithm of the power required.

Ex. 1.—What is the square of 64?

Given number 64	log	1.806180
Index of the power		2
Square 4096		3.612360

Ex. 2.—What is the third power of 24?

Given number 24	log	1.380211
Index of the given power		3
Third power 13824		4.140633
		508
		125

Ex. 3.—Required the fourth power of 0.05?

Log 0.05	$\bar{2}.698970$
	4
	$\overset{+}{2}.795880$
	$\bar{8}$
Fourth power = 0.00000625	$\bar{6}.795880$

Problem VII.—*To perform Evolution by Logarithms.*

Rule.—Divide the logarithm of the given number by the index of the root, supposed to be expressed by an integer, as, for example, the square root by 2, the cube root by 3, and the quotient will be the logarithm of the root.

If the given number be a decimal, and the arithmetical complement of the negative index be used, then prefix 1 to that index for the square root, 2 for the cube root, 3 for the fourth root, &c.

If the index of the root be expressed by a fraction of which the numerator is not unity, then multiply the logarithm of the given number by the numerator, and divide it by the denominator of that index.

Ex. 1.—What is the square root of 1296?

Given number	1296	log	3.112605
Square root	36		1.556302

Ex. 2.—Required the cube root of 0.0009261?

Given number	0.009261	log $\bar{3}.966658$, or 7.966658
Cube root	0.21	$\bar{1}.322219$, or 9.322219

What is the fourth root of 0.00007634?

Given number 0.00007634 log	$\bar{5}.882752$
Given index	$\frac{1}{4}$
Log of the root 0.0934734	$\bar{2}.970688$

In this example, because the index of the root 4 is not contained in the negative index 5 a certain number of times exactly, the logarithm $\bar{5}.882752$ is resolved into its equivalent $\bar{8} + 3.882752$, and the product of this by $\frac{1}{4}$ is $\bar{2}.970688$ the logarithm of the root required.

TABLE III.—*The Angles which every Point and Quarter-Point of the Compass makes with the Meridian.*

This table is useful for reducing the points of the mariner's com-

pass to degrees, and conversely. It is divided into seven columns; in the two first and two last columns are contained the names of the several points; the third and fifth contain the corresponding points and quarter-points reckoned from the meridian; and the fourth the degrees, minutes, and seconds, answering to them. Its use is obvious.

TABLE IV.—*Logarithmic Sines, Tangents, and Secants, to every Point and Quarter-Point of the Compass.*

In performing calculations relative to navigation, it will be found convenient to take the logarithmic sines, tangents, and secants, from this table, thereby saving the trouble of reducing them to degrees, &c., by the preceding table. The manner of using it is easy, and will be readily understood from the explanation of the table which immediately follows.

TABLE V.—*Logarithmic Sines, Tangents, and Secants.*

This table contains the logarithms of the natural sines, tangents, and secants, to each degree and minute of the quadrant in the usual manner. To facilitate calculations in which time is involved, the degrees and minutes have been converted into time at the rate of 15° to an hour, and annexed at the top and bottom of the page and in two additional side-columns.* These, together with proportional parts to each second of time, or to every fifteen seconds of a degree, at the bottom of each page, will, it is hoped, render this table still more easy and general in its use than those of a similar kind usually given.

The degrees are numbered at the top of the table, in a direct order, from 0° to 45°, and, at the bottom of the table, in a retrograde order, from 45° to 90°. The minutes are contained in two of the marginal columns. The minutes in the left-hand column belong to the degree at the top of the page, and those in the right-hand column belong to the degree at the bottom. In like manner, the minutes and seconds of time in the first left-hand column belong to the hour at the top, and those in the right-hand column belong to the hour at the bottom. To promote perspicuity, it is recommended to mark minutes and seconds of the circle always by accents, and those of time by *m* and *s*, as is done in the tables.

PROBLEM I.—*To find the Sine, Cosine, &c. answering to any given Degree or Minute.*

RULE.—Find the given degrees at the top of the page if less than 45°, and the minutes in the left-hand column; opposite to which, and under the word sine, cosine, &c. is the number required. But if the given degrees be greater than 45° and less than 90°, find them at the bottom, and the required sine, cosine, &c. will be found above the word sine, cosine, &c. opposite to the given number of minutes in the right-hand column. If the given arc exceed 90°, find the sine, cosine, &c. of its supplement, or, which comes to the same thing, and will be more easy in practice, to find the *sine* of an arc above 90°, reject 90°, and take the *cosine* of the remainder. To find the *cosine* of an arc above 90° reject 90°, and take the *sine* of the

* This table will therefore convert degrees into time, and conversely.

remainder. The same method may be pursued for the tangents and secants both for arcs and time, recollecting that 90° corresponds to 6^h.

Ex. 1.—Required the log sine of 23° 28′?

Under the word sine in the page marked 23° on the top, and opposite to 28′ in the left-hand column, is 9.600118, the sine required.

Ex. 2.—What is the cotangent of 55° 57′?

In the page marked 55°, at the bottom, and opposite 57′ in the right-hand side-column, is 9.829805, the cotangent of 55° 57′.

Ex. 3.—Required the secant of 125° 40′?

The supplement of 125° 40′ is 54° 20′, the secant of which is 10.234280, or, which comes to the same thing, the cosecant of 35° 40′ the excess of 125° 40′ above 90° is 10.234280, the secant required. Hitherto the given arc has been supposed not to exceed 180°; but, in several astronomical calculations, it frequently happens that arcs through the whole circle are employed; consequently, if the arc lie between 180° and 270°, diminish it by 180°; if between 270° and 360°, take its explement to 360°, and take the logarithmic sines, &c. as before. Otherwise, for the log *sine*, &c. of an arc between 270° and 360°, take the log *cosine*, &c. of its excess above 270°, and for the log *cosine*, &c. of an arc between 270° and 360°, let the *sine*, &c. of its excess above 270° be taken. And for the log *sine*, &c. of an arc between 180° and 270° let the log *sine* of its excess above 180° be taken. Thus the log sine of 300° 28′ is the log sine, &c. of 30° 28′, the excess above 270°; and the log sine of 220° 18′ is the same as that of 40° 18′, and so on. The same may be done when time is employed, recollecting that 6^h corresponds to 90°, 12^h to 180°, 18^h to 270°, and 24^h to 360°.

Problem II.—*To find the Sine, Tangent, &c. of an Arc expressed in Degrees, Minutes, and Seconds.*

Rule.—Find the sine, tangent, &c. corresponding to the given degree and minute, and also that answering to the next greater minute, multiply the difference between them by the given number of seconds, and divide the product by 60; then the quotient added to the sine, tangent, &c. of the given degree and minute, or subtracted from the cosine, cotangent, &c. will give the quantity required nearly. To facilitate this process the difference, to 100″, has been given in the column marked D. Multiply this difference by the number of seconds, cut off two figures from the right, and add the remainder to the sine, tangent, &c. of the given degree and minute, or subtract it from the cosine, &c., and the quantity required will be obtained nearly.

When the powers or roots of sines, &c., are required, multiply the sine, &c. of the arc by the index.

It may be observed, that the difference *opposite the minute*, in the side-column, strictly speaking, must be taken to find the P. P. for seconds when the degrees are found at the top of the page, but the Diff. next above the minute when the degrees are found at the bottom of the page, though this nicety need only be attended to when great accuracy is required, and the differences vary rapidly.

Ex. 1.—Required the log sine of 23° 27′ 40″?

Log sine of 23° 27′ is		9.599827
23 28 is		9.600118
Difference	. .	291
Seconds	. .	40
		60|11640
		194
Log sine of 23° 27′		9.599827
Proportional part for 40″		194
Log sine of 23° 27′ 40″ is		9.600021

Or difference under D., and opposite 27′, is	485
Multiply by 40″, and . . .	40
Striking off two figures on the right gives	194.00

The same as before.

If no very great precision is required, then the proportional part for the nearest fifteen seconds may be taken from the small table at the bottom of the page, and by taking aliquot parts, a sufficient degree of accuracy for most purposes may be attained.

Ex. 2.—Required the logarithm tangent of $2^h\ 24^m\ 46^s$?

Log tangent of $2^h\ 24^m\ 44^s$ is	9.864180
Proportional part for 2^s is	132
Log tangent of $2^h\ 24^m\ 46^s$ is	9.864312

Ex. 3.—Required the *secant* of $9^h\ 45^m\ 36^s$?

The *cosecant* of its excess above 6^h, or $3^h\ 45^m\ 36^s$, gives 10.079396.

Ex. 4.—Required the sine of $20^h\ 44^m\ 56^s$?

The cosine of 2 44 56 is 9.876236 being the sine of $20^h\ 44^m\ 56^s$.

Ex. 5.—Required the sine of 28° 57′ 18″?

Sine of 28° 57′	0″	. .	9.684887
	15	. .	58
$\frac{1}{5}$ of 15″	3	. . $\frac{1}{5}$=	12
Sine of 28 57	18	. .	9.684957

Ex. 6.—Required the sine ^{2} of 30°? *Ans.* 9.397940.

Ex. 7.—Required the cos $^{\frac{2}{3}}$ of 18°? *Ans.* 9.967309.

PROBLEM III.—*To find the Sine or Tangent of a small Arc, less than three Degrees.*

1. To find the sine.

To the logarithm of the arc reduced to seconds, with the decimal annexed, add the constant quantity 4.685575, and from the sum *subtract* the third of the arithmetical complement of the log cosine, or, which comes to the same thing, one-third of the secant; the remainder will be the logarithmic sine of the given arc.

2. To find the tangent.

To the logarithm of the arc in seconds and constant quantity 4.685575, add two-thirds of the secant, the sum is the log tangent of the given arc.

Ex. 1.—What is the log sine of the sun's mean horizontal parallax, supposed to be 8″.68?

Logarithm of 8″.68 is	0.938520
Constant . . .	4.685575
One-third of sec 8″.68 is	0.000000
Log sin of 8″.68 is	5.624095

Or, since in very small arcs the sine and tangent are each very nearly equal to the length of the arc when it does not exceed 10′, and the length of an arc of one second is 0.0000048481368; multiply the length of one second by the number of seconds and parts of a second making the index positive by the former rules, and the sine or tangent will be obtained, thus,—

0.0000048481368 × 8″.68 = 0.0000420818274; the log of this is 5.624094, the log sine or tangent required.

Ex. 2.—Required the tangent of 1° 24′ 30″.46?

To the constant logarithm . .	4.685575
Add log of 1° 24′ 36″.46=5076″.46	3.705561
And $\frac{2}{3}$ × 0.000132= . .	88
Log tang. of 1° 24′ 36″.46 .	8.391224

PROBLEM IV.—*To find the Degrees, Minutes, and Seconds answering to any given Log Sine, or Tangent.*

RULE.—In its respective column find the nearest sine, tangent, &c. to that given, and take the degrees from the top or bottom of the page, according as the quantity is found in a column, with the proper title at the top or bottom, and the minute is found in the same horizontal line, in the left or right hand marginal columns, according as the quantity is found in a column titled at the top or at the bottom of the page.

Ex. 1.—Required the arc, or degrees and minutes corresponding to the log sine 9.584665?

This is found in a column marked sine at the top under 22 degrees, and opposite 36 minutes, or 1 hour, 30 minutes, and 24 seconds of time.

Ex. 2.—What is the arc in degrees or time answering to the log tangent 10.358430, making use of the tables of proportional parts at the bottom of the page.

Given log tangent . .	10.358430
66° 20′ 0″ corresponds to . .	10.358253
Difference . .	177.
And 0 0 30 to	173

Hence 66 20 30 is the arc required.

Or, 4ʰ 25ᵐ 20ˢ answer to .	10.358253
And 2 to 173, or nearly	177

Hence 4 25 22 is the time nearly.

Or to 177 add two ciphers, and divide by 572, the number under D, and opposite to 10.358253, or rather by 573, the number above it, as the form in which the tables are printed requires, and we have

66° 20′ 31″ very nearly; and this method must be followed in al similar cases.

PROBLEM V.—*To find the Degrees, Minutes, and Seconds answering to the Logarithmic Sine or Tangent of a very small Arc.*

RULE.—To the given log *sine* add the constant 5.314425 and one-third of the corresponding secant, the sum, rejecting 10 in the index, will be the logarithm of the number of seconds in the required arc.

To the given log *tangent* add the constant 5.314425, and from the sum subtract two-thirds of the corresponding secant, rejecting 10 in the index, the result will be the logarithm of the seconds of the required arc.

Ex. 1.—Required the arc whose log sine is 6.497655?

Constant . . .	5.314425
Given log sine . .	6.497655
⅓ of 0.000000 is . .	0.000000
Log arc 64″.8756 . .	1.812080
Or 1′ 4″.8756	

Ex. 2.—What is the arc whose log tangent is 7.164440?

Constant . . .	5.314425
Given log tangent . .	7.164440
⅔ of 0.000000 is . .	0.000000
Log arc 301″.207 . .	2.478865
Or 5′ 1″.207	

TABLE VI.—*Natural Sines, Tangents, Secants, and versed Sines to every Degree of the Quadrant.*

The method of taking out the numbers required from this table will be readily comprehended from what has already been said relative to the preceding. When minutes or seconds occur, proportional parts must be taken by means of the differences found by actual subtraction.

Ex.—What is the natural sine of 5° 48′ 56″?

Natural sine of 5° is . . .	0.87156
Prop. part of diff. 17372 for 48′ 56″ is	14168
Natural sine for 5° 48′ 56″ . .	101324

In almost all the computations throughout this work the rules are so compressed as to avoid the use of natural sines, &c.; for in most cases the logarithmic sines, &c., render the operations more easy.

TABLE VII.—*Meridional Parts to every Degree of the Quadrant.*

The degrees are found under the letter D, and the meridional parts under M.P., and when minutes and seconds occur, proportional parts of the difference must be taken in the manner shown above.

Ex.—Required the meridional parts answering to 45° 36′?

Meridian parts to 45° . . .	3929.9
Prop. part of diff. 85.7 to 36′ is .	50.6
Meridian parts to 45° 36′ is .	3080.5

TABLE VIII.—*Traverse Table, or Difference of Latitude and Departure.*

This table contains the measures of the sides and angles of right-angled plane triangles, the distance being represented by the hypotenuse, and the difference of latitude and departure by the legs or sides about the right angle, and the course and its complement by the acute angles. Hence, if any two of these be known, except the two acute angles, the rest are found by inspection. The course is given in degrees or points in the two exterior marginal columns, the distance is found at the top or bottom of the page, according as the course is less or greater than four points or 45°; and the difference of latitude and departure is found in columns under or above these words respectively.

If there are minutes in the course, proportional parts may be taken where great accuracy is required, otherwise they may be omitted if less than 30′, but, if more than 30′, the degrees in the course must be increased by 1°. The distances 1, 2, 3, 4, &c., at the top and the bottom may be accounted 10, 20, 30, &c., or 100, 200, 300, &c. if the difference of latitude and departure be increased in the same proportion by removing the decimal point a corresponding number of places to the right. If the distance consist of several effective figures, the difference of latitude and departure must be found for each figure separately, and the sum of the results taken.

PROBLEM I.—*The Course and Distance being given, to find the Difference of Latitude and Departure.*

Find the course in the right or left hand column, and in a line with it, under or above the given distance, the difference of latitude and departure will be obtained.

Ex. 1.—A ship sails N. N. E. 60 miles, what difference of latitude and departure has she made?

Course.	Dist.	Diff. Lat.	Departure.
2 points	60	55.433	22.961

Ex. 2.—A ship sails S. E. b. S. ½ S., or S. S. E. ½ E. 244 miles, required her difference of latitude and departure?

Course.	Dist.	Diff. Lat.	Departure.
2½ points	200	176.38	94.28
	40	35.277	18.856
	4	3.5277	1.8856
	244	215.1847	115.0216

Ex. 3.—A ship sails 300 miles S., 54° 30′ W., what is her difference of latitude and departure?

Course.	Dist.	Diff. Lat.	Departure.
54°	300	176.34	242.71
55	300	172.07	245.75
Mean 54½	300	174.20	244.23

When several courses and distances are given, the results must be placed in a table, the sum of the several northings and southings,

eastings and westings taken, and placing the less sums under the greater, the differences will show how much the ship has, upon the whole, changed her situation, and in what direction she has moved.

TABLE IX.—*Diurnal Logarithms.*

This table, to which I have ventured to give the title of Diurnal Logarithms, is useful for making computations in which time is concerned, particularly for reducing the right ascension and declination, &c. of the sun or moon to any intermediate time between those times given in the Nautical Almanac, where the proportional parts to daily differences are required. It has two sets of arguments, the one answering to 12^h, since the moon's place is given in the Nautical Almanac for every noon and midnight; the other corresponding to 24^h for the sun.

Rule.—To the logarithm from this table corresponding to the Greenwich apparent time add the proportional logarithm (Table X.) of the variation on the given day for 24^h or 12^h, as the case may be, the sum will be the proportional logarithm of the part of it for the given time, which, added to or subtracted from the number corresponding to the preceding noon or midnight, according as it is increasing or decreasing, will give its value at the instant required.*

Ex. 1.—Required the sun's right ascension March 20, 1826, at $20^h\ 46^m\ 40^s$ apparent Greenwich time?

Greenwich time .	$20^h\ 46^m\ 40^s$	D. L.	0.06262
Change of R. A. in 24^h	$3^m\ 38^s.2$	P. L.	1.69457
Prop. part for $20^h\ 46^m\ 40^s$	3 9.0		1.75719
R. A. at preceding noon	23 57 42.0		
R. A. at $20^h\ 46^m\ 40^s$	0 0 51.0		

Ex. 2.—Required the moon's declination, September the 15th, 1826, at $7^h\ 48^m\ 30^s$ P. M. apparent time on the meridian of Greenwich?

Moon's declination at noon . .	2° 7′ 8″ S.	
at midnight	0 9 19 N.	
Sum = diff. in 12 hours . .	2 16 27	
App. time $7^h\ 48^m\ 30^s$ diurnal log . .		18662
Change of dec. in 12^h, 2° 16′ 27″ prop. log		12030
Change in $7^h\ 48^m\ 30^s$ +1 28 47 prop. log		30692
Dec. at noon . . −2 7 8		
Dec. at $7^h\ 48^m\ 30^s$ −0 38 21 S.		

When the differences are very irregular, a correction on that account becomes necessary. This will be exemplified in the explanation of Table XXVII.

TABLE X.—*Proportional Logarithms.*

This table is chiefly useful for facilitating the method of finding

* The whole operation may be performed by Table IX. alone when the quantities fall within the limits of the table, by reckoning hours minutes, and minutes seconds, &c.

the apparent time at Greenwich, answering to a given central distance between the moon and the sun, a fixed star or a planet, by the assistance of the Nautical Almanac. It is extended to three hours on account of the distances being given in various ephemerides to every three hours of time. As degrees and hours are similarly divided, it answers equally well for either, and is marked accordingly. To this table proportional parts have been added at the bottom of each page to every tenth of a second, which may be useful where great accuracy is required. The table is very useful in calculations where sexagesimal divisions are employed. The method of taking out the log of any quantity will be readily understood from what has already been said.

TABLE XI.—*Depression or Dip of the Horizon.*

The dip of the horizon is an angle contained between a horizontal line passing through the eye of the observer, and a line from his eye to the visible horizon, when these lines are in the same vertical plane. This table contains the dip answering to a free unobstructed horizon, and the numbers corresponding to the height of the eye are to be subtracted from the observed altitude when taken by the fore observation, but added to it in the back observation.*

TABLE XII.—*The Dip at different Distances from the Observer.*

If the land is not sufficiently distant to afford a free horizon, it may be sometimes necessary to obtain an altitude referred to the surface of the sea at some known or estimated distance. Under such circumstances the dip may be taken from this table.

TABLE XIII.—*Correction to be* added *to the observed Altitude of the Sun's lower Limb when taken by a Fore Observation to find the true Altitude.*

This table was computed by the author a good many years ago, for the purpose of combining the usual corrections, namely, dip, refraction, parallax, and semidiameter. The variation of the sun's semidiameter from 16′ is given at the bottom of the table, which, unless considerable accuracy be required, may be neglected. The arithmetical complement of the numbers from this table to 32′, will be the correction to be *subtracted* when the upper limb is observed.

TABLE XIV.—*Correction to be* subtracted *from the observed Altitude of a fixed Star to find the true.*

This table is similar to the last, and contains the sum of the two corrections, dip, and refraction, to be subtracted when the fore observation is employed.

TABLE XV.—*Sun's Semidiameter, &c.*

This table, taken from the Nautical Almanac for 1826, will answer for most purposes for a considerable number of years to come.

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* If the sun's semidiameter be taken in general at 16′, the whole correction at sea, or $c = 16' - \left\{ \sqrt{h} + t' - \frac{t'}{30} \right\}$ in which h is the height of the eye in feet, and t' the natural cotangent of the altitude. This in most cases will be + 12′.

It contains the time of the sun's semidiameter passing the meridian, the sun's semidiameter, hourly motion in longitude, and the log of the sun's distance from the earth, for every sixth day in the year.

The time of the sun's passing the meridian in solar time is useful for reducing an observation of a passage of the preceding or subsequent limb over the meridian taken with a transit instrument, to that of the centre. If the clock by which observations are made be regulated according to sidereal time, this quantity must be increased in the ratio of 365 to 366, or at a mean by about 0ˢ.18, where great precision is required. The semidiameter of the sun is necessary to reduce an observation of the limb to that of the centre, whether in altitudes or angular distances. It is also useful for determining the index-error of a sextant, or the exactness of the scale of micrometers.

The hourly motion is useful for computing eclipses. The log of the sun's distance is requisite in the calculation of the places of the planets and comets, and for some other purposes.

TABLE XVI.—*The Sun's Parallax in Altitude and Zenith Distance.*

The author computed this table from a mean of the determinations of Delambre from the observations of the transit of Venus over the sun's disk in June, 1769. He found the mean horizontal parallax to be 8″.68. It is hoped it will prove useful where great accuracy is required.

TABLE XVII.—*Mean Refractions.*

For the elements of this table the author is indebted to the liberality of Mr Ivory, the most distinguished mathematician in the British islands. On comparing it with that given in the Transactions of the Royal Society of London, it will be seen that it has been expanded considerably, so as to render its application more easy by giving the mean refraction, and its logarithm for every 10′ from the zenith to the horizon, subjoining the differences of the logarithms for the purpose of computing proportional parts more readily.

TABLES XVIII. XIX. and XX.—These tables are employed to correct the preceding according to the state of the barometer and thermometer, as shown in the explanation at the bottom of page 89 of the tables. Table XVIII. gives the correction for the state of the air, and may be denominated the exterior thermometer; Table XIX. gives the correction for the barometer; and Table XX. serves to correct the barometer for the temperature shown by the attached thermometer, or interior thermometer as it is generally called. The refraction must be added to the apparent zenith distance or subtracted from the apparent altitude to obtain the true.

Ex. 1.—Required the mean refraction for 21° 40′ of zenith distance or 68° 20′ of altitude?

Opposite to 21° 40′ in Table XVII., and under 30, will be found 0′ 23″.21, the refraction required when the barometer stands at 30 inches, and the thermometer at 50°; and this is sufficient for most purposes when great accuracy is not required.

Ex. 2.—Required the true refraction when the zenith distance is 70° 41′.7, the barometer 30.045, and thermometer 34°?

Zenith distance 70° 40′ log $\delta\theta$ Table XVII.		2.21752
1.7		68
Thermometer 34° Table XVIII.		0.01472
Barometer 30.0 Table XIX.		0.00000
.045		64
Thermometer 34 Table XX.		70
Log r	2′ 51″.50 = 171″.50	2.23426
Observed refraction	2 51 .50	
Error of the table	0 .00	

Ex. 3.—Let $\theta = 82° 41' 47''$, the thermometer 66° Fahrenheit, and barometer 29.87, required the true refraction?

θ	= 82° 41′ 47″ log		2.63224
Ther. = 66		Table XVIII.	9.98575
		XX.	9.99931
Barometer	29.87		9.99812
r'	=	6′ 52″.50 = 412″.50	2.61542
$\frac{d\,\delta\theta}{d\tau} \times (66° - 50°) =$			
$-0.067 \times +16$	=	$-$ 1.072	
$\frac{d\,\delta\theta}{dp} \times (29.87 - 30) =$			
$+0.10 \times -0.13 =$	=	$-$ 0.013	
r	=	6 51.415	
Obs. r	=	6 52.260	
Error of table	=	$-$ 0.845	

Ex. 4.—Let $\theta = 87° 42' 10''$, thermometer 35°, and barometer 29.5 inches, what is the true refraction?

$\theta = 87° 40' 0''$ log, $\delta\theta$			3.00466
2 10			390
Ther.	35°		0.01379
Bar.	29.5		9.99270
Ther.	35		65
Log r'		17′ 16″.81 = 1036″.81	3.01570
$\frac{d\,\delta\theta}{d\tau} \times (35° - 50°)$	=		
-0.606×-15	=	+ 9 .09	
$\frac{\delta\, d\theta}{dp} \times (29.5 - 30.0) =$			
$+1.04 \times -0.5$	=	$-$ 0 .52	
$r =$		17 25 .38	
Observed refraction		17 26 .50	
Error of the table		$-$ 1 .12	

Examples for Exercise.

	Z. D.	Bar.	Therm.		Obs. Ref.	Error.
		In.	In.	Out.		
1.	70° 46′ 30″.0	29.686	46°	44°.17	2′ 44″.83	+ 1″.51
2.	76 17 0 .0	29.850	45	43 .00	3 55 .85	+ 0 .61
3.	76 55 31 .2	29.686	40	37 .10	4 8 .98	+ 1 .86
4.	81 27 18 .6	29.924	61	58 .19	6 1 .90	+ 1 .55
5.	83 58 6 .7	29.810	36	29 .95	8 48 .52	+ 0 .53
6.	86 14 42 .0	29.174		47 .75	12 4 .20	+ 0 .28
7.	87 23 44 .0	30.000	60	56 .08	15 32 .80	— 1 .15
8.	88 39 32 .0	29.800	38	34 .40	23 7 .94	—15 .70
9.	89 26 51 .4	29.907	39	33 .46	30 16 .60	—39 .70

Hence at moderate zenith distances the error of the table is small, sometimes + and at other times —. From 70° to about 85°, the error is generally +, but from 85° to 90° it becomes —, and is considerable near the horizon. We may therefore infer that the horizontal refraction, 34′ 17″.5, given by the table in a mean state is, in general, too small, though, from the uncertainty and irregularity to which it is subject, it is very difficult to estimate accurately its true quantity. Perhaps from the irregularity of temperature in various parts of a line near the surface of the earth through which the ray of light must pass to reach the eye of the observer, it will be impossible ever to assign the true quantity of the horizontal refraction under given circumstances. In fact, no instrument as yet has been employed to ascertain the effects of aqueous vapour floating in the atmosphere, on the exact quantity of the horizontal refraction; and we suspect that the barometer and thermometer alone are inadequate to that purpose.*

TABLE XXI.—*Augmentation of the Moon's Semidiameter in Altitude and Zenith Distance.*

The apparent magnitude of any object being in the inverse ratio of its distance, and as the moon is nearer the observer in the zenith than in the horizon, by the earth's radius her apparent semidiameter must be greater in the former situation than in the latter. This table contains that increase corresponding to six different values of the semidiameter, at different degrees of the true altitude. If the quantity is not found to the accuracy required by inspection, it may be determined by proportional parts in the usual manner.

TABLE XXII.—*Reduction of the Moon's Parallax in the Spheroid.*

As the earth differs somewhat considerably from a sphere, the eccentricity being about $\frac{1}{310}$, it follows that the equatorial parallax must be greater than that at the various intermediate latitudes from the equator to the pole. This table contains the quantity to be subtracted from the equatorial parallax given in the Nautical Almanac to reduce it to what it ought to be at any other latitude.

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* It is said that an admixture of vapour in the atmosphere has little or no effect on the amount of refraction, which, I think, is hardly correct, since β is certainly affected by it. See page 89, Tables.

TABLE XXIII.—*Logarithms of the Earth's Radii in each Parallel of Latitude; the Equatorial Radius being Unit, and Compression* $\frac{1}{300}$.

This table will be found useful in some nice observations in astronomy, where the spheroidal figure of the earth must be taken into account.

Example.—To Greenwich in latitude 51° 28′ 38″.4 the radius is 9.9991159.

TABLE XXIV.—*Angles which, the Vertical to any Point of the Earth's Surface, makes with the Radius drawn from that Point to the Centre, or, as it is usually called, the Reduction of the Latitude to* $\frac{1}{300}$ *of Compression.*

This table is useful in several astronomical observations, such as the computation of eclipses, occultations, &c.

Example.—The apparent latitude of Greenwich is 51° 28′ 38″.4, required that reduced to the centre?

Latitude .	51° 28′ 38″.4
Reduced .	— 11 11 .6
Reduced latitude	51 17 26 .8

From this table the reduction of the altitude may be obtained by the following rule:—

To the secant of the azimuth reckoned from the meridian of an opposite name from the latitude, add the proportional logarithm of the reduction of latitude, the sum will be the reduction of the altitude, to be reckoned positive when the azimuth is less than 90°, and negative when greater.

Example.—Required the reduction of altitude corresponding to an azimuth of 36° 42′ in the latitude Greenwich 51° 28′ 38″ N.

Azimuth	36° 42′ 0″	Secant	0.09595
Reduction of lat.	11 11 .6	Prop. log	1.20631
Reduction of alt.	8 58 .4	Prop. log	1.30226

In computing time, &c., if the reduced latitude be used, the reduced altitude must be employed also; but, in general, unless absolutely necessary in such computations as that of time, it is easier not to employ either of these reductions. In lunars, when great accuracy is required, the reduced altitude and reduced parallax must be employed with the reduced latitude.

TABLE XXV.—*For determining the Latitude at any Time by the Pole-Star.*

This table was computed by Mr Littrow of Vienna, and will be found very useful for determining the latitude of a place by the pole-star. A full explanation is given at the bottom of the page immediately under the table.

Ex. 1.—In latitude 56° N. nearly, the zenith distance (Z) of the pole-star, by an astronomical circle, was found to be 35° 20′ 50″ when its apparent polar distance (p) was 1° 36′.7, and the star just $14^h\ 26^m\ 56^s$ from the time of upper culmination; required from these data the exact colatitude of the place of observation?*

* Mr Littrow neglected to state, that for 1′ variation of p the tabular quantity must vary 0.03 N. This is, however, of less consequence, as the direct method, upon the whole more simple, is given at page 85.

Now $14^h\ 26^m\ 56^s$ gives $M = 31''.23$, and $N. = -$			0°	0′	0″.48
And $31''.23 \times -3'.3 \times 0.02 = -2''.06 = -3.3 \times .02M$					
Then $31''.23 - 2''.06 = 29''.17 = M'$, log	1.4649				
Cot Z 35° 21′	0.1491				
	1.6140 =	—0	0	41	.12
Cos t. $14^h\ 26^m\ 56^s = 9.9039$					
p 96′.7 log 1.9854					
—1.8893 = —77′.5 =		—1	17	30	.00
		—1	18	11	.60
	Z	35	20	50	.00
Colatitude of Edinburgh Observatory		34	2	38	.40
Latitude		55	57	21	.60

TABLE XXVI.—*Delambre first calculated this Table for finding the Augmentation of the Semidiameter of the Moon in Solar Eclipses and Occultations, without computing the Altitude. It is used as follows:*

To the altitude of the nonagesimal in signs, *add* the distance of the moon from it, and from that altitude *subtract* the moon's distance from it; then take the equations from this table, Part I., answering to the sum and difference, and take the sum of these, regard being had to the signs. To this add the equations corresponding from Part II. If the observation be that of an occultation, the equation answering to the true latitude and parallax in latitude of the moon is to be taken from Part III. In a solar eclipse this part vanishes. Then enter Part IV. with the sum of the former equations in the first vertical column, and the horizontal semidiameter at the top; and take out the corresponding number, which being applied to the former aggregate, according to its sign will give the augmentation of the moon's semidiameter.

Ex.—Let the altitude of the nonagesimal be 55° 18′, the apparent distance of the moon from it 14° 42′, the moon's true latitude 24′ 2″ S., the parallax in latitude 35′ 40″, and the horizontal semidiameter 15′ 30″; what is the augmented semidiameter?

Altitude of nonagesimal	1^s 25° 18′	
App. dist. of moon from it	0 14 42	
Sum	2 10 0	Part I. +7″.70
Remainder	1 10 36	I. +5.33
		+13.03
		Part II. + 0.17
Moon's true lat. 24′ 2″ S., and par. in lat. 35′ 40″		Part III. — 0.12
Sum		+13.08
To moon's semidiameter 15′ 30″, and Sum 13″.08		Part IV. — 0.82
Augmentation		12.26
Semidiameter		15′ 30.00
Augmented semidiameter		15 42.26

TABLE XXVII.—*Equations of Second Differences for twelve Hours.*

In computing the moon's place from the Nautical Almanac for any given time by proportion, a correction resulting from the moon's unequal motion must be applied to the proportional part of the moon's motion in longitude or latitude, answering to the given time after noon or midnight. This correction is contained in the table, the arguments of which are the mean of the two second differences of the moon's motion at the top, and the apparent time after noon or midnight in the respective side-column. This equation must be *added* to, or SUBTRACTED from, the proportional part of the first difference of the moon's motion in twelve hours, according as that difference is *decreasing* or INCREASING.

Hence the correct change, corresponding to the given interval, will be obtained.

If the given second difference is not found in the table exactly, the sum of the equations answering to the several terms, which make up the second difference collectively, is to be taken.

This table may be applied in the computation of the place of a planet. And as the sun's declination varies somewhat irregularly about the solstices, a column has been added to the lower half of the table on the right side for differences in twenty-four hours, to determine the exact declination for any given time where great accuracy is required.

Ex. 1.—Required the moon's declination on the 15th of September, 1826, at $7^h\ 48^m\ 30^s$ P. M. apparent time on the meridian of Greenwich?

In the explanation of Table IX. this is found to be 0° 38′ 21″ S. by proportion; it is only now required to find the correction depending on second differences. For this purpose two declinations must be taken out preceding the given time, and two after it, from which the mean second difference must be found.

The Moon's declination.

1826.		*First Diff.*	*Sec. Diff.*	*Mean.*
Sept. 14th at midnight is	4° 23′ 24″ S.			
		2° 16′ 16″		
15th at noon	2 7 8 S.		0′ 11″	
		2 16 27		34″
15th at midnight	0 9 19 N.		1 19	
		2 15 8		
16th at noon	2 24 27 N.			

If the first differences first increase and then decrease, or *vice versa*, half the difference of the two second differences is the mean, instead of the half sum, as would have been the case had the differences regularly increased or decreased.

In this case the equation must be added or subtracted, according as the *first* first difference is greater or less than the *third* first difference.

Now to 30″ and $7^h\ 48\frac{1}{2}^m$ the equation is .	3″.4
to 4	0 .4
The whole equation is	3 .8

Which, according to the rule above, must be *added* to the proportional part formerly found under the explanation of Table IX.; that

is, to 1° 28′ 47″ we must add 4″, and the true proportional part becomes . . . . + 1° 28′ 51″ N.
And declination at noon being . — 2 7 8

The true declination is . . — 0 38 17 S.

Unless the declinations are all north or all south, it is almost unnecessary to use the equation of second differences.

Ex. 2.—Required the moon's right ascension on the 20th November, 1826, at 9ʰ 36ᵐ 30ˢ P. M.?

The Moon's right ascension,

1826.		*First Diff.*	*Sec. Diff.*	*Mean.*
Nov. 19th at midnight is	116° 20′ 7″			
		6° 11′ 40″		
20th at noon	122 31 47		1′ 51″	
		6 9 49		1′34½″
20th at midnight	128 41 36		1 18	
		6 8 31		
21st at noon	134 50 7			

App. time 9ʰ 36ᵐ 30ˢ		Diurnal log	.09653
Change of dec 6° 9′ 49″		Prop. log	1.46543
Or ÷ by 60=6′ 9″.82		Prop. log	
Prop. part	4′ 56′.12	Prop. log	1.56196
Or	4° 56′ 7″.2		

In this example we have considered the degrees minutes, the minutes seconds, and the seconds have been converted into a decimal by dividing by 6, since the change of declination exceeds the limits of the table. This comes to the same thing as dividing by 60; but any other aliquot part might have been taken, such as a half, a third, &c. provided the proportional part be doubled, trebled, &c. as derived from this table.

Now to 9ʰ 36ᵐ 30ˢ and 1′ the equation is 0′ 4″.5
and 0 30″ . . . 2 .3
and 0 4½ . . . 0 .4

Amount of the whole equation is . . 7 .2

Which must be added to 4° 56′ 7″.2, because the first differences are decreasing, consequently the corrected proportional part is 4° 56′ 14″.4.

Therefore, if to the right ascension at noon on the 20th, that is to . . . . 122° 31′ 47″
There be added . . . . 4 56 14 .4

The true right ascension required is . . 127 28 1 .4

In very nice operations, when the moon's motion is remarkably irregular, it is sometimes necessary to take into account the equations of third and fourth differences; thus:—

1826.		1st Diff.	2d Diff.	3d Diff.	4th Diff.
Nov. 19th at noon	110° 6′ 18″	+ 6° 13′ 49″	—2 9		
19th at midnight	116 20 7	+ 6 11 40	—1 51	+ 18	+ 15
20th at noon	122 31 47	+ 6 9 49	—1 18	+ 38	+ 12
20th at midnight	128 41 36	+ 6 8 31	—1 33	+ 45	+ 13½
21st at noon	134 50 7	+ 6 7 58			
21st at midnight	140 58 5	Mean S. Diff. 1 34½		T. D.+33″	M. F. D.13½

R. A. at noon 20th		122° 31′ 47″.0
Prop. part	+4° 56′ 7″.2	
Equation of Second Difference	+ 7 .2	
Equation of Third Difference	— 0 .2	
Equation of Fourth Difference	+ 0 .2	
	+4 56 14 .4	+ 4 56 14 .4
True Right Ascension		127 28 1 .4

This happens to be the same as before, from the circumstance that the equations of third and fourth differences are equal, and have contrary signs. They may sometimes have the same sign, and amount to a few seconds, but in general they are very small, frequently destroying one another, and seldom need be attended to, since they are always much less than the probable error in the lunar tables.

Ex. 3.—Required the sun's declination at noon, on the 20th of June, 1826, at Otaheité, in longitude $9^h\ 58^m$ W.?

Sun's declination at noon				23° 27′ 11″ N.
Time $9^h\ 58^m$ diurnal log	0.38166			
Var. 0′ 25″ prop. log	2.63548			
P. P. 0 10″.4	3.01714			+ 10 .4
	First Diff.	*Second Diff.*	*Mean.*	
Diff. for 19th	51	26	25	+ 3 .0
20th	25	24		
21st	1			
True declination				23 27 24 .4

In this example the argument in time is found in the right-hand column in the lower half of the table.

TABLE XXVIII.—*Reduction to the Meridian, Parts I. and II.*

In the course of the great trigonometrical survey lately performed in France, the repeating circle was much used in the determination of latitudes and other operations. Latitudes were determined by observing repeatedly, near the meridian, the altitudes or zenith distances of a celestial object, reducing those taken *off* the meridian by appropriate formulæ or tables to what they would have been *on* the meridian. This method may be successfully practised by smaller instruments,—such as Troughton's reflecting circle, or even a good sextant; and Dr Brinkley, with his large eight-feet circle in the observatory at Dublin, takes three or four observations each day as near noon as possible, which are afterwards reduced to noon.

To facilitate these operations, this table has been computed, Part I. by Delambre, and Part II. by Schumacher.

Ex. 1.—Application of the preceding table to observations of the star Arcturus at the observatory of Dublin, on May 12th, 1820, made with the eight-feet circle, having three microscopes, one on the right side of the instrument, one at the bottom, and one on the left.

The latitude of the observatory from numerous observations of Dr Brinkley, corrected by his own very accurate table of refractions, which are peculiarly adapted to his observatory, is 53° 23′ 13″.46

Mean N. P. D. of Arcturus for 1820	60 52 31 .89
Mean right ascension	211 51 51 .6
Place of moon's node	11^s 29 26

Time by Clock.	Left Micros.	Z.D. Bottom Microscopes.	Right Micros.	Mean of the three Microscopes.	Refraction.
h. m. s.	"	° ′ ″		° ′ ″	″
13 56 28	49.7	33 19 50.5 E.	4.3	33 19 54.83	37.82
14 0 28	31.7	33 17 32.6 E.	47.1	17 37.13	37.77
14 9 51	50.6	33 14 54.5 W.	45.0	14 50.03	37.74
14 14 52	38.0	33 16 41.0 W.	31.7	16 36.90	37.77
Barometer 29.67 in		Inter. Ther. 52.5 Ext. —— 48.0	Mean.	33 17 14.72	37.775

Time of Star's Transit by Clock.	Time of Observation.	Difference.	Reduction. Part I.	Reduction. Part II.
h. m. s.	h. m. s.	h. m. s.		
14 7 3.3	13 56 28	0 10 35.3	220″.10	0″.12
14 7 3.3	14 0 28	0 6 35.3	85 .22	0 .02
14 7 3.3	14 9 51	0 2 47.7	15 .32	0 .00
14 7 3.3	14 14 52	0 7 48.7	119 .80	0 .04
		Sums	440 .44	0 .18
			110 .11	0 .045

Now, if the tabular quantity in Part I. be called m, and that in Part II. be called n, the latitude λ, the declination δ, the approximate zenith distance z, the declination and zenith distance being $+$ if north, and $-$ if south, and the true zenith distance Z;

$$\text{then } Z = z - \frac{\cos\lambda\cos\delta}{\sin Z}\cdot m + \left(\frac{\cos\lambda\cos\delta}{\sin Z}\right)^2 \cot Z\cdot n$$

$$\text{or } Z = z - \frac{\cos\lambda\cos\delta}{\sin Z}\left(m - \frac{\cos\lambda\cos\delta}{\sin Z}\right)\cot Z\cdot n \text{ nearly.}$$

In the formula it is supposed that the latitude of the place and declination of the star, and consequently its zenith distance, are previously known; but in all cases where the latitude alone, or the declination alone, is known, z must be substituted for Z in the formula, and then the resulting reduction, which will not differ materially from the truth, when applied to z will give Z and λ very nearly correct; after which, the operation pointed out by the formula must be repeated with Z and λ as if they had been previously known. This repetition which, as appears by the following example, is easily performed, will give the reduction correct enough for all observations made near the meridian; but, if the horary distance be great, a second repetition may be necessary, though scarcely when the observations are kept within the extent of our table, and, unless from necessity, they should not be taken more distant, as in that case a small error in the time will produce a considerable error in the zenith distance. On this account observations very distant from the meridian are not to be recommended, as they may tend to vitiate those made near it, if the distance of the object from the pole be considerable. In the case of stars whose polar distance is not great, the observations may be continued longer; and observations may be made on the pole star at any time of its diurnal revolution.

The distance of the sun or star from the meridian in time is determined by a clock or chronometer whose motion should correspond with that of the object observed, that is showing mean solar time if the sun be the object, or sidereal time if a star be observed. This method, however, cannot always be adopted, and therefore if the sun be the object, and the clock regulated to sidereal time, m must be multiplied by 0.9945542, of which the log is 9.997628, and

if a star be observed, and the clock regulated to mean solar time, then m must be multiplied by 1.0054756 of which the log is 0.002372. When the clock does not go accurately to either times, the value of m must be further multiplied by $1+0.00002315\ r$, whose log is $0.000010053 \times r$, in which r denotes the daily rate of the clock expressed in seconds, to be accounted *plus* when losing, and *minus* when gaining. When r is negative, the arithmetical complement of the log denoted by $0.000010053\ r$ must be taken.

In observations with the repeating circle, for determining the latitude, it is necessary to attend to the *verticality* of the circle, since an inclination of the circle will cause a corresponding error in the results. But if the amount of the inclination be known, the error in the result may be ascertained by the following equation in which e is the error, i the inclination, and z the zenith distance:—

$$e = \tfrac{1}{2} \sin 1'',\ i^2 \cot z, \text{ or}$$

$$\log e = \text{const. log } 4.384545 + 2 \log i + \log \cot z.$$

The *position of the level* must also be attended to, and the bubble must either be brought by the proper screw to the zero of its scale, or an account of the place of the bubble in the two opposite positions of the circle, must be taken, and an allowance made for the difference, according to the divisions of the scale. If the bubble is *nearer the observer* than zero, the sign of the divisions is +, if farther from the observer it is —.

λ 53° 23′ 13″ cos . .	9.775544		
δ 20 7 28 cos . .	9.972641		
z 33 17 15 cosec . .	0.260554 (a)	cot . .	0.182722
	0.008739	× 2 = . . .	0.017478
m 110.11 log . . .	2.041787	n 0.045 log	8.653213
	38	2d, cor + .0713	8.853413
1st, Cor — 112″.35 (e) . .	2.050564 (c)		
or — 1′ 52 .35	380		
2d, Cor + 0 .071	184		
—1 52 .279			
z 33° 17 14 .720			
z′ 33 15 22 .441			
Ref. + 37 .775			
z″ 33 16 0 .216 (f) cosec	0.260794 (b)		
	240 (b—a)		
— 112 .41 (d) . .	2.050804 {c+(b—a)}		
— 0 .06 (d—e)	766		
z‴ 33 16 0 .156 {f—(d—e)}	38		

This result scarcely differs from Dr Brinkley's, which is 38° 16′ 0″.17, to which the aggregate of precession, aberration, and nutation, amounting to—13″.53, being applied, gives 33° 15′ 46″.64 for the mean zenith distance on January 1, 1820.

TABLE XXIX.—*Reduction to either Solstice, the Obliquity of the Ecliptic being* 23° 27′ 40″.

The obliquity of the ecliptic is determined by a number of meridian altitudes, or zenith distances near either solstice. If the sun's longitude were three or nine signs exactly at noon, the operation would be very simple; but as that seldom happens, it is necessary to reduce the actual observations to what they would have been under these circumstances. To accomplish this object, this table has been constructed. In the table the obliquity is supposed to be 23° 27′ 40″, and the reduction is the difference between this quantity and the sun's declination at the several points of the ecliptic corresponding to the observed right ascensions. With the differences and variations for 100″ change of obliquity the table may be adapted to any time within the limits of the table's variation of obliquity. Both quantities will thus be additive till the year 1835. The table is extended to 30ᵐ, and consequently observations may be reduced by it for about seven days before and as many after the solstice.

Ex. 1.—On the 15th of June, 1826, the sun's declination was observed to be 23° 18′ 51″.7, when the right ascension was 5ʰ 25ᵐ 51ˢ.4, and the obliquity 23° 27′ 39″, what was the reduction to the solstice?

	Tabular obliquity	23° 27′ 40″
6ʰ 0ᵐ 0ˢ	Estimated obliquity	23 27 39
5 32 51.4		
	Excess	1
27 8.6 = distance from the solstice,		
27 0.0 gives . .		8′ 42″.73
8.6 gives . .		5 .564
1″.0 var. obl. gives . .		0 .005
Reduction . .		8 48 .299
Sun's declination .		23° 18 51 .7
True obliquity . .		23 27 39 .999

By operating in this way for several days near either solstice, the true obliquity may be obtained from a mean of a number of observations, and consequently likely very near the truth. It may be observed, however, that the sun's latitude from Delambre's tables, taken with a contrary sign, should be applied to the obliquity determined in this manner.

Calculation of the Obliquity of the Ecliptic, June, 1765.

FROM MASKELYNE'S OBSERVATIONS, VOL. I.

Greenwich, 1765.

Days.	Bar.	Thermometer.		Observed Zenith Distance.		Error of Collim.	Corrected Z. D.
June	In.	In.	Out.				
16	29.57	68°.0	62°.0	⊙ L. L.	28° 20′ 27″.2	+ 1″.1	28° 20′ 28″.3
				⊙ U. L.	27 48 50 .9	+ 1 .1	27 48 52 .0
19	30.04	65 .5	60 .5	⊙ L. L.	28 16 20 .3	+ 1 .1	28 16 21 .4
				⊙ U. L.	27 44 47 .9	+ 1 .1	27 44 49 .0
20	30.04	69 .0	62 .0	⊙ L. L.	28 15 53 .2	+ 1 .1	28 15 54 .3
21	29.96	70 .5	64 .0	⊙ L. L.	28 15 43 .3	+ 1 .1	28 15 44 .4
				⊙ U. L.	27 44 6 .4	+ 1 .1	27 44 7 .5
22	29.87	65 .5	62 .5	⊙ L. L.	28 16 2 .0	+ 1 .1	28 16 3 .1
23	29.97	71 .0	62 .5	⊙ L. L.	28 16 43 .7	+ 1 .1	28 16 44 .8
				⊙ U. L.	27 45 10 .3	+ 1 .1	27 45 11 .4
24	29.91	69 .0	64 .0	⊙ L. L.	28 17 50 .1	+ 1 .1	28 17 51 .2
				⊙ U. L.	27 46 16 .2	+ 1 .1	27 46 17 .3

Longitude of moon's node by Tables I. and II. = 11S. 11° 20′.

R. A. of α Serpentis for 1770, Table X.		15ʰ	32ᵐ	57ˢ.14
Reduction to June ~~21~~ 16, 1765, or for 4.55 years	—			13.34
Mean R. A.		15	32	43.80
Correction by Maskelyne's Table XVII.	+			2.46
XVIII.	—			0.38
True R. A. of Star		15	32	45.88
Time of star's transit by clock		15	32	23.80
Error of clock slow by α Serpentis, 16th	—			22.08
α Ophiuchi, 16th	—			21.99
Mean error	—			22.035
Mean error 24th	—			25.710
Loss in 8 days				3.675
Daily rate	—			0ˢ.46
Transit of sun's second limb by clock		5ʰ	40ᵐ	53ˢ.6
Semidiameter in time	—		1	8.7
Transit of centre		5	39	44.9
Error of clock slow	+			22.0
Sun's R. A. June 16th		5	40	6.9
In like manner 19th		5	52	35.4
20th		5	56	44.8
21st		6	0	54.2
22d		6	5	3.6
23d		6	9	13.0
24th		6	13	22.7

Latitude of Greenwich by Ivory's Refractions, 51° 28′ 38″.4 N.

June	Refraction.	Parallax.	Reduction.	Declination.	Apparent Obliquity.
16	29″.05	4″.05	4′ 43″.77	23° 23′ 32″.73	23° 28′ 16″.50
19	30 .09	4 .05	39 .40	23 27 37 .16	23 28 16 .56
20	30 .12	4 .05	7 .58	23 28 3 .73	23 28 11 .31
21	29 .70	4 .05	0 .59	23 28 16 .80	23 28 17 .39
22	29 .90	4 .05	18 .37	23 27 55 .15	23 28 15 .52
23	29 .71	4 .05	1 0 .31	23 27 14 .64	23 28 14 .95
24	29 .80	4 .05	2 8 .44	23 26 8 .40	23 28 16 .84
				Mean . .	23 28 15 .41
				Rejecting 20th	23 28 15 .96
			Table LI.	Lunar Eq.	— 8 .77
			L.	Solar Eq.	+ 0 .43
				Sun's latitude	— 0 .58
				Mean obliquity	23 28 6 .84

TABLE XXX.—*To change Mean Solar into Sidereal Time.*

As a clock regulated by sidereal time is indispensable in every observatory, it is necessary to convert solar into sidereal time in order to know by the clock when any phenomena, such as eclipses, occultations, &c., calculated in mean solar time should take place. This table is employed for that purpose, as will appear by the following example.

An immersion of θ Aquarii by the moon will, by calculation, take place on January 5, 1824, at $3^h\ 46^m\ 50^s$, mean solar time by the meridian of Greenwich; what will be the time by a sidereal clock which shows $0^h\ 0^m\ 0^s$ when the point Aries is on the meridian, and whose error that day was $36^s.54$ fast?

In this case the clock would be a right-ascension clock; and if she went true, would show the right ascension of the celestial bodies as they passed the meridian when observed by a transit instrument. Now on the 5th of January, 1824, the sun's right ascension at noon is $19^h\ 1^m\ 37^s.0$, the same as would be shown by a clock truly regulated.

But as the clock was $36^s.54$ fast on that day, this must be added to give the time shown by the clock, that is, she shows $19^h\ 2^m\ 13^s.54$ at noon. As the immersion will happen at $3^h\ 46^m\ 50^s$ P. M. this must be converted into sidereal time, and added to the preceding to give the time shown by the clock, so that an astronomer may be prepared to observe it.

This operation may be accomplished by the table.

Time.		*Acceleration.*
$3^h\ 0^m\ 0^s$	gives	$0^m\ 92^s.569$
46 0	.	7 .557
50	.	0 .138
3 46 50		0 37 .264

Hence to the time shown by the clock at noon	19^h	2^m	$13^s.54$
There must be added	0	0	37.26
And	3	46	50.00
Whence the time of occultation shown will be	22	49	40.80

TABLE XXXI.—*To change Sidereal into Mean Solar Time.*

This table may be useful for finding the rate of a clock or chronometer. As the transit of a fixed star advances $3^m\ 55^s.908$ daily on mean solar time, if the passage of a star be observed with a transit instrument each day for several successive days, or the disappearance of a star during several successive nights behind a fixed object, such as the vane of a steeple or the body of the steeple itself, nearly in the meridian, the position of the eye of the observer being also fixed, the rate of the clock becomes known on sidereal, and consequently, by this table, on mean solar time.

1. Required the retardation on $10^d\ 5^h\ 48^m\ 56^s$ of sidereal time?

For 10 days we have	$0^h\ 39^m\ 19^s.080$
$0^d\ 5^h\ 0^m\ 0^s$	49.147
48 0	7.864
56	0.153
1 5 48 56	—0 40 16.244
	10^d 5 48 56.000
Mean solar time	10 5 8 39.756

2. Let $21^h\ 10^m\ 0^s$ sidereal time be converted into mean time on the meridian of Greenwich.

Sidereal time given	21^h	10^m	$0^s.0$
Sidereal time at mean noon, January 1, 1828, by Supplement to the Nautical Almanac	18	40	29.33
Interval in sidereal time	2	29	30.67

Retardation or mean sidereal time for this interval.

$2^h\ 0^m\ 0^s$ gives	$19^s.659$			
29 0	4.751			
30	0.082			
0.67	0.002			
Retardation	—24.494	—		24.494
Mean solar time required		2	29	6.176

3. Conversely to convert $2^h\ 29^m\ 6^s.176$ mean time, January 1, 1828, into sidereal time on the same meridian we have

Mean interval from mean noon, Jan. 1,	2^h	29^m	$6^s.176$
Acceleration of sidereal or mean time for the interval $2^h\ 29^m\ 6^s.176$ is	+		24.494
	2	29	30.670
Sidereal time at mean noon, January 1,	18	40	29.330
Sidereal time required	21	10	0.000

which ought to be the time shown by the clock at the instant in question.

If the place of observation be not on the meridian of Greenwich, the sidereal time in column second of the Supplement to the Nautical Almanac must be corrected for the longitude by *addition* if *west*, but by subtraction if east.

Suppose, on January 1, 1828, the sidereal time in longitude 9^h 10^m 6^s west were required, then . . . 18^h 40^m $29^s.33$

9^h 0^m 0^s gives in Table XXX,	1^m $28^s.708$		
10 0 . . .	1.643		
6 . . .	0.017		
9 10 6 gives . .	+1 30.368		+ 1 30.37
The time to be used in that longitude is .			18 41 59.70

TABLE XXXII.—*To convert Mean Time into Parts of the Equator.*

This table may be useful for converting into degrees, &c. the hours, minutes, &c. shown by a clock or chronometer regulated according to mean time; and the method of using it will be readily understood from the examples to the two preceding tables, and that of Captain Kater in the article finding the latitude, page 110.

TABLE XXXIII.—*Lengths of Circular Arcs.*

The method of using this can be no difficulty to those acquainted with the preceding tables, as they are employed in a similar manner.

TABLES XXXIV., XXXV., XXXVI., XXXVII., XXXVIII., XXXIX., XL., XLI., and XLII. are abridged from a series of tables by Mr Fallows, astronomer at the Cape of Good Hope, and were transmitted to the Admiralty, along with an approximate catalogue of stars which he had formed there, and are very convenient for finding at once the amount of the corrections for precession, aberration, and nutation for any given observation, both in right ascension and declination. In addition to these, however, another table must be computed annually. Since the tables are only given to every ten minutes of right ascension, proportional parts are added for every single minute as far as 6 indicated by the figure in the place of tens in the side-column. If the odd minutes exceed 6, the proportional part must be taken at twice, or the complementary proportional part to the next minute of even tens, must be applied with a contrary sign when necessary. If the arguments are found at the top of the tables, the P. P. must be applied according to the signs; if at the bottom, with a contrary sign.

To understand the method of applying these tables is premised the following

Synopsis:—

		Constants.
Table XXXIV.	$= - 1^s.3362 \sin \text{R. A.} \tan \text{dec.} + 3^s.0678$	$= a$
XXXV.	$= - 1.3500 \sin \text{R. A.} = p$, and $p \times \sec \text{dec}$	$= b$
XXXVI.	$= - 1.2390 \cos \text{R. A.} = q$, and $q \times \sec \text{dec}$	$= c$
XXXVII.	$= + 0.6430 \cos \text{R. A.} = s$, and $s \times \tan \text{dec}$	$= d$
XXXVIII.	$= - 20.0436 \cos \text{R. A.} =$ annual precession	$= a'$
XXXIX.	$= - 20.2550 \cos \text{R. A.} = p'$, and $p' \times \sin \text{dec}$	$= b'$
XL.	$= + 18.5800 \sin \text{R. A.} = q'$, and $q' \times \sin \text{d} + r'$	$= c'$
XLI.	$= + 8.0659 \cos \text{dec.} =$ . . .	$= r$
XLII.	$= - 9.6480 \sin \text{R. A.} = s'$ . . .	$= d'$

Annual Table, part 1st $= t - \dfrac{\sin \Omega}{3} - \dfrac{\sin 2 \odot}{40}$ . . $= A$

part 2d $= 0.93046 \left(\cos \Omega - \dfrac{\cos 2 \Omega}{100} + \dfrac{2}{31} \cos 2 \odot\right)$ . . . . . . $= D$

where t is the time elapsed since the commencement of the year, considered unity when the sun's mean R. A. is supposed to be $18^h\ 40^m$. This t may be taken for most purposes from Table LV., corrected for the difference between the sun's mean right ascension at the beginning of the year and $18^h\ 40^m$.

Table of sines of sun's longitude at the time of culmination $= B$

Table of cosines of the same . . . $= C$

Then the whole correction in R. A. $= A a + B b + C c + D d$ (1)

in dec. $= A a' + B b' + C c' + D d'$ (2)

Ex.—Required the corrections of Fomalhaut in right ascension and declination for July 20th, 1824, at the time of his passing the meridian of Greenwich, the R. A. of the star being $22^h\ 48^m$, and declination 30° 33′ south.

The sun's longitude for this time is 118° 12′, of which the natural sine is .881 = B, and the cosine is .473 = E. A and D must be taken from an annual table, or computed from the formulæ given above for that purpose.

Then from Table XXXIV., &c. take the proper numbers for the R. A. of the star, and complete the multiplications indicated by formula (1) the sum of the results will be the total correction in R. A., and those by formula (2) will be that in declination.

Thus Table XXXIV. 414	B	C	A	D
Tan. dec. .590				
	.881	.473—	.901+	.112+
207	.418	1.178—	3.312+	.616
37				
	.352	471	2.981	
.244+	. 9	82	3	061
Constant . 3.068	. 6	3.		6
			2.984+	1
Prec. in R. A. 3.312	.367+	.556+		
	.556+			.068+
			Tan. dec.	.590
	.923+			
Nat. sec. dec. .	1.161			.034
				6
	923			
	92			.040+
	55			
	1			
	$1.071+ = x$			
	$2.984+ = y$			
	$0.040+ = z$			

$4.095+ = x+y+z$ = the sum of corrections in right ascension.*

* See the catalogue published by the Astronomical Society of London, into which Mr Baily has introduced considerable improvements; and the variable quantities for each day in the year are given in a supplement to the Nautical Almanac, published annually. The characters do not signify the same things in the two sets of formulæ.

Annual precession for 22ʰ 48ᵐ, table XXXVIII. is = 19″.062.—

	B	C	C	A	D
	.881 + 19.264 —	.473 — 5.742 +	.473 — 6.945 +	.901 + 19.062—	.112+ 2.981+
	15.411 1.541 .019	2.297 .402 .017	2.778 .486 .021	17.156 019	0.298 30 6
	16.971 — 2.716 +	2.716 +	3.285 —	17.175—	0.334+
sin. dec.	14.255 — .508 +				
	7.127 .114				
	7.241 — = x' 3.285 — = y' 17.175 — = z' 0.334 +				
	27″.366 — = sum of corrections in declination.				

And in this manner the total corrections for any number of stars may be readily computed.

TABLES XLIII., XLIV., XLV., XLVI., XLVII. and XLVIII., are general for the same purpose as those above. By the former are computed more readily the corrections of a number of stars near one another than by the present, though they are convenient and very accurate for computing the corrections for any single star.

Ex.—Required the true apparent right ascension and declination of α Aquilæ on the 1st of January, 1828; the mean R. A. being 19ʰ 42ᵐ 23ˢ.6 and declination 8° 25′ 15″ N.?

1st, To find the Nutation in R. A. and Declination.

R. A. of star	9ˢ 25° 36′				
Lon. moon's node	7 1 54				
Remainder	2 23 42	tab. XLIII.—0″.97,	5ˢ 23° 42	tab. XLIII.+8″.72	
Sum	4 27 30	tab. XLIV.+1 .09,	7 27 30	tab. XLIV.+0 .69	
		+0 .12		nut. in dec.+9 .41	
Declination 8° 25′ tangent		.15			
Product, or part first		+0 .018			
Long. moon's node, part second		+9 .14			
Nutation in R. A.		+9 .158	= 0ˢ.61		

To find the Aberration in R. A. and Declination.

R. A. of star	$9^s\ 25°\ 36'$				
Sun's lon.	9 10 8				
Remainder	0 15 28	Tab. XLVI.	— 18″.72		
Sum	7 5 44	Tab. XLVII.	— 0 .68		
			— 19″.40	log	1.2878
Declination of star	8° 25′	secant	.	.	0.0047
Aberration in R. A.	— $1^s.3$ = — 19″.51			log	1.2925
Remainder	+ 3^s = 3^s 15° 28′	Tab. XLVI.	+5″.18		
Sum	+ 3 = 10 5 44	Tab. XLVII.	+0 .48		
			+5 56		L. 0.7528
Star's declination	8° 25′	Sine			9.1654
Sun's longitude	9^s 10 8	Part 1st	+0″.83		L. 9.9182
Sum	9 18 33	Table XLVIII.	—1 .28		
Remainder	9 1 43	Table XLVIII.	—0 .12		
Aberration in declination	. . .		—0 .57		

Mean R. A.	$19^h\ 42^m\ 23^s.60$	Declination	.	8° 25′ 15″.0 N.
Nutation in R. A.	+ 0.61	Nut. in dec.	+	9 .4
Aber. in R. A.	— 1.30	Aber. in dec.	—	0 .6
True R. A.	19 42 22.91	True dec.		8 25 23 .8

TABLE XLIX. *contains the mean obliquity of the ecliptic for the beginning of the year,* which I have determined from all the most accurate observations I could obtain, together with the annual and monthly diminutions for the purpose of computing it at any other time.

TABLES L. and LI. *give the necessary corrections to determine the apparent obliquity at any given time,* which will be easily applied, and the mode of application is obvious.

TABLES LII. and LIII. *contain the lunar and solar equations of the equinoxes in time,* which are sometimes more convenient than in space.

TABLE LIV. *contains the mean right ascensions and declinations of the principal fixed stars for* 1830, together with their annual variations for reducing them to any other time required.

TABLE LV.—*Decimal Numbers for each Day in the Year.* It is useful wherever the fraction of the year is wanted, as in reducing the places of stars, &c. to any given day in the year. This is accomplished by multiplying the annual variation by the number of

years and decimal for the given day. The result applied with its proper sign will give its mean place after the given time to which the corrections for precession, nutation, and aberration, being also applied with their proper signs, will give the apparent place at that time.

TABLE LVI.—*The Right Ascension of the Sun.*

This table is adapted to leap year, particularly the year 1828, and is only intended to answer the purposes of instruction when no great degree of accuracy is required and the Nautical Almanac is not at hand. It may be reduced to subsequent leap years by adding 7^s.3 for every period of four years.

In order to adapt it to common years, *one-fourth* of the difference between the given and preceding days is to be subtracted from the right ascension in the table for the first after leap year, *one-half* for the second after leap year, and *three-fourths* for the third; and in the months of January and February, the right ascension is to be taken for the day following that given.

This table may be employed in finding the apparent time by the altitude of a star, for finding the time of a star's transit when that is required, for obtaining the latitude by a meridian altitude, &c.

I. *To find the Time of Transit.*

Rule.—From the R. A. of the star, increased by 24^h if necessary, subtract that of the sun; the remainder will be the approximate time of transit. To this time apply the longitude of the given place in time by *addition* or subtraction, according as it is *west* or east; the result may be called the reduced time. To this reduced time compute the right ascension of the sun, which will be the sun's true R. A. at the time of transit. Now from the star's right ascension for the given time subtract the sun's true R. A.; the remainder will be the apparent time of transit.

II. To find the apparent time of rising and setting of a known star; the latitude and longitude of the place, and the year and day of the month being known.

Rule.—Find the apparent time of the transit of the star by the preceding rule; then find half the time of the continuance of the star above the horizon, by the method shown in Problem VI. of Spherical Trigonometry in the Introduction, pages 91 and 97, which, being applied to the time of transit by subtraction and addition, will give the apparent times of the rising and setting of the star respectively.*

TABLE LVII.—*Declination of the Sun.*

This table contains the sun's declination for the noon of each day on the meridian of Greenwich for the year 1828, or leap year. By this table the declination, sufficiently correct for many purposes, may be found for other years. For the first year after leap year, take one-fourth of the difference between the declinations for the given

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* Mr Thomas Lynn has given, in his extensive collection of Nautical Tables, the times of transits of 60 principal stars for every day in the year, which, in many calculations, are very useful.

and preceding days, which is to be *added* to the declination for the given day, if at that time the declination is *decreasing*, but subtracted if increasing. In the second after leap year take the half, in the third take three-fourths of the difference, and apply this correction in the same manner as before; the result will be the declination required. And in the months of January and February the declination is to be taken for the day following that given.

TABLE LVIII.—*The Equation of Time.*

This table contains the equation of time for 1828, or leap year; and is to be found for any other year in the same manner as the declination above explained.

Time, deduced from observations of the sun, is called *apparent time*, to which the equation of time, being applied according to its title in the table, gives *mean time*. Since a clock or chronometer is constructed upon the supposition of a uniform motion, this table will be useful for ascertaining the rate and error on mean solar time. Also, if a clock be regulated to mean solar time, the instant when the sun's meridian altitude ought to be observed to find the latitude, is known by applying the equation of time to 12^h, with a contrary sign to that in the table. These applications will be more readily understood by consulting the article on finding the longitude by chronometers in the introduction.

TABLE LIX.—*Correction of the Longitude by Chronometers.*

This table is on the same principles as that given by Rossel in the third volume of Biot's Astronomie Physique, only substituting for the natural numbers their logarithms, as being more convenient in practice.

Ex.—At Tongatabou, in latitude 21° 7′ 35″ S., and longitude 175° 14′ 30″ W., on the 6th April, 1793, at $19^h\ 53^m\ 31^s.44$, the daily rate of a chronometer was + $5^s.24$, with an original error of + $1^m\ 20^s.93$. The ship sailed W. from Tongatabou, and arrived at Ballade harbour, on the 22d of April, 16 days afterwards, when, by observation, the daily rate was + $8^s.56$, and the error $1^h\ 24^m\ 23^s.71$ fast for mean time at noon.

Daily rate at Tongatabou	$+5^s.24$
at Ballade	$+8\ .56$
Sum	13.80
Half, or mean daily rate	6.9

Difference of longitude between Tongatabou and Ballade by the first daily rate of $5^s.24$ . . . . . . 20° 24′ 34″
Difference of longitude by the mean rate of $6^s.9$. 20 17 55

Difference easterly . . . . 6 39 E.
because the difference of longitude ought to be diminished.

From these data, what is the correction of the observed longitude, on the 17th of April, at $7^h\ 34^m$?

Correction of the longitude of Ballade for 16 days

6′ 39″ = 399″ log	2.60097
Log for 16 days, Table LIX., ar. co.	7.86646
From 6th April to 17th, or 11 days, log Table LIX.	1.81954
Correction 3′ 14″ = 194″ log	2.28697

The correction of the longitude of the 17th gives the place of observation more easterly, because Ballade ought to be the east of the position calculated by the daily rate determined at Tongatabou.

Since the first two logarithms are constant, the correction of the longitude for other days in the same run is easily obtained by substituting for the last logarithm that from Table LIX. for the given number of days elapsed from the time at which the rate was originally determined, and in this manner ought all longitudes to be corrected in a long run, where the rate of the chronometer has experienced considerable alterations.

The same thing may be done without the table, as in the following example taken from Captain Hall's observations on the coast of South America:—

"San Blas, West Coast of Mexico.

"Corrections to be applied to chronometrical measurements of the longitude of places between Acapulco and San Blas.

"The rate of the chronometer, by which the differences of longitude was obtained, of places between Acapulco and San Blas, was that determined at Acapulco, or $\pm$ 0ˢ.0 *per diem*.

"On arriving at San Blas, however, after an interval of 18 days from Acapulco, the rate was found to be + 2ˢ.6 per day. It became necessary, therefore, to make a proportional allowance at intermediate places for the increase of rate, which increase may be taken as uniform during the interval. This is effected by computing the whole difference of longitude by the mean of the two rates + 0ˢ.0 and 2ˢ.6, namely 1ˢ.3, and taking the difference between this determination and that by the first rate, whence are obtained 351″ for the accumulated error in longitude in 18 days' interval.

"Now the sum of a series of 18 terms in arithmetical progression, having 1″ for the first term, and 1″ for the common difference, is 171, consequently $\frac{351''}{171} = 2''.053$ nearly for the daily increase in the error of longitude, and this multiplied by the sum of the terms in the series before designed, according to the number of the days elapsed since the rate was first determined, will give the respective corrections in longitude to be applied to those deduced by chronometer, with the Acapulco rate. Whence we get 2′ 15″, for an interval of 11 days, to be deducted from the longitude of Colima, west of Acapulco; and the correction for an interval of 15½ days is 4′ 21″, to be taken from the longitude of Cape Corrientes, west of Acapulco."

TABLE LX.—*Latitudes and Longitudes of Places.*

This table contains the latitudes and longitudes of a few of the principal places in the world, given with all the accuracy in my power. It also contains the time of high water at the times of new and full moon, and the depth of the water at spring and neap tides, which are necessary to find the time of high water at any particular place on a given day, as well as the depth of the water of any tide, and at any hour of the tide, which may sometimes be necessary. The height of the *neap tides* is seldom given in tide tables, though for these purposes the one should be given as well as the other.

Indeed, it were to be wished that officers of the Royal Navy as well as others should carefully mark all these circumstances; so

that a complete tide table, embracing all the necessary data, might at last be formed.

TABLES LXI. and LXII. serve to convert space into time, and conversely, and their use is so easy to those acquainted with many of the foregoing tables that any farther explanation is unnecessary.

TABLE LXIII. contains a selection of useful numbers frequently wanted in calculation, which have their logarithms and arithmetical complements subjoined.

TABLES LXIV., LXV., LXVI., LXVII., and LXVIII. are given for the purpose of computing the time of high water at such places where the heights of *spring* and *neap* tides are known.

TABLE LXIV. has been added to this second edition on account of its great simplicity, and where a reference to the Nautical Almanac is unnecessary, or where that work is not at hand. It is therefore quite accessible to practical seamen of the humblest pretensions, is sufficiently accurate for most purposes, and will seldom deviate an hour from the truth. It is also useful to find the time of high water approximately, in order to take the necessary quantities from the Nautical Almanac for the succeeding methods, which are more accurate, particularly the last.

TABLE LXV. is deduced from Bernoulli's solution of the problem of the tides, to which has been applied the constant number -22^m, for the hour of high water on the day of full and change, for a harbour that would otherwise have high water when the sun and moon are on the meridian.*

TABLE LXVI., Part I., contains the necessary factors for finding the height of the tide at high water, and Part II. the height at any hour after the nearest high water. In this table a is the rise at spring tides, and b that at neap. More accurate numbers might have been deduced from the solution of Laplace; but as there is great uncertainty arising from the direction and strength of the winds, it has been thought unnecessary to introduce tables so bulky and complex for such an object.

TABLES LXVII. and LXVIII., derived from the solution of Laplace in the *Mécanique Céleste,* taking into account the declinations of the sun and moon, will give the *time* of high water very accurately; and the *height* may be found from the preceding table with as much accuracy as, from the uncertain nature of the problem, the subject seems to admit, and they are used in the following manner:—

To find the time of high water at any place.

Rule 1. Find the given month and day in Table LXIV. in the first

* In some tables similar to ours, such as Dr Mackay's, this number — 22^m, supposed constant, but really about the mean of the accurate numbers at the bottom of Table LXVIII. has been omitted.

part on the left, and under the year on the right, will be obtained the correction in time to be added to the time of high water at full and change, from a tide-table such as LX., and the sum will be the time of high water required.

If the sum exceeds 12^h 25^m, subtract this number from it; if it exceed 24^h 50^m, subtract as before, and the remainder will be the time of high water in the afternoon of the given day. The time of high water of the tide preceding may be found nearly by subtracting 25^m, and that of the succeeding tide by adding 25^m. This degree of accuracy will be sufficient to enable vessels to be ready to enter a harbour or pass a bar, provided they are prepared rather before the computed time, in order to take advantage of the highest rise determined by observation or signal.

Rule 2. Let the approximate time be found by the preceding rule. To this time and the given longitude take from the Nautical Almanac the moon's horizontal parallax.

Again, to the time of the moon's transit over the meridian of Greenwich, as given in the Nautical Almanac, take the equation from Table LXX., answering to the longitude and daily variation of transit between the *given* and *preceding* day, if the *longitude is east.* Subtract it from the transit over the meridian of Greenwich, and the remainder will be the transit over the meridian of the given place. But, if the longitude be *west,* the equation answering to the longitude and daily variation of transit between the *given* and *following day* must be *added* to the time of transit over the meridian of Greenwich to obtain the time of transit of the meridian of the given place. To the time of high water at new and full moon, add the reduced time of transit over the meridian of the given place, and to the sum apply the equation from LXV., answering to time of transit and horizontal parallax previously found; and the result will be the true time of high water required.

Rule 3. To the approximate time of high water found by either of the preceding rules, and the longitude of the place, find the moon's transit over the meridian of the given place preceding the time required as formerly, and take out also, in like manner, the semidiameters of the sun and moon to the nearest noon or midnight. Now, from Table LXVII. take the diminution of the semidiameters answering to the declination, of which the degrees are found in the left-hand column, and every ten minutes at the top; subtract it from the semidiameters found from the Nautical Almanac, and the remainders will be the reduced semidiameters.

Find the difference between the reduced semidiameters of the sun and moon, and prefix the sign +, if the moon's diminished semidiameter be the greater; but the sign —, if it be less. Look for this at the top or bottom of Table LXVIII., and the time of transit in the side columns, and take out the corresponding equation, which must be applied, according to the title above or below, to the reduced time of transit. Lastly, to the difference of the diminished semidiameters at the top, take out the number from the lowest line in the table, and subtract it from the above result, and the remainder will be the apparent time of high water, to which, if the equation of time be applied, it will give the mean time required. The height of the tide at any time may be found by Table LXVI.

Should the time of high water by computation be that of the following morning, half the sum of the transits on the given and preced-

ing days must be taken; and if the time of high water by computation exceed 24^h, the transit on the preceding day must be taken, to obtain the time of high water on the evening of the given day; or, instead of recomputing in this manner, which is only necessary in cases of great accuracy, it will in general be near enough the truth to subtract $12^h\ 25^m$ in the first case, and $24^h\ 50^m$ in the second.

Ex.—Required the time of high water at Leith in longitude 3° 10′ W. on the first of March, 1830?

1. Time of high water at new and full moon at Leith . . . . $+2^h$ 20
Correction for March 1st, 1830, Table LXIV. +5 40

Time of high water nearly . . 8 0 P. M.
Half the mean lunar retardation . . — 25

Time of high water approximately . . 7 35 A. M.

2. To this time and longitude 3° 10′ W. the moon's horizontal parallax is . . 58′.1.
Time of the moon's passing the meridian of Greenwich by the Nautical Almanac, March 1st, 1830, . . 5^h 46
Cor. to 3° 10′ W., and daily retardation 55^m, Table LXX. + 0

Reduced time of transit of upper meridian . 5 46 P. M.
Half the daily retardation . . — 27

Reduced time of transit of under meridian . . 5 19 A. M.
Time of high water at new and full moon at Leith . . $2^h\ 20^m$ . . $2^h\ 20^m$
Time of passing the upper meridian . . 5 46 P. M. lower, 5 19 A. M.
Equation to $5^h\ 46^m$, and hor. par. 58′1 (T. LXV.) — 1 8 to $5^d\ 19^m$ and 58′.5 — 1 13

True app. time of high water 6 58 P. M. . . 6 26 A. M.

3. Sun's semidiameter to this time 16′ 10″— 6″ dim.=16′ 4″
Moon's semidiameter at $6^h\ 58^m$ P.M. 15 51 —27 dim.=15 24

Difference by Table LXVII. . : . — 40 P. M.
Sun's semidiameter as before 16′ 10″— 6″ dim.=16′ 4″
Moon's semidiameter at $5^h\ 19^m$ A.M. 15 57 —24 dim.=15 33

Difference, ... — 31 A. M.
Time of high water at N. and F. moon, $+2^h\ 20^m$ . . $+2^h$ 20
Time of upper transit . . +5 46 lower +5 19
Cor. to diff. semids., — 40″, and transit — 40,—31″ — 41
Number from bottom of Table LXVIII.— 20 — 20

True apparent time of high water 7 6 P.M. 6 38 A. M.
Equation of time . . + 13 + 13

Mean times . . . 7 19 6 51

Whence the first method is erroneous about one hour in this instance, and the two last agree very nearly. The third method, how-

ever, is, on the whole, the more accurate, as allowance is made for the declinations of the sun and moon; and the number to be *always subtracted* is taken truly from the Table at the bottom of page 116, instead of applying the mean —22^m, as is done in method second.

4. To the time of upper transit, $5^h\ 46^m$, and the moon's horizontal parallax, 58'.1, are obtained from Table LXIV., Part I., the factors 0.165*a*, and 0.934*b*. But by Table LX., *a* is 16 feet, and *b* is 9 feet, whence the height of the tide is $=0.165\times16+0.934\times9=$ 11.046 feet.

Also to lower transit $5^h\ 19^m$, and the moon's horizontal parallax 58'.5, the factors are 0.261*a*, and 0.859*b*, and the height will therefore be 11.907 feet.

5. Again, the height of the former tide, at $2^h\ 30^m$ after high water, is from Part II., $=11.046\times0.67=7.40$ feet, and that of the latter, $=11.907\times0.67=7.98$ feet.

TABLE LXIX. has been introduced for the purpose of computing the mean longitude of the moon's ascending node, which is necessary to determine the true places of the stars, contained in Table LIV., nearly comprehending all the fixed stars to the third magnitude inclusive. By means of this Table, and Table LXXIV., containing the sun's longitude, the arguments for entering the Tables, in order to determine the Nutation and Aberration, may be deduced without a reference to any other Tables, which may sometimes be inaccessible. The second column contains the mean longitude of the moon's ascending node on the first of January each year, on the meridian of Greenwich, when the sun's longitude is 281°, from 1820 till 1850. The remaining columns contain the *retrograde* motion of the node for months, days, and hours, which is always to be subtracted from the longitude of the given epoch.

Ex.—Required the longitude of the moon's ascending node on December 25, 1838, at 11 P. M., the time of upper culmination of Rigel?

In this example the retrograde motion of the node for the months, days, and hours, being greater than the mean longitude at the beginning of the year, on the meridian of Greenwich, to which all our astronomical tables are adapted, 360° are to be added to it before the subtraction takes place. In the computations of the places of the stars, contained in the Nautical Almanac, no account has hitherto been taken of the 29th day of February in leap year, though, in cases where extreme accuracy is required, this ought not to be neglected. Should the longitude of the place for which the calculations are made, differ considerably from Greenwich, the time must be reduced to that of Greenwich, in the usual manner.

Add			360.000
1838, January 1st,			18.223
			378.223
Dec. 25,	16^d	—18.534	
	9^d	— 0.477	
	11^h	— 0.024	
			—19.035
Longitude of Node, $11^s\ 29°\ 11'$			= 359.188

The degrees may be converted into signs, by dividing by 30°, and the value of the decimal will give the minutes near enough the truth, for the purpose here required.

TABLE LXX. contains a correction of the moon's passage of the meridian of Greenwich, to reduce to it that of any other meridian. It must be added in west longitude, and subtracted in east, according to the title at the top.

The arguments are the longitude in the left-hand column, and the daily difference of the times of passage, at the top.

TABLE LXXI. contains the contraction of the semidiameters of the sun and moon, on account of refraction, according to the altitude of the object found in the left-hand column, and the inclination of the measured semidiameter at the top.

TABLE LXXII. contains the effect of the solar nutation of a star in right ascension and declination, to be applied according to the signs. The second part of the solar nutation, in R. A., must be multiplied by the tangent of declination.

TABLE LXXIII. consists of numbers for finding the length of a degree in latitude, longitude, and of the length of the pendulum at any latitude, to an ellipticity of $\frac{1}{308}$, those on the equator being known.

Length of an arc of one degree of latitude at the equator, is	362755 feet.
One degree of longitude there . .	365144 feet.
Length of equatorial pendulum . . .	39.01326 inches.

Whence, by multiplying any of these by the factor, for the latitude, from the Table, the result will be the length at the given latitude.

Thus, at Edinburgh, in latitude 55° 57′ 18″ N.

A degree of latitude	$= 362755 \times 1.00685 =$	365169 feet.
longitude	$= 365144 \times 0.56069 =$	204733 feet.
Length of pendulum	$= 39.01326 \times 1.00370 =$	39.15761 inches.
Increase of the number of vibrations	$= 159.78$	sec.

TABLE LXXIV. gives the true longitude of the sun at apparent noon, on the meridian of Greenwich, for leap year, particularly 1828, and it may be adapted to other years, with sufficient precision, for several purposes, especially for finding the necessary arguments for computing the aberration of the fixed stars, &c., as exemplified in the explanation of Tables XLVI., XLVII., and XLVIII., by subtracting *one-fourth* of the difference between the given and preceding days from the longitude in the Table for the first after leap year, *one-half* for the second, and *three-fourths* for the third; or it will be near enough to subtract 15′ for the first, 30′ for the second, and 45′ for the third; and, in the months for January and February, the longitude is to be taken for the day following that given.

TABLE LXXV. consists of the log. versed sine of the corresponding arc, or time diminished by the constant log. of 2, or 0.301030, and is therefore the logarithms of half the versed sines.

But as 2 sine $^2 \frac{1}{2}$ P, is the versed sine of the arc P, it is evident that the Table is also the log. sine $^2 \frac{1}{2}$ P. If P be the angle at the pole, made by any meridian passing through a given object, with the meridian of the place of observation, the hour angle from noon will thus, from computation, become known. If Z, the angle at the zenith, be substituted for P, the azimuth will in like manner become known by the rules in page 99, illustrated by the examples in page 100. The quantities are taken from this Table, similarly to those in Table V.; and the proportional parts, for seconds of time, are very easily obtained from the differences to 100^s by annexing two ciphers to the difference between the tabular and given logarithms and dividing this by the number in the column of differences; the quotient will be the seconds and decimal. As this 100^s corresponds to 25′, or the *fourth part* of 100′,—to find the minutes, multiply the difference given in that column in the Table, by 4, annex two ciphers to the difference between the nearest logarithm found in the Table and the given logarithm, and, dividing the latter by the former, the result will be the minutes and decimal, which may be reduced to seconds if necessary; or, after annexing two ciphers, divide first by 4, and afterwards by the tabular difference, the same result will be obtained as before, and perhaps rather more simply.

Ex. 1. Let the given log (page 124) be .		9.334685
Next greater gives	$20^h\ 18^m\ 0.00^s$	9.336053
Now 136800 ÷ 6019 =	+ 22.72	
		1368
Time from preceding noon	20 18 22.72	
Ex. 2. Let the given log (page 97) be . .		9.697321
Next less gives . . .	89° 45.′00	9.697071
$\frac{25000}{4 \times 31.65} =$	+ 1.97	250
Azimuth . .	89 46.97 =	89°46′ 58″

The following Tables, containing the solutions of Plane and Spherical Triangles, which, by Mathematicians conversant with Formulæ, will be always preferred to Rules, were drawn up by my friend, Mr Andrew Girvan, accountant, Edinburgh.

In using these Formulæ, the signs of the Trigonometrical lines marked in the Table, page 12, must be carefully attended to.

APPENDIX.

Table for the Solution of Plane and Spherical Triangles.

Let A, B, and C denote the three Angles of any Triangle, and a, b, and c the sides respectively opposite to these Angles.

I. Right-angled Plane Triangles, right angled at A.

Given.	Required.	Formulæ.	No
I. A side and an angle.			
a and B	b and c	$b = a \sin B$, and $c = a \cos B$ . .	1
b and B	a and c	$a = b \operatorname{cosec} B$, and $c = b \cot B$ . .	2
c and B	a and b	$a = c \sec B$, and $b = c \tan B$ . .	3
II. Two sides.			
a and b	c	$c = \sqrt{(a^2-b^2)} = \sqrt{\left\{(a+b)(a-b)\right\}}$.	4
	B and C	$\sin B = \cos C = \frac{b}{a}$, and $\operatorname{cosec} B = \sec C = \frac{a}{b}$	5
	$\frac{1}{2}$ C	$\sin \frac{1}{2} C = \sqrt{\left(\frac{a-b}{2a}\right)}$, $\cos \frac{1}{2} C = \sqrt{\left(\frac{a+b}{2a}\right)}$, and $\tan \frac{1}{2} C = \sqrt{\left(\frac{a-b}{a+b}\right)}$ . .	6
b and c	a	$a = \sqrt{(b^2+c^2)} = b\sqrt{\left(1+\frac{c^2}{b^2}\right)}$ . .	7
	B	$\tan B = \frac{b}{c}$, and $\cot B = \frac{c}{b}$ . .	8
	B—C	$\tan (B-C) = \frac{(b+c)(b-c)}{2bc}$ . .	9
III. The three sides.			
a, b, and c	B—C	$\sin (B-C) = \frac{(b+c)(b-c)}{a^2}$, and $\cos (B-C) = \frac{2bc}{a^2}$ . .	10

II. Plane Triangles in general.

Given.	Required.	Formulæ.	No
I. A side and the three angles a, A, B, & C	b and c	$b = a \text{ cosec A sin B}$, and $c = a \text{ cosec A sin C}$	11
II. Two sides and angle opposite to either a, b, and A	c	$c = b \cos A \pm \sqrt{(a^2 - b^2 \sin^2 A)}$.	12
	B	$\sin B = \frac{b}{a} \sin A$ . . .	13
III. Two sides and the contained angle. b, c, and A	a	$a = \sqrt{(b^2 + c^2 - 2\,b\,c \cos A)} = \sqrt{\left\{(b+c)^2 - 4\,b\,c \cos^2 \tfrac{1}{2} A\right\}}$.	14
		$\frac{2 \cos \frac{1}{2} A}{b+c} \sqrt{(b\,c)} = \sin \Phi$, then $a = (b+c) \cos \Phi$	15
	$\frac{1}{2}(B \backsim C)$	$\frac{b}{c} = \tan \Phi$, then $\tan \frac{1}{2}(B \backsim C) = \tan(\Phi - 45^\circ) \cot \frac{1}{2} A$ . . .	16
		$\tan \frac{1}{2}(B \backsim C) = \frac{b \backsim c}{b+c} \cot \frac{1}{2} A = \cot (C + \frac{1}{2} A)$	17
	C	$\tan C = \frac{c \sin A}{b - c \cos A}$, and $\cot C = \frac{b}{c} \text{ cosec A} - \cot A$ . . .	18
IV. The three sides. $a, b, \& c = 2s$	A	$\cos \frac{1}{2} A = \sqrt{\left\{\frac{(s-a)\,s}{b\,c}\right\}}$ . .	19
		$\sin \frac{1}{2} A = \sqrt{\left\{\frac{(s-b)(s-c)}{b\,c}\right\}}$, $\sin A = \frac{2}{b\,c} \sqrt{\left\{(s-a)(s-b)(s-c)\,s\right\}}$	20
		$\tan \frac{1}{2} A = \sqrt{\left\{\frac{(s-b)(s-c)}{(s-a)\,s}\right\}}$ . .	21
Area.		area of the triangle $= \frac{1}{2} b\,c \sin A = \frac{1}{2} b^2 \sin A \sin C \text{ cosec B}$ . .	22

III. Right-angled Spherical Triangles, right-angled at A.

Note.—In the following Formulæ, as already observed, the rules of the signs in Algebra require to be attended to. The quantities marked * are of the same affection, and those marked ^ are ambiguous, in which case the quantities required may be either those found directly, or their supplements.

Given.	Required.	Formulæ.	No
I. A side and an angle.	b	$\sin^* b = \sin a \sin^* B$ . . .	23
a and B	c	$\tan c = \tan a \cos B$ . . .	24
The hypotenuse and an angle.	C	$\cot C = \cos a \tan B$ . . .	25
b and B	a	$\sin \hat{a} = \sin b \operatorname{cosec} B$ . . .	26
An oblique angle and the side opposite.	c	$\sin \hat{c} = \tan b \cot B$ . . .	27
	C	$\sin \hat{C} = \sec b \cos B$ . . .	28
b and C	a	$\tan a = \tan b \sec C$, and $\cot a = \cot b \cos C$	29
An oblique angle and the side adjacent.	c	$\tan c = \sin b \tan C$ . . .	30
	B	$\cos B = \cos b \sin C$, and $\sec B = \sec b \operatorname{cosec} C$	31
II. Two sides.	c	$\cos c = \cos a \sec b$, and $\sec c = \sec a \cos b$	32
a and b	B	$\sin^* B = \operatorname{cosec} a \sin^* b$ . .	33
The hypotenuse and a side.	C	$\cos C = \cot a \tan b$, and $\sec C = \tan a \cot b$	34
b and c	a	$\cos a = \cos b \cos c$, and $\sec a = \sec b \sec c$	35
The sides opposite to the oblique angles.	B	$\tan B = \tan b \operatorname{cosec} c$ . . .	36
	C	$\tan C = \operatorname{cosec} b \tan c$ . . .	37
III. The two oblique angles.	a	$\sec a = \tan B \tan C$, and $\cos a = \cot B \cot C$	38
B and C	b	$\cos b = \cos B \operatorname{cosec} C$, and $\sec b = \sec B \sin C$	39
	c	$\cos c = \operatorname{cosec} B \cos C$, and $\sec c = \sin B \sec C$	40

IV. Spherical Triangles in general.

Given.	Required.	Formulæ.	No
I. Two sides and an angle opposite to either. a, b, and A	B	$\sin \overset{A}{B} = \operatorname{cosec} a \sin b \sin A$ . .	41
	C	$\left\{\begin{array}{l}\cos b \tan A = \cot \Phi \\ \cot a \tan b \cos \Phi = \cos \Psi\end{array}\right\}; C = \Phi \pm \Psi$	42
	c	$\left\{\begin{array}{l}\tan b \cos A = \tan \varphi \\ \cos a \sec b \cos \varphi = \cos \psi\end{array}\right\}; c = \varphi \pm \psi$	43
	c	$\sin c = \sin a \operatorname{cosec} A \sin C$ . .	44
II. Two sides and the contained angle. a, b, and C	A	$\left\{\begin{array}{l}\tan a \cos C = \tan \varphi, \text{ and} \\ \quad \tan A = \sin \varphi \operatorname{cosec} (b-\varphi) \tan C\end{array}\right\}$	45
	A and B	$\left\{\begin{array}{l}\tan \frac{1}{2} (A+B) = \\ \quad \sec \frac{1}{2} (a+b) \cos \frac{1}{2} (a-b) \cot \frac{1}{2} C\end{array}\right\}$.	46
		$\left\{\begin{array}{l}\tan \frac{1}{2} (A-B) = \\ \quad \operatorname{cosec} \frac{1}{2} (a+b) \sin \frac{1}{2} (a-b) \cot \frac{1}{2} C\end{array}\right\}$	47
	c	$\left\{\begin{array}{l}\tan a \cos C = \tan \varphi, \text{ and} \\ \quad \cos c = \cos a \sec \varphi \cos (b \backsim \varphi)\end{array}\right\}$	48
III. Two angles and a side adjacent to either. c, A, and B	a	$\left\{\begin{array}{l}\cos c \tan A = \cot \Phi, \text{ and} \\ \quad \tan a = \tan c \cos \Phi \sec (B-\Phi)\end{array}\right\}$	49
	a and b	$\left\{\begin{array}{l}\tan \frac{1}{2} (a+b) = \\ \quad \tan \frac{1}{2} c \sec \frac{1}{2} (A+B) \cos \frac{1}{2} (A-B)\end{array}\right\}$	50
		$\left\{\begin{array}{l}\tan \frac{1}{2} (a-b) = \\ \quad \tan \frac{1}{2} c \operatorname{cosec} \frac{1}{2} (A+B) \sin \frac{1}{2} (A-B)\end{array}\right\}$	51
	C	$\left\{\begin{array}{l}\cos C = \cos A \operatorname{cosec} \Phi \sin (B-\Phi); \\ \quad (\Phi \text{ as above, formula 49})\end{array}\right\}$	52
	C	$\sin C = \operatorname{cosec} a \sin c \sin A$ . .	53

IV. continued.

Given.	Required.	Formulæ.	N
IV. Two angles and a side opposite to either a, A, & B	b	$\sin b = \sin a \operatorname{cosec} A \sin B$ . .	54
	c	$\left\{\begin{array}{l}\tan a \cos B = \tan \varphi, \\ \sin \varphi \cot A \tan B = \sin \psi\end{array}\right\}; c = \varphi \pm \psi$	55
	$\frac{1}{2} c$	$\tan \frac{1}{2} c = \tan \frac{1}{2}(a+b) \cos \frac{1}{2}(A+B) \sec \frac{1}{2}(A-B)$	56
	$\frac{1}{2}$ C	$\cot \frac{1}{2} C = \cos \frac{1}{2}(a+b) \sec \frac{1}{2}(a-b) \tan \frac{1}{2}(A+B)$	57
	C	$\left\{\begin{array}{l}\cos a \tan B = \cot \Phi, \\ \cos A \sec B \sin \Phi = \sin \Psi\end{array}\right\}; C = \Phi \pm \Psi$	58
V. The three sides. a, b, and $c = 2s$		$\sin \frac{1}{2} A = \left\{\operatorname{cosec} b \operatorname{cosec} c \sin (s-b) \sin (s-c)\right\}^{\frac{1}{2}}$	59
	A	$\cos \frac{1}{2} A = \left\{\operatorname{cosec} b \operatorname{cosec} c \sin s \sin (s-a)\right\}^{\frac{1}{2}}$	60
		$\tan \frac{1}{2} A = \left\{\operatorname{cosec} s \operatorname{cosec} (-a) \sin (s-b) \sin (s-c)\right\}^{\frac{1}{2}}$	61
VI. The three angles A, B, and C, $= 2$ K		$\sin \frac{1}{2} a = \left\{-\cos K \operatorname{cosec} B \operatorname{cosec} C \cos (K \backsim A)\right\}^{\frac{1}{2}}$	62
	a	$\cos \frac{1}{2} a = \left\{\operatorname{cosec} B \operatorname{cosec} C \cos (K \backsim B) \cos (K \backsim C)\right\}^{\frac{1}{2}}$	63
		$\tan \frac{1}{2} a = \left\{-\cos K \cos (K \backsim A) \sec (K \backsim B) \sec (K \backsim C)\right\}^{\frac{1}{2}}$	64

V.—*Particular Solutions.*

I. To reduce the length of an inclined base to its horizontal measure.

Let B be the length of the base on the inclined plane, b that reduced to the horizontal plane, and θ the inclination, then

$$b = B \cos \theta \quad . \quad . \quad . \quad . \quad (1)$$

But as θ is generally a small angle, and need not be known with extreme precision, it is better to calculate the excess of B above b, and supposing θ to be given in minutes,

$$B - b = B(1 - \cos \theta) = 2 B \sin^2 \frac{\theta}{2} = \frac{1}{2} B \theta^2 \sin^2 1' = \frac{\sin^2 1'}{2} \theta^2 B,$$

$$\text{or } B - b = 0.00000004231\, \theta^2 B \quad . \quad (2)$$

Logarithmically,

$$\text{Log.}(B - b) = \text{const. log. } 2.626422 + 2 \log. \theta + \log B \quad . \quad (3)$$

II. In measuring a base, it sometimes happens that all its parts do not lie in the same straight line, but are inclined to one another at very obtuse angles. In this case there are given the two sides and contained angle to find the third side.

1. When the contained angle is very obtuse.

Let a and b be the given sides, and C the contained angle in the given triangle ABC. Then put $C = 180° - \theta$, in which case θ is very small, and is the defect of C from two right angles.

Whence $$A'' = \left\{ \sin A + \frac{1}{6} \sin^5 A \right\} \frac{1}{R''} = \frac{a\theta}{a+b} \left\{ 1 + \frac{b(a-b)}{6(a+b)^2} \left(\frac{\theta}{R''} \right)^2 \right\} \quad (1)$$

2. Supposing that the side b is very small in comparison of a, then,

$$B'' = \left\{ \frac{b}{a} \sin C + \frac{b^2}{2a^2} \sin 2C + \frac{b^3}{3a^3} \sin 3C + \&c. \right\} R'' \quad . \quad (2)$$

3. $$\text{Log.}\, c = \log.\, a - M\frac{b}{a}\cos C - M\frac{b^2}{2a^2}\cos 2C - M\frac{b^3}{3a^3} - \&c. \quad (3)$$

in which M is the logarithmic modulus, or 0.4342945.

III. Having given the hypotenuse of a right-angled spherical triangle, and one of the oblique angles, to find the value of the side adjacent to this angle expressed in series.

Let a be the hypotenuse, c one of the sides about the right angle, and B the contained angle, then

$$a - c = \tan^2 \tfrac{1}{2} B \sin 2a - \tfrac{1}{2} \tan^4 \tfrac{1}{2} B \sin 4a + \tfrac{1}{3} \tan^6 \tfrac{1}{2} B \sin 6a - \&c. (1)$$

Let $\tan c = \theta \tan a$, and

$$a - c = \left(\frac{1-\theta}{1+\theta} \right) \sin 2c + \tfrac{1}{2} \left(\frac{1-\theta}{1+\theta} \right)^2 \sin 4c + \tfrac{1}{3} \left(\frac{1-\theta}{1+\theta} \right)^3 \sin 6c \, . \quad (2)$$

IV. Having given a and b, the two sides of a spherical triangle, little different from a quadrant, together with the side c, it is required to find C from the three given sides a, b, c.

By hypothesis $a = 90° - \alpha$, $b = 90° - \beta$; and as α and β are very small, the angle C is measured by an arc nearly equal to c.

Let $C = c + x$, and

$$x = \tfrac{1}{4} \frac{(\alpha+\beta)^2}{R''} \tan \tfrac{1}{2} c - \tfrac{1}{4} \frac{(\alpha-\beta)^2}{R''} \cot \tfrac{1}{2} c.$$

V. In some cases of trigonometrical surveying, one side of the spherical triangle is very small in comparison of the other two, and therefore to obtain the requisite accuracy the following series must be employed.

$$\begin{aligned} 180° - B = A + c \sin A \cot b &+ \tfrac{1}{2} c^2 \sin A \cos A\,(1 + 2\cot^2 b.) \\ &+ \tfrac{1}{3} c^3 \sin A \cos^2 A \cot B\,(3 + 4\cot^2 b.) \\ &- \tfrac{1}{6} c^3 \sin A \cot B\,(1 + 2\cot^2 b) \quad . \quad (1) \end{aligned}$$

$$\begin{aligned} C = \frac{c}{\sin b} \sin A + \frac{c^2}{\sin b} \sin A \cos A \cot B \\ + \tfrac{1}{3} \frac{c^3}{\sin b} \sin A \cos^2 A\,(1 + 4\cot^2 b) - \tfrac{1}{3} \frac{c^3}{\sin b} \sin A \cot^2 b \end{aligned} \quad (2)$$

$$a = b - c \cos A + \tfrac{1}{2} c^2 \cot b \sin^2 A + \tfrac{1}{2} c^3 \cos A \sin^2 A\,(\tfrac{1}{3} + \cot^2 b.) \quad (3)$$

Spheroidal and Geodesic Formulæ.

I.

The following formulæ, given by Mr Ivory in the Philosophical Magazine of 1828, for determining the relations of arcs and angles upon the surface of the terrestrial spheroid, are introduced here on account of their simplicity and general utility.

Let A denote the length of any arc on the earth's surface in fathoms or feet.
- Δ, the length of a degree of longitude on the equator in the same measure $= 60856$ fathoms, or 365136 feet.
- γ, the length of the chord between any two points on the earth's surface.
- E, the excess of A above γ, or of the arc above its chord.
- R, the mean radius of the earth $= 20887680$ feet.
- r, the length of an arc in degrees equal to the radius.
- δ, the chord of the elliptical meridian between two places near to each other.
- ι, the ellipticity of the meridian $= 0.00324$.
- a, the radius of the equator.
- $a\,(1-\iota)$, the polar semiaxis.
- λ', the latitude of any place nearest the equator.
- λ, that of another more distant.
- m, the azimuth at one of these places, or the angle between the meridian and the arc passing through the other.
- m', that at the other place.
- ω, the difference of longitude between them.
- M, the logarithmic modulus $= 0.4342945$.

(1) Put $\beta = \dfrac{A}{\Delta}$

(2) Sin $\dfrac{\beta}{2} = \dfrac{\gamma}{2a} = \dfrac{\gamma}{2r\Delta}$

(3) $$E = \frac{A^3}{24\,R^2},\ \gamma = A - \frac{A^3}{24\,R^2},\ A = \gamma + \frac{A^3}{24\,R^2}$$

(4) $$\delta = (\lambda - \lambda')\left\{1 - \iota\left[\frac{1}{2} + \frac{3}{2}\cos(\lambda + \lambda')\right]\right\}$$

(5) $$\text{Log}\cos\lambda' = \log\left(\frac{\cos\lambda\sin m}{\sin m'}\right) + M\iota\,(\sin^2\lambda - \sin^2\lambda')$$

(6) $$\text{Log}\sin\omega = \log\left(\frac{\sin\beta\sin m}{\cos\lambda'}\right) - M\iota\sin^2\lambda'$$

(7) $$\text{Log}\sin\frac{\omega}{2} = \frac{1}{2}\log\left\{\frac{\sin\frac{\beta+\delta}{2}\sin\frac{\beta-\delta}{2}}{\cos\lambda\cos\lambda'}\right\} - \frac{M\iota}{2}(\sin^2\lambda + \sin^2\lambda')$$

$$(8)\ \text{Log}\tan\frac{\omega}{2} = \log\left\{\frac{\cos\frac{\lambda-\lambda'}{2}}{\sin\frac{\lambda+\lambda'}{2}}\cot\frac{m+m'}{2}\right\} + M\iota\cos^2\frac{\lambda+\lambda'}{2}$$

(9) Tan $A' = \cos\lambda\tan\omega$, A' being an arc perpendicular to the meridian.

II.

The following formulæ of Oriani have been employed by Captain Kater in the new survey detailed in the Philosophical Transactions for 1828, to determine the difference of longitude between Greenwich and Paris.*

Let a denote the radius of the equator, - $= 20921180$ feet.

b, the polar semiaxis, - - - $= 20853180$ feet.

e, the eccentricity $= \sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{1-\frac{b^2}{a^2}} = 0.08056$

ι, the ellipticity $= \frac{e^2}{2} = \frac{0.00649}{2}$ - $= 0.003245$

M, the distance in feet from the perpendicular to the meridian of Greenwich.

P, the distance in feet to the meridian of Greenwich.

Put $m = \frac{M}{b\sin 1''}$, $p = \frac{P}{b\sin 1''}$, and L the latitude of Greenwich.

Let λ be the latitude of the foot of the perpendicular let fall from the given station on the meridian of Greenwich.

φ, the required latitude of the given station, and

u, the longitude of the given station, then

$$(10)\qquad \lambda = L \pm m\left\{1 - 2\iota + 3\iota\cos^2\left(L \pm \frac{m}{2}\right)\right\}$$

$$(11)\qquad \psi = p\,(1 - 2\iota\sin^2\lambda)$$

$$(12)\ \text{Sin}\ \varphi = \sin\lambda\cos\psi$$

$$(13)\ \text{Tan}\ u = \frac{\tan\psi}{\cos\lambda}(1 - \iota\cos^2\lambda)$$

Though Mr Ivory's formulæ are, in general, as simple as the nature of the subject will admit, yet I have been induced to add the formulæ of Oriani in order to enable Computers to check their calculations by different methods.

Explanation and Application of the foregoing Formulæ.

Example 1.—The chord, or γ, between Beachyhead and Dunnose, by the trigonometrical survey, is 339397.6 feet, what is the length of the arc, or A ?

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* The author is indebted to Captain Kater for a copy of this interesting survey, of which an abstract is given in the following pages.

By formula (3) $A = \gamma + \dfrac{A^3}{24\,R^2}$

$\gamma = 339397.6$	$\log \times 3 =$	- -	16.592127
$24\,R^2$	ar. co. log	- -	3.980009
$E =$	3.7 log	- - -	0.572136
$A = 339401.3$			

Ex. 2.—Required from the new survey the difference of longitude between the Observatories of Greenwich and Paris?

The length of an arc drawn from Dover perpendicular to the meridian of Greenwich is 303804 feet, or 50634 fathoms, as stated in the Philosophical Transactions, 1828, page 180.

If the point where this arc intersects the meridian of Greenwich be considered as the first station in formula (6), and Dover as the second, then $m = 90°$, and $\lambda' = 51°\ 7'\ 45''.6$. Since the given distance is not a chord, but an arc on the earth's surface, β may be found by reducing the given length taken as an arc on the earth's equator to degrees. Formula (1)

Now a degree on the equator to an ellipticity of 0.00324 is 60856 fathoms, whence

$$\beta = \frac{A}{\Delta} = \left(\frac{50634}{60856}\right)^{0} \times 60' \times 60'' = 49'\ 55''.3.$$

By substituting these in formula (6) we have

$\beta = 0°\ 49'\ 55''.3$	sine 8.162000	$M = 0.4342945$	log 9.637784
$m = 90\ \ 0\ \ 0\ .0$	sine 10.000000	$\epsilon = 0.00324$	log 7.510545
$\lambda' = 51\ \ 7\ \ 45\ .6$	sec 0.202342	$\lambda' = 51°\ 7'\ 45''.6$	$\sin^2$ 9.782589

$\log\left(\dfrac{\sin\beta\,\sin m}{\cos\lambda'}\right) =$ 8.364342 $M\epsilon\,\sin^2\lambda' = 0.000853$ log 6.930918

$-M\epsilon\,\sin^2\lambda'$ - -0.000853

$\omega = 1°\ 19'\ 23''.79$ sin 8.363489, which is the longitude of Dover east from Greenwich.

Ex. 3.—As no azimuth either at Dover or at Dunkirk is given, formula (7) must be employed in the following solution.

General Roy makes the distance from Dover to Dunkirk 244916 feet according to his scale, and, taking the ratio of his scale to the imperial standard to be as 1.0000691 to 1, according to Captain Kater, this distance would be 40822 imperial fathoms.

Now the distance being an arc on the surface of the spheroid, we have, as before, $\beta = \left(\dfrac{40822}{60856}\right)^{0} = 40'\ 14''.87$.

The latitude of Dover, or	$\lambda =$	$51°\ 7'\ 45''.6$
Dunkirk, or	$\lambda' =$	$51\ \ 2\ \ 8\ .5$
	$\lambda + \lambda' =$	$102\ \ 9\ \ 54\ .1$
	$\lambda - \lambda' =$	$5\ \ 37\ .1$

Whence by formulæ (4) and (7),

$\frac{3}{2} = 1.5 \log \qquad + 0.176091$

$\lambda + \lambda' = 102^\circ 9' 54''.1 \quad \cos - 9.323625$

$-0.3160 \quad \log - 9.499716$

$\frac{1}{2} = +0.5000$

$+0.1840 \quad \log + 9.264818$

$- \quad \iota = -0.00324 \log - 7.510545$

$-0.00060 \log - 6.775363$

$+1.00000$

$+0.99940 \log + 9.999739$

$\lambda - \lambda' = 537''.1 = 337''.1 \log 2.527759$

$\vartheta = 536.9 = 336.9 \log 2.527498$

$\lambda = 51^\circ 7' 45''.6 \quad \sin^2 \text{ N.N. } 0.68616$

$\lambda' = 51\ 2\ \ 8.5 \quad \sin^2 \text{ N.N. } 0.60456$

$\log 0.083044 \quad \text{N.N. } 1.21072$

$M \log 9.637784$

$\iota \log 7.510545$

$\log 7.231373 \quad \text{N.N. } 0.001704$

$$\frac{M\iota}{2}(\sin^2\lambda + \sin^2\lambda') = 0.000852$$

$\beta = 0^\circ 40' 14''.9$

$\vartheta = 0\ \ 5\ 36\ .9$

$\beta + \vartheta = 0\ 45\ 51\ .8$	half $= 22'\ 55''.9$	sine - -	7.824159
$\beta - \vartheta = 0\ 34\ 38\ .0$	half $= 17\ 19\ .0$	sine - -	7.702189
$\lambda = 51\ \ 7\ 45\ .6$	secant	- - - -	0.202342
$\lambda' = 51\ \ 2\ \ 8\ .5$	secant	- - - -	0.201463
		2	15.930153

$$\frac{1}{2}\log\left\{\frac{\sin\frac{\beta+\vartheta}{2}\sin\frac{\beta-\vartheta}{2}}{\cos\lambda\cos\lambda'}\right\} \quad - \ - \ - \ - \quad 7.965076$$

$$-\frac{M\iota}{2}\left(\sin^2\lambda + \sin^2\lambda'\right) \quad - \ - \ - \ - \ - \ - \quad -0.000852$$

$\frac{1}{2}\omega = 0^\circ 31' 39''.57 \quad . \quad \sin \frac{1}{2}\omega \quad - \ - \ - \quad 7.964224$

$\qquad\qquad 2$

$\omega = 1\ \ 3\ 19\ .14 =$ long. of Dunkirk E. from Dover.

$\quad\ \ 1\ 19\ 23\ .79 =$ long. of Greenwich W. from Dover.

Sum $2\ 22\ 42\ .93 =$ long. of Dunkirk E. from Greenwich.

$\quad\ \ 2\ 22\ .00 =$ long. of Dunkirk E. from Paris.

Diff. $2\ 20\ 20\ .93 = 9^m\ 21^s.39 =$ longitude of Paris in time east from Greenwich.

Ex. 4. Required the difference of longitude between Greenwich and Paris by the formulæ of Oriani, from the following data taken from the new survey?

By Table III., page 180, Phil. Trans., 1828, $M = 184282.44$, $P = 427511.43$ when referred to Notre Dame at Calais, and taking the values of a, b, and ι, as before stated, we have,

$$m = \frac{184282.44}{20853180 \sin 1''} = 1822.79 \qquad \iota = 0.003245$$

$$2\iota = 0.006490$$

$$p = \frac{427511.43}{20853180 \sin 1''} = 4228.36 \qquad 3\iota = 0.009735$$

$L = 51° 28' 38''.5$, whence by (10,) (11,) (12,) and (13,) we have, $\lambda = 50 \quad 58 \quad 24 .8$, $\psi = 1° 10' 12''.1$, $\varphi = 50° 57' 32''$, and $u = 1° 51' 19''$. Taking the longitude of Calais to be 22′ 59″ W. of Paris, as stated in the Connaissance des Temps, the longitude of Paris will be $2° 20' 18'' = 9^m 21^s.2$, or $0^s.19$ less than before. In this manner the latitudes and longitudes in Table V. were computed by Captain Kater.

Ex. 5. The latitude of Beachy Head, or $\lambda = 50° 44' 21''$ N.
Dunnose, or $\lambda' = 50 \quad 37 \quad 5$
and the perpendicular from Beachy Head upon the meridian of Dunnose is 56020 fathoms by the Trigonometrical Survey, consequently

$$\beta = \frac{56020}{60856} \times 3600'' = 55' 13''.92, \text{ and } m = 90°$$

Now by formula, (6,) as in example, (1,) we have $\omega = 1° 27' 6''.65$, and formula, (9,) gives $A' = 55' 8''.11$. Whence extending this arc to a degree, or $55' 8''.11 : 60' :: 56020 : 60963$, the length of the perpendicular degree in fathoms.

Ex. 6. Again let $\lambda = 50° 44' 21''$ and $\lambda' = 50° 37' 5''$, $m = 96° 55' 58''$, and $m' = 81° 56' 53''$, and employing formula (8,) ω will be found to be $1° 26' 54''\cdot 76$, and $A' = 55' 0''.59$. Whence the perpendicular degree is 61102 fathoms, or 80 fathoms *less* than that given in the Trigonometrical Survey, and 139 fathoms *greater* than the result by formulæ (6) and (9) in ex. (5) stated above.

In fact ω is found by other methods from independent data to be $1° 27' 5''.62$, giving the perpendicular degree 60974 fathoms, exceeding the result in Ex. 5 by about 11 fathoms only, and agreeing exactly with a spheroid of 0.00324 of ellipticity.

Ex. 7. The latitude of Crowborough station or λ is $51° 3' 18''.30$ N. and longitude $9' 21''.45$ E. from Greenwich, and the latitude of Fairlight station or λ' is $50° 52' 36''.88$ N. At Crowborough the observed angle between the meridian of Crowborough and Fairlight or m is $121° 4' 58''.36$, and at Fairlight the angle between the meridian and Crowborough, or m' is $58° 33' 26''.14$. Whence by formula (8) the difference of longitude is $27' 49''.90$ E.

Ex. 8. Again, the latitude of Blancnez or λ is $50° 55' 29''.36$ N., and at Fairlight, of which the latitude or λ' is $50° 52' 36''.88$, the angle between the meridian and Blancnez or m is $85° 36' 39''.73$, and at Blancnez, the angle between the meridian and Fairlight or m' is $93° 32' 31''.11$; then by formula (8) the difference of longitude between Fairlight and Blancnez is $1° 5' 34''.08$ E.

Hence the longitude of Crowborough being .	0°	9′	21″	.45 E.
The difference between Crowborough and Fairlight	0	27	49	.90
And between Fairlight and Blancnez .	1	5	34	.08
Longitude of Blancnez . . .	1	42	45	.43
Captain Kater's result by Oriani's formulæ from independent data, ϵ being $\frac{1}{300}$, is .	1	42	47	.45
Defect by formula (8) . . .	—		2	.02

If a compression of $\frac{1}{300}$, that adopted by Captain Kater, had been used, the difference would have been $-1''.78$ only, which may fairly be attributed to unavoidable errors of observations, and a diversity of methods.

TRIGONOMETRICAL SURVEYING AND GEODESIC OPERATIONS.

1. To apply the various formulæ for geodetical purposes, and trigonometrical surveying to practice, the greater part of the survey performed in 1821, 1822, and 1823, for determining the difference of longitude between the Royal Observatories of Greenwich and Paris, communicated by Captain Kater, to whom I have been considerably indebted, is here subjoined.

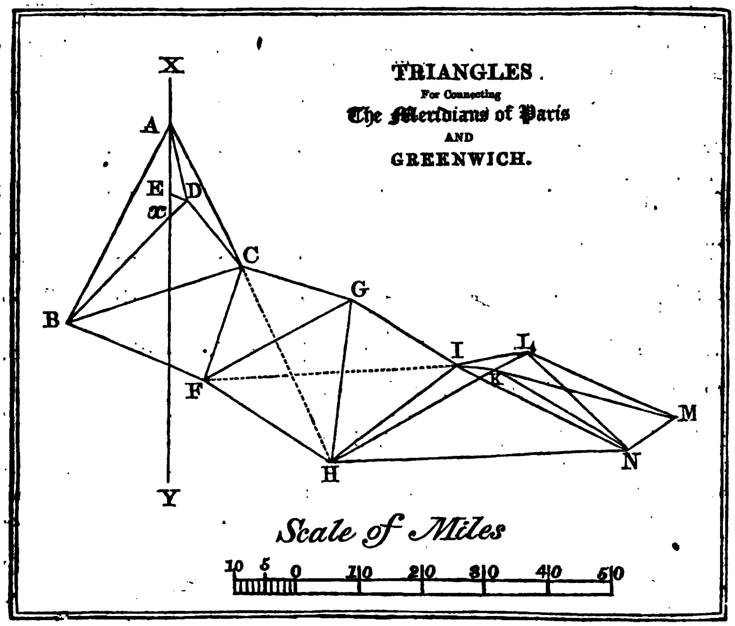
2. Triangles measured on the surface of the earth, of which the sides are small in proportion to the radius, may be considered as spherical triangles, and the sides may therefore be computed by the rules of Spherical Trigonometry. This method would in most cases be too laborious, and consequently other plans have been adopted, susceptible of greater simplicity, and of sufficient accuracy. First, the arcs may be reduced to the chords, and the sides calculated by Plane Trigonometry. Secondly, the observed angles may be reduced as has been proposed by Legendre, so that the computations may be performed by Plane Trigonometry, and of these the last is the more simple method.

3. In this case, if, from each of the observed angles of a small spherical triangle, one-third of the spherical excess be deducted, the sines of the angles thus diminished will be proportional to the lengths of the opposite sides, so that the triangle may be resolved as if it were rectilineal. The spherical excess besides being applied in this manner to prepare the angles for calculation, is useful to show the degree of accuracy of the observed angles, and from the two observed angles, to derive the third when it cannot be conveniently observed, and is therefore given in a separate column.

4. The spherical excess may be readily obtained by adding to the constant logarithm 0.373660 the logarithmic sine of the contained angle, and the logarithms of the containing sides, the sum will be the logarithm of the spherical excess in seconds; or to the same constant logarithm 0.373660, add the logarithmic cosecant of the angle opposite the given side, the logarithmic sines of the adjacent angles, and twice the logarithm of the interjacent side, the sum will be the logarithm of the spherical excess in seconds; or it may be found by any of the rules, page 193, sect. VI.

5. If, however, a plan of the triangles be constructed, to which a scale of English miles is adapted, the spherical excess may be readily found by scale and compasses, or more readily by the usual feather-edged rules according to sect. VI., rule 4, by multiplying the base and perpendicular in miles together, and dividing the product by 152, the quotient will be the spherical excess in seconds.—*Leslie's Geometry*, p. 418. This may be very conveniently performed by the sliding rule. Set 152 on the sliding line of numbers to the base of the triangle on the fixed line in miles; then opposite the perpendicular, also in miles, on the slide, will be found the spherical excess in seconds on the fixed line. Thus in the triangle DBC, formed by Wrotham, Severndroog, and Leith Hill, the side BC will measure about 30 miles on the scale, and upon it the perpendicular from D will be about 13 miles. Now, set 152 on the sliding line of numbers, to 30 on the fixed line, then opposite 13 on the slide will be found 2″.56 on

the fixed line, the spherical excess, agreeing with the computation of Captain Kater.



XY is the meridian of Greenwich.
A Chingford.
B Leith Hill.
C Wrotham Hill.
D Severndroog Castle.
E The centre of the Transit in the Royal Observatory.
F Crowborough.
G Stede Hill.
H Fairlight Down.
I Tolsford Hill.
K Folkstone Turnpike.
L Dover Castle.
M Calais.
N Blancnez.

6. To illustrate these rules and observations, let one of the first of the following series of triangles be selected in which the observed angles are derived from the mean of a considerable number taken with the great theodolite belonging to the Royal Society of London, and similar to that of the Board of Ordnance, now employed by Colonel Colby in the Trigonometrial Survey of the British Islands.

Stations.	Observed Angles.	Spherical Excess.	Reduced Angles for Calculation.	Distances, or opposite sides in feet.
Wrotham Station,	65° 26′ 47″.68	. . .	65° 26′ 46″.85	144760.96
Severndroog Castle,	86 25 58 .40	. . .	86 25 57 .57	158844.37
Leith Hill Station,	28 7 16 .42	. . .	28 7 15 .58	75014.27
	180 0 2 .50	2″.56	180 0 0 .00	

The first column contains the names of the stations, the second the observed angles, the third the spherical excess computed by some of the foregoing rules, the fourth the reduced angles for com-

putation, and the fifth the distances or lengths of the sides opposite the stations whose names are given in the first column.

7. The whole of the reduced angles are found in the following manner. See pages 190 and 191. The sum of the three observed angles amounts to - - - - - 180° 0′ 2″.50

But the spherical excess being 2″.56, this sum should have been - - - - - -		180 0 2 .56
Difference, or E - - - - -	—	0 .06
And $\frac{E}{3}$ is - - - - - -	—	0 .02
The spherical excess, or e, is - -	+	2 .56
And $\frac{e}{3}$ is - - - - - -	+	0 .85

Whence the whole correction, or $\frac{E}{3}+\frac{e}{3}=0''.85-0''.02=0''.83$, to be applied with a contrary sign, to reduce the angles for computation, when all the angles are taken with equal care. Thus, 65° 26′ 47″.68 — 0″.83 = 65° 26′ 46″.85, the angle at Wrotham, prepared for calculation; and in a similar manner the others are found. The sides are then proportional to the sines of the opposite angles, and when one side is obtained by measurement, the rest are readily determined.

8. In the Philosophical Transactions for 1828, page 176, it is stated that the staff marking the station at Chingford, was about 20 *inches*, or 1.73 feet, more correctly, to the west of the meridian of the centre of the transit of the Royal Observatory at Greenwich, and bearing N. 6″.17 W., which must be added to the angle DAE in the foregoing figure, to obtain the true bearing of Severndroog Castle from Chingford, which will be found to be S. 12° 46′ 20″.12 E. Also if from AED 106° 10′ 36″.48, there be subtracted 6″.17, and the difference 106° 10′ 30″.31, be taken from 180°, the remainder, 73° 49′ 29″.69, will be the bearing of Severndroog Castle, S. E. from the Royal Observatory, as recorded in Table II. Again, if the above angle, 12° 46′ 20″.12, be added to ADx, or ADB, 124° 7′ 48″.85, making 136° 54′ 8″.97, and this sum taken from 180°, there will be obtained the angle BxY, S 43° 5′ 51″.03 W, the bearing of Leith Hill from Severndroog Castle. In like manner the bearings of all the other stations from one another may be found.

9. Now supposing a parallel to the meridian of Greenwich, and to its perpendicular, to be drawn through each station, we have the bearings and distances of the other stations from such parallels, calculated by means of a right angled plane triangle, of which the hypotenuse and one angle (the bearing) are given to find the other two sides.

Thus, let K be the distance between the given stations, M the distance from the parallel to the perpendicular on the meridian, P the distance from the parallel to the meridian, and θ the bearing or angle, with the parallel to the meridian; then,

1. $M = K \cos\theta$; and 2. $P = K \sin\theta$. - (A)

The base from which the sides of the triangles are obtained, was measured on Hounslow Heath by General Roy; and Captain Kater, after reducing the measure to the imperial standard, found the distance from Leith Hill to Severndroog Castle, to be 144760.96

English feet. From this side, then, all the others are to be found. A side near the termination of the series, is generally connected by angles, with a base of verification, measured with equal care as the first, for the express purpose of checking the final result.

TABLE I.

Having prepared the angles for computation, as directed, the results are given in the following Table :—

No	Stations.	Angles.	Sides in Feet.	No	Stations.	Angles.	Sides in Feet.
	A	12° 46′ 13″.95	14612.73		G	Mean G from 8,	
1	E	106 10 36.48	63489.32			11, & 13, above.	134038.70
	D	61 3 9.56	57847.66				
	A	16 35 2.00	75014.27		F	36° 5′ 23″.75	134038.70
2	D	149 26 13.58	133640.58	14	I	33 24 6.63	125267.72
	C	13 58 44.42	63489.32		H	110 30 29.62	213125.73
	A	39 31 34.96	144760.41		F	25 38 37.62	105080.58
3	D	124 7 48.85	187883.00	15	G	118 37 34.12	213127.05
	B	16 14 36.19	63489.32		I	35 43 48.26	141791.16
	C	65 26 46.85	144760.96		K	36 17 56.85	134038.70
4	D	86 25 57.57	158844.37	16	I	136 51 46.41	154811.39
	B	28 7 15.58	75014.27		H	6 50 16.74	26957.63
	F	87 5 14.32	158844.37		N	27 45 38.34	134038.70
5	B	38 56 55.25	99982.55	17	I	118 34 47.32	252708.40
	C	53 57 50.43	128615.26		H	33 39 34.34	159500.18
	G	44 44 51.73	99982.55		N	31 22 30.02	154811.39
6	F	41 58 19.84	94980.95	18	K	121 48 12.43	252705.93
	C	93 16 48.43	141790.75		H	26 49 17.55	134168.12
	H	65 2 34.54	141790.75		K	Mean I and K from 17 and 18.	252707.16
7	G	53 13 23.05	125267.46				
	F	61 44 2.41	137745.52		H	Mean H from 18 and another.	134167.74
	I	69 7 57.10	137745.52		L	117 18 16.41	134167.74
8	H	45 27 52.81	105080.02	19	K	50 37 49.97	116726.89
	G	65 24 10.09	134038.00		N	12 3 53.62	31560.06
	H	33 6 29.69	99982.55		M	38 42 1.54	134167.74
9	C	43 11 7.27	125267.87	20	K	9 21 18.23	34880.94
	F	103 42 23.04	177831.03		N	131 56 40.23	159605.30
	G	53 13 23.05	125267.87		M	8 42 38.35	31560.06
10	F	61 44 2.41	137745.95	21	K	41 16 30.44	137471.95
	H	65 2 34.54	141791.20		L	130 0 51.21	159605.30
	I	69 7 57.10	137745.95		M	47 24 37.27	116726.89
11	G	65 24 10.09	134038.43	22	N	119 52 50.04	137472.03
	H	45 27 52.81	105080.36		L	12 42 32.69	34880.94
	H	31 56 2.41	94980.95		N	Mean N 21 & 22.	137471.99
12	C	50 5 40.54	137747.27			General Roy.	137459.40
	G	97 58 17.05	177832.66			Mean of these.	137465.70
	I	69 7 57.10	137747.27				
13	G	65 24 10.09	134039.68				
	H	45 27 52.81	105081.36				

TABLE II.

Employing the Formulæ 1 and 2, (A) section 9, the following Table, containing Distances from Parallels to the Meridian, and to the Perpendicular to the Meridian, of Greenwich will be obtained.

No	Stations.	Objects.	Bearings.	P. Feet.	M. Feet.
1	T. R. Obs.	Severndrg.	S 73° 49′ 29″.69 E	14034.28 E	4070.72 S
2	Severndg.	Wrotham,	S 43 20 6.54 E	51479.66 E	54561.77 S
3	Wrotham,	Stede Hill,	S 76 1 32.25 E	92169.87 E	22936.75 S
4	StedeHill,	Tolsford,	S 59 23 57.12 E	90446.52 E	53491.64 S
5	Tolsford,	Folkstone,	S 85 23 40.63 E	26870·59 E	2164.49 S
6	Folkstone,	Dover,	N 65 52 20.12 E	28802.86 E	12900.88 N
		Calais,	S 72 51 8.14 E	152510.49 E	47057.49 S
		Blancnez,	S 63 29 49.91 E	120068.38 E	59871.23 S
7	Blancnez,	Dover,	N 51 25 56.29 W	91265.50 W	72772.10 N

TABLE III.

From the preceding Table is derived the following, containing Distances from the Meridian of Greenwich, and from its Perpendicular.

No	Stations.	Distance from the Meridian of Greenwich. Feet.	Distance from the Perpendicular to the Meridian of Greenwich. Feet.
1	Severndroog Castle,	14034.28 E	4070.72 S
2	Wrotham, - -	65513.94 E	58632.49 S
3	Stede Hill, - - -	157683.81 E	81569.24 S
4	Tolsford, - -	248130.33 E	135060.88 S
5	Folkstone, - - -	275000.92 E	137225.37 S
6	Dover Castle, - -	303803.78 E	124324.49 S
7	Notre Dame, Calais, -	427511.41 E	184282.86 S
8	Blancnez, - -	395069.30 E	197096.60 S

TABLE IV.

The following Table contains the Distance of each Station from the Meridian, and from the Perpendicular to the Meridian of Greenwich, obtained from a mean of all the triangles given in Table I.

No	Stations.	Distance from the Meridian of Greenwich. Feet.	Distance from the Perpendicular to the Meridian of Greenwich. Feet.
1	Centre of Transit R. Obs.	0.00	0.00
2	Severndroog Castle, -	14034.26 E	4070.71 S
3	Wrotham, - -	65513.85 E	58632.80 S
4	Stede Hill, - - -	157683.94 E	81568.92 S
5	Leith Hill, - -	84872.55 W	109773.90 S
6	Dover Castle, - -	303803.83 E	124324.08 S
7	Tolsford, - - -	248130.86 E	135060.49 S
8	Folkstone, - - -	275000.91 E	137225.12 S
9	Crowborough, - -	35857.53 E	154115.76 S
10	Notre Dame, Calais, -	427511.43 E	184282.44 S
11	Blancnez, - - -	395069.29 E	197076.10 S
12	Fairlight, - - -	143276.72 E	218559.79 S

TABLE V.

LATITUDES AND LONGITUDES,

BY THE FORMULÆ OF ORIANI,

The compression being $\frac{1}{300}$, and the Latitude of the centre of the Transit 51° 28′ 38″.96, which appears to be about half a second too great.

No	Stations.	Latitudes. N.	Longitudes. Arcs.	Longitudes. Time.
		° ′ ″	° ′ ″	m s
1	Centre of Transit R. Obs.,	51 28 38.96	0 0 0.00	0 0.00
2	Severndroog Castle,	51 27 58.74	0 3 41.64 E	0 14.77 E
3	Wrotham, - -	51 18 59.35	0 17 11.33 E	1 8.75 E
4	Stede Hill, - -	51 15 7.00	0 41 18.86 E	2 45.26 E
5	Leith Hill, - -	51 10 34.00	0 22 12.01 W	1 28.80 W
6	Dover Castle, -	51 7 45.59	1 19 23.45 E	5 17.55 E
7	Tolsford, - -	51 6 8.65	1 4 48.19 E	4 23.21 E
8	Folkstone, - -	51 5 43.18	1 11 48.61 E	4 47.24 E
9	Crowborough, -	51 3 18.30	0 9 21.45 E	0 37.43 E
10	Church of Notre Dame,	50 57 27.95	1 51 18.73 E	7 25.24 E
11	Blancnez, -	50 55 29.36	1 42 47.45 E	6 51.16 E
12	Fairlight, - -	50 52 36.88	0 37 14.23 E	2 28.94 E

In consequence of that part of the survey performed by the French not being yet published, in the absence of better authority, we may take the longitude of Calais as stated in the Connaissance des Temps, to be about $0^\circ\ 28'\ 59''$ W. from Paris, and to this adding $1^\circ\ 51'\ 18''.73$, the longitude of Calais E. from Greenwich by the preceding table, there will be obtained $2^\circ\ 20'\ 17''.73$, the longitude of Paris E. from Greenwich. This converted into time gives $9^m\ 21^s.18$.

The longitude of Paris, by example 3, page 272, is	9^m	$21^s.39$	E
By the present calculation it is - -	9	21 .18	
By fire signals it was found to be - -	9	21 .46	
Mean - - - -	9	21 .34	

ON TRIGONOMETRICAL LEVELLING.

1. The triangle formed in a vertical plane above the earth's surface, in this operation, is called a *hypsometrical triangle*. It is formed by the chord K of the terrestrial arc, comprised between the verticals of the two stations where the reciprocal zenith distances δ, δ' have been observed, by the straight line which joins the two points of observation, and the difference of level $dE = \theta$, that is, by the portion of the vertical of the most elevated point, intercepted between that point and the arc of a great circle of the earth passing through the other point. In this rectilineal triangle the sides respectively opposite the sides K and θ are, in designating by u the angle of the vertical of the two stations, $90^\circ - \frac{u}{2} - \frac{\delta' - \delta}{2}$, and $\frac{\delta' - \delta}{2} = v$, as will readily appear, and thus

$$K \sin v = \theta \cos\left(v + \frac{u}{2}\right) \quad . \quad . \quad . \quad . \quad (1)$$

By developing this by the formula of Maclaurin,

$$dE = K \sec \tfrac{1}{2} u \tan\left(\frac{\delta' - \delta}{2}\right) + K \sec \tfrac{1}{2} u \tan \tfrac{1}{2} u \tan^2\left(\frac{\delta' - \delta}{2}\right) \quad (2)$$

This formula is independent of refraction. The two terms of the second member ought always to have the same sign when K is taken for the base of the hypsometrical triangle, of which one of the extremities is the least elevated station above the sea. Thus these two terms are additive or subtractive according as the zenith distance δ' is greater or less than δ.

2. When one of the zenith distances, as δ', only is known, $\delta = 180^\circ - \delta' + u$, and therefore $\frac{\delta' - \delta}{2} = \delta' - \frac{u}{2} - 90^\circ$, hence $-\tan \frac{1}{2}(\delta' - \delta) = \cot(\delta' - \frac{1}{2} u,)$ and supposing δ' acute, formula (2) becomes

$$dE = K \sec \tfrac{1}{2} u \cot\left(\delta' - \frac{u}{2}\right) + K \sec \tfrac{1}{2} u \tan \tfrac{1}{2} u \cot^2\left(\delta' - \frac{u}{2}\right) \quad (3)$$

In this last formula, γ being supposed the apparent zenith distance ought to be augmented by the refraction $r = n\, u$ to obtain the true difference of level, which, in this case, gives

$$dE = K \sec \tfrac{1}{2} u \cot \left(\gamma + \frac{2n-1}{2} u\right) +$$
$$K \sec \tfrac{1}{2} u \tan \tfrac{1}{2} u \cot^2 \left(\gamma + \frac{2n-1}{2} u\right) \quad . \quad . \quad (4)$$

In this formula attention must be paid to the sign of the cotangent, and that the two terms of the second member, as before, have the same sign, when K answers to the lower station of the two compared.

When the observations do not give the refractions at the time they are made, n is generally taken $= 0.08$, and then $\frac{2n-1}{2} = -0.42$.

In practice, the first terms of formulæ (3) and (4) may be considered sufficient.

3. The following formula may be advantageously employed instead of formula (4).

$$dE = K \sec \tfrac{1}{2} u \cot \gamma + q\, K^2 \sec \tfrac{1}{2} u \operatorname{cosec}^2 \gamma \quad . \quad . \quad . \quad (5)$$

in which the log q is 2.301455.

The second term is always positive, and in general $\sec \frac{1}{2} u = 1$, and $\operatorname{cosec} \gamma = 1$, for the greater K is the nearer does γ become a right angle.

4. If k designate, at the level of the sea, the shortest curvilineal distance of the two stations whose difference of level is required, and h be the smallest absolute height known nearly, by a provisory calculation, the logarithm of the base K of the hypsometrical triangle which will give the difference of level by one of the formulæ (3), (4), or (5), will be

$$\text{Log } K = \log k + \frac{Mh}{\varrho} - \frac{Mk^2}{24\varrho^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which M is the log modulus $= 0.4342945$, and ϱ the mean radius of the earth in feet. Whence $\log \frac{M}{\varrho} = 2.317894$, and $\log \frac{M}{24\varrho^2} = 3.617793$.

Let $h = 3280.9$ feet, and $k = 118112.4$ feet, $\log k = 5.072295$, consequently in this case $\log \frac{Mh}{\varrho} = 5.833887$, $\frac{Mh}{\varrho} = 0.000068$, $-\frac{Mh^2}{24\varrho^2} = -0.000001$ nearly.

$$\text{Also } u = \frac{k}{N \sin 1''} = Fk \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which u is the angle of the verticals, k the length of the curve passing through them, N the radius of curvature of the arc perpendicular to the meridian, therefore Fk is the angle of the verticals,

or arc in seconds which measures them. Puissant's Géodésie, Table IX., Vol. I.

Let N be the normal, then $N = t + e \sin^2 \lambda$, where t is the radius of the equator, and e the eccentricity. (See page 195.)

Ex. 1.—$\text{Log } k = 5.072295$ $\quad \delta' = 90° \; 46' \; 26''.4$

$$\frac{Mk}{\varrho} = +000068 \qquad \delta = 89 \; 33 \; 0.0$$

$$-\frac{Mk^2}{24\varrho^2} = -000001 \qquad \delta' - \delta = 1 \; 13 \; 26.4$$

$$\text{Log K} = 5.072362 \qquad \tfrac{1}{2}(\delta' - \delta) = 36 \; 43.2$$

Log K		= 5.072362		
Log secant $\frac{1}{2}u$	9′ 41″.4	= 0.000002	log tangent .	7.4501
Log tan $\frac{1}{2}(\delta' - \delta)$	36 43 .2	= 8.028645	log tangent .	8.0286
Log (1) 1261.86 feet		= 3.101009	. . .	3.1010
Log (2) + 0.03 log Part II.		. . . . . .		8.5797
dE		= 1261.89		

This calculation supposes the refraction to be the same at both stations. From the result also it is clear, that the first term is sufficiently accurate for any purpose to which the method can be applied. If the series of operations be terminated at both extremities by the ocean, the algebraic sum of the dE, or $\Sigma d\text{E} = 0$, if the operations be correct, and the two points of the ocean on the same level. To arrive at satisfactory conclusions, it is necessary to procure two equations of condition by means of two stations both to the right and left.

5. Formula (4), in like manner may be employed, on the supposition that the mean coefficient of refraction is $0.08 = n$, then $\frac{2n-1}{2} = -0.42$.

Let $K = 164045$ feet, $\delta' = 88° \; 39'$, then $u = 1614''.8$, $\frac{1}{2}u = 807''.4 = 13' \; 27''.4$.

$$\delta' = 88° \; 39' \; 0''$$

$$\frac{2n-1}{2}u = - \; 11 \; 18$$

$$\delta_{,} = 88 \; 27 \; 42$$

$$90$$

$$90° - \delta_{,} = 1 \; 32 \; 18$$

Log K		164045 feet	$= 5.214963$	
Log sec $\frac{1}{2}u$	$0^\circ\ 13'\ 27''.4$		$= 0.000003$ tan . .	7.5926
Log cot δ,	88 27 42		$= 8.429032$ . .	8.4290
1st, dE'	$=$	4405.53 feet	$= 3.643998$ . .	3.6440
		0.46	. . Second term,	9.6656
dE	$=$	4405.99 feet		

6. The same result nearly may be obtained from formula (5).

Log K as before	$= 5.214963$ Log $K^2 =$	0.429926
Log sec $\frac{1}{2}u$	$= 0.000003$	0.000003
Log cot δ' $88^\circ\ 39'$	$= 8.372292$ cosec^2 . .	0.000242
Log 1st term 3865.97	$= 3.587258$ const. log q .	2.301455
Log 2d term 539.05		2.731626
dE $=$ 4405.02 feet.		

From these formulæ, investigated by Puissant, Lieut. Col. Corabœuf, and Lieut. Peytier, have completed a series of levels along the Pyrenees from the Atlantic to the Mediterranean, though the results have not yet been published.

7. It has been shown, page 39, that $R^2\ CT =$
AB $\tan(A + \frac{1}{2}\text{comp. }A)\tan A$. Let $\pi = 180^\circ$, then $\left(\frac{\pi}{2} - A\right) + \frac{1}{2}A$
$= \frac{\pi}{2} - \frac{1}{2}A$, or half an arc added to its complement is equal to the complement of that arc, whence D being the depression,

$$R^2TC = AB \cot D \cot \tfrac{1}{2}D \quad . \quad . \quad . \quad (8)$$

It more frequently happens, however, that AB is the quantity required, and TC the mean radius of the earth is known as in the case of trigonometrical surveying, when the elevation of the stations above the sea forms a necessary part of the operations.

8. Let TC the mean radius of the earth be denoted by ϱ, AB the elevation by dE, the zenith distance of the horizon by δ, or depression $\delta - 90^\circ$ by D, and R'' an arc equal to the radius in seconds; then when the depression is small, or not exceeding a few minutes, as is usually the case, the value of D may be taken in seconds denoted by D'', and supposing the radius R equal to unity formula (8) will become, $\varrho = dE \times \frac{R''}{D''} \times \frac{R''}{\frac{1}{2}D''} = \frac{2dE\ R''^2}{D''^2}$ . . . . (9)

From this is derived, $dE = \frac{\varrho}{2} \times \frac{D''^2}{R''^2}$ . . . . (10)

But if AT be designated by Δ, or the arc BT, which is nearly equal to it, then, $dE = \frac{\Delta.D''}{2R''}$ nearly . . . . . (11)

The height of an object above the sea is therefore determined from the observed depression and mean radius of the earth, (3,) or distance of the visible horizon, (4,) though this last method will seldom be applicable.

9. Let the mean effect of refraction = 0.08, as formerly, be denoted by n, then formula (3) will become

$$dE = \frac{\varrho}{2}(1+n)^2 \tan {}^2D \quad . \quad , \quad . \quad . \quad (12)$$

$$\text{Or, } dE = \frac{\varrho}{2}(1+n)^2 \times \frac{D''^2}{R''^2} \quad . \quad . \quad . \quad . \quad (13)$$

10. When the upper station is very high the difference of refraction on that account formerly supposed to be the same at both stations must be attended to.

Let $\alpha = 0.000293876$, the coefficient of refraction according to Delambre, p the barometric pressure at the upper station, τ the temperature of the mercury in the barometer by the attached thermometer, β the expansion of air for 1° of Fahrenheit by the detached thermometer, and t the temperature of the atmosphere; then

$$dE = \frac{\varrho}{2}\tan^2 D + \alpha\varrho\left\{1 - \frac{1}{1+\beta\,(t-50^\circ)} \times \frac{p}{30} \times \frac{1}{1 + \frac{\tau - 50^\circ}{10000}}\right\} \quad (14)$$

in which $\frac{\varrho}{2} = 10443840$ feet; log . . . 7.018860

and $\alpha\varrho = 6138.38785$ feet; log . . . 3.788054

The logarithms of the three last factors may be taken from XVIII, XIX, and XX of the General Tables, (page 89.)

Ex. 1. From example 33, (page 51,) $D = 19' 51''.28$ after being corrected for refraction, and $dE = 350$ feet, whence by formula (9) ϱ will be found thus :—

$2\,dE = 700$ feet, log		2.845098
$R''^2 = 2 \log R''$ (Table LXIII. page 112) — 10.	=	0.628850
$D''^2 = 2 \log D'' = 2 \log 1191''.28$, ar. co.	=	3.847972
$\varrho = 3974.5$ miles $= 20985500$ feet log	=	7.321920

and $2\varrho = 7949$ miles the mean diameter of the earth.

Ex. 2. Again suppose the mean radius of the earth, or ϱ, to be known, and the depression, D, to be found by observation, then formula (10) will give the height, or dE.

$\frac{\varrho}{2}$ log (Table LXIII. or formula 13)	7.018860
R''^2 . . ar. co. log , . . .	9.371150
$D''^2 = 2 \log D'' = 2 \log 1191''.28$ . . .	6.152028
$dE = 348.37$ feet log . . . , .	2.542038

the height of the Caltonhill above the sea in feet, about 1.63 feet less than 350.

Ex. 3. Captain Kater observed the depression of the sea from the station at Folkstone, or D, to be 23′ 37″, whence by formula (13) dE will be found as follows :—

$\frac{\varrho}{2}$ log	. . . .	7.018860
$(1+n)^2 = (1.08)^2 = 2 \log 1.08 =$	. . . .	0.066848
R''^2	ar. co. log as before	9.371150
D''^2	$= 2 \log D'' = 2 \log 1417''$	6.302740
dE	$= 575$ feet	2.759598

Ex. 4. The depression of the horizon of the sea from the station on Leith hill was 30′ 42″, required the elevation ?

Since the first three logs are constant and their sum is 6.456858, consequently

To constant log	. . . . . .	6.456858
Add log D''^2, or $2 \log D'' = 2 \log 1842'' =$	.	6.530580
And $dE = 971.5$ feet	. . . . . .	2.987438

In this manner any number of heights may be computed very expeditiously.

Ex. 5.	At Tolsford . .	D = 23′ 39″.7	H = 577.1 feet
6.	Fairlight . .	= 23 48 .5	= 585.1
7.	Stede Hill . .	= 23 56 .0	= 590.0
8.	Crowborough .	= 27 58 .0	= 806.2
9.	Blancnez . .	= 20 33 .3	= 435.5

11. In strictness, the normal should be used in place of the mean radius of the earth, when the direction of the horizon is perpendicular to the meridian, and the radius of curvature when on the meridian, or when it is oblique to it; but the difference of the results in all ordinary cases, derived from these more accurate suppositions, will in general be much less than the errors of observation.

Should it be thought necessary to adopt these refinements, the normal $n = \iota + e \sin^2 \lambda$ . . . . . (1)

the radius of curvature, $\varrho' = \iota\,(1 - 2\epsilon + 3\epsilon \sin^2 \lambda)$ . . (2)

See pages 195, 196, and 197 for an explanation of the characters.

Since the length of a degree is proportional to the radius of curvature, by substituting n for p, and ϱ' for m in formula (15,) page 197, the value of ϱ'' the oblique radius of curvature will be obtained, or

$$\varrho'' = \frac{n\varrho'}{n - (n - \varrho') \sin^2 a} = \frac{\varrho'}{1 - \dfrac{n - \varrho' \sin^2 a}{n}} \quad . \quad (3)$$

It may be remarked, that when the height is determined in this manner from observations on the depression of the horizon of the sea, the time of the tide should be known in order to make the necessary allowance for the rise or fall.

TABLE VI.

For obtaining A, B, C, and D, in formulæ (1) and (2), page 251, and taking the natural sines, &c. to three places of decimals from Table VI., page 63 of the General Tables, which is sufficiently extensive for this purpose, the first series of tables for correcting the mean places of the fixed stars, after the manner of Bessel and Fallows, will be found remarkably convenient.

ARGUMENTS.							
	⊙	☊	⊙	☊	⊙	⊙	
°	A′	A″	D′	D″	B	C	°
0	—0.000+	—0.000+	+0.060+	+0.921+	+0.000—	+1.000+	360
10	0.009	0.057	0.057	0.908	0.174	0.985	350
20	0.017	0.112	0.046	0.867	0.342	0.940	340
30	0.023	0.164	0.030	0.801	0.500	0.866	330
40	0.026	0.211	+0.010+	0.711	0.643	0.766	320
50	0.026	0.252	—0.010—	0.600	0.766	0.643	310
60	0.023	0.286	0.030	0.470	0.866	0.500	300
70	0.017	0.311	0.046	0.325	0.940	0.342	290
80	—0.009+	0.327	0.057	0.170	0.985	+0.174+	280
90	0.000	0.333	0.060	+0.009+	1.000	0.000	270
100	+0.009—	0.329	0.057	—0.153—	0.985	—0.174—	260
110	0.017	0.315	0,046	0.311	0.940	0.342	250
120	0.023	0.292	0.030	0.461	0.866	0.500	240
130	0.026	0.259	—0.010—	0.597	0.766	0.643	230
140	0.026	0.217	+0.010+	0.714	0.643	0.766	220
150	0.023	0.169	0.030	0.810	0.500	0.866	210
160	0.017	0.116	0.046	0.881	0.342	0.940	200
170	0.009	0.059	0.057	0.925	0.174	0.985	190
180	+0.000—	—0.000+	+ 0.060+	—0.940—	+0 000—	—1.000—	180
	A′	A″	D′	D″	B	C	

$$\text{Formulæ, } \begin{cases} 1 . A = t + A' + A'' \\ 11 . D = D' + D'' \end{cases}$$

In the example, page 251, ☊ will be found by Table LXIX. to be 278°.4, and ⊙ by Table LXXVI. to be 117°.2; hence

t for July 19th, by Table LV. is + 0.546

A′ by the table above for 117°.2, + 0.021; and D′ is = —0.034

A″ . . . 278 .4, + 0·328 D″ = +0.144

A = t + A′ + A″ . . = + 0.895; D = D′ + D″ = +0.110

Also to 117°.2, B is + 0.887, and C is — 0.456.

To exemplify these tables, and the series from XXXIV. to XLVIII. inclusive, let the apparent right ascension and declination of Rigel on the 25th of December, 1838, at the time of upper culmination, or about 11^h P. M., be computed by both sets of tables, taking its mean place from Table LIV.; and the solar nutation from Table LXXII, when the second series is employed, recollecting that when the declination is south to change the sign.

Ans. True R. A. 5^h 6^m 49.41. Dec. 8° 23′ 27″.0 S. by first series.
5 6 49.42 8 23 26 .8 by second.

It ought to be remembered, that when the first series of tables are used, the mean place of any star is brought up by its annual variations to the beginning of the year only, as the part of a year is allowed for by introducing t into the formulæ; but when the second set are used, the mean place must be obtained for the time required.

EXAMPLE.

Mean R. A. 1830, Table LIV.	$= 5^h\ 6^m\ 22^s.20$	dec.	$8°\ 24'\ 15''.0$ S.
Variation in 8 years .	+ 23.04		— 36 .8
Mean R. A. 1838 . .	5 6 45.24	dec.	8 23 38 .2 S.
Correction in R. A. .	+ 4.17	dec.	— 11 .2
True R. A. Decr. 25th, at 11^h P. M.	5 6 49.41	dec.	8 23 27 .0 S.

Here $\odot = 273°.8$ Table LXXIV; ☊ $= 359°.2$ Table LXIX; $*$'s R. A. $= 76°.7$; $2 = \odot\ 187°.6$; $2\odot - * = 111°$ nearly.

t for December 25.5th is .	$= + 0.982$	
A′ for 273°.8 . .	$= - 0.003$ and	D′ $= - 0.059$
A″ for 359 .2 . .	$= + 0.005$.	D″ $= + 8\ 920$
A is therefore . .	$= + 0.984$.	D $= + 0.861$
B $= -0.994$		C $= + 0.066$

Whence, after the examples in pages 251 and 252,

$Aa = +2^s.843$ . . .	$Aa' = -4''.526$
$Bb = +1.326$ . . .	$Bb' = +0.645$
$Cc = -0.020$ . . .	$Cc' = +0.700$
$Dd = +0.019$ . . .	$Dd' = -8.050$

Corrections $= +4.168$ in R. A. and in Dec. . $= -11.231$

TABLE VII.

For adapting the augmentation of the moon's semi-diameter from Table XXI., to the true or apparent altitude, or zenith distance.

Moon's Alt.	For Moon's True Alt. or Z. D. Moon's Semidiameter. 14′ 30″	15′ 30″	16′ 30″	For Moon's App. Alt. or Z. D. Moon's Semidiameter. 14′ 30″	15′ 30″	16′ 30″	Moon's Z. D.
0°	−0″.10	−0″.14	−0″.17	+0″.10	+0″.14	+0″.17	90°
10	−0 .09	−0 .12	−0 .15	0 .11	0 .14	0 .17	80
20	−0 .07	−0 .09	−0 .11	0 .12	0 .15	0 .19	70
30	−0 .03	−0 .04	−0 .05	0 .13	0 .17	0 .21	60
40	+0 .02	+0 .03	+0 .04	0 .15	0 .20	0 .24	50
50	+0 .08	+0 .10	+0 .13	0 .17	0 .22	0 .27	40
60	+0 .13	+0 .16	+0 .21	0 .18	0 .24	0 .29	30
70	+0 .17	+0 .23	+0 .28	0 .21	0 .26	0 .32	20
80	+0 .20	+0 .26	+0 .32	0 .21	0 .27	0 .33	10
90	+0 .21	+0 .28	+0 .34	0 .21	0 .28	0 .34	0

TABLE VIII.

Equations of Third Differences for Twelve Hours.

Time after Noon or Midnight.		Third Differences.														
+	—	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	10″	20″	30″	40″	50″
h m	h m	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0 0	12 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0 30	11 30	0.2	0.4	0.5	0.7	0.9	1.1	1.3	1.5	1.6	1.8	0.0	0.1	0.1	0.1	0.2
1 0	11 0	0.3	0.6	1.0	1.3	1.6	1.9	2.2	2.5	2.9	3.2	0.1	0.1	0.2	0.2	0.3
1 30	10 30	0.4	0.8	1.2	1.6	2.1	2.5	2.9	3.3	3.7	4.1	0.1	0.1	0.2	0.3	0.3
2 0	10 0	0.5	0.9	1.4	1.9	2.3	2.8	3.2	3.7	4.2	4.6	0.1	0.2	0.2	0.3	0.4
2 30	9 30	0.5	1.0	1.4	1.9	2.4	2.9	3.4	3.8	4.3	4.8	0.1	0.2	0.2	0.3	0.4
3 0	9 0	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.7	4.2	4.7	0.1	0.2	0.2	0.3	0.4
3 30	8 30	0.4	0.9	1.3	1.7	2.2	2.6	3.0	3.4	3.9	4.3	0.1	0.1	0.2	0.3	0.4
4 0	8 0	0.4	0.7	1.1	1.5	1.9	2.2	2.6	3.0	3.3	3.7	0.1	0.1	0.2	0.2	0.3
4 30	7 30	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.3	2.6	2.9	0.0	0.1	0.1	0.2	0.2
5 0	7 0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	0.0	0.1	0.1	0.1	0.2
5 30	6 30	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	0.0	0.0	0.1	0.1	0.1
6 0	6 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0
+	—															

TABLE IX.

Equations of Fourth Differences for Twelve Hours.

Time after Noon or Midnight.		Fourth Differences.									
		1′	2′	3′	4′	5′	10″	20″	30″	40″	50″
h m	h m	″	″	″	″	″	″	″	″	″	″
0 0	12 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0 30	11 30	0.2	0.4	0.6	0.8	1.0	0.0	0.1	0.1	0.1	0.3
1 0	11 0	0.4	0.8	1.2	1.6	2.0	0.1	0.1	0.2	0.3	0.2
1 30	10 30	0.6	1.2	1.7	2.3	2.9	0.1	0.2	0.3	0.4	0.5
2 0	10 0	0.7	1.5	2.2	3.0	3.7	0.1	0.2	0.4	0.5	0.6
2 30	9 30	0.9	1.8	2.7	3.6	4.5	0.1	0.3	0.4	0.6	0.7
3 0	9 0	1.0	2.1	3.1	4.1	5.1	0.2	0.3	0.5	0.7	0.9
3 30	8 30	1.1	2.3	3.4	4.6	5.7	0.2	0.4	0.6	0.8	0.9
4 0	8 0	1.2	2.5	3.7	4.9	6.2	0.2	0.4	0.6	0.8	1.1
4 30	7 30	1.3	2.6	3.9	5.2	6.5	0.2	0.4	0.7	0.9	1.1
5 0	7 0	1.4	2.7	4.1	5.4	6.8	0.2	0.5	0.7	0.9	1.1
5 30	6 30	1.4	2.8	4.2	5.6	7.0	0.2	0.5	0.7	0.9	1.2
6 0	6 0	1.4	2.8	4.2	5.6	7.0	0.2	0.5	0.7	0.9	1.2

TABLE X.

To Reduce a Distance Measured on an Inclined Plane to Horizontal Measure.

Angle.	Red.	Angle.	Red.	Angle.	Red.	Angle.	Red.	Angle.	Red.	Angle.	Red.
° ′		° ′		° ′		° ′		° ′		° ′	
0 20	.0000	4 20	.0029	8 20	.0106	12 20	.0231	16 20	.0404	20 20	.0623
0 40	.0001	4 40	.0033	8 40	.0114	12 40	.0243	16 40	.0420	20 40	.0643
1 0	.0002	5 0	.0038	9 0	.0123	13 0	.0256	17 0	.0437	21 0	.0664
1 20	.0003	5 20	.0043	9 20	.0132	13 20	.0270	17 20	.0454	21 20	.0685
1 40	.0004	5 40	.0049	9 40	.0142	13 40	.0283	17 40	.0472	21 40	.0707
2 0	.0006	6 0	.0055	10 0	.0152	14 0	.0297	18 0	.0489	22 0	.0728
2 20	.0008	6 20	.0061	10 20	.0162	14 20	.0311	18 20	.0508	22 20	.0750
2 40	.0011	6 40	.0068	10 40	.0173	14 40	.0326	18 40	.0526	22 40	.0772
3 0	.0014	7 0	.0075	11 0	.0184	15 0	.0341	19 0	.0545	23 0	.0795
3 20	.0017	7 20	.0082	11 20	.0195	15 20	.0356	19 20	.0564	23 20	.0818
3 40	.0020	7 40	.0089	11 40	.0207	15 40	.0372	19 40	.0583	23 40	.0841
4 0	.0024	8 0	.0097	12 0	.0219	16 0	.0387	20 0	.0603	24 0	.0865

SINCE the preceding part of the Appendix on Trigonometrical Surveying was printed, I have had a correspondence with Mr Ivory, communicating some remarks on his formulæ, and a new solution of a very useful problem in geodetical operations, of which no direct solution had been previously given. It is contained in the following

PROPOSITION.

To find the shortest distance between two points on the surface of the earth, considered an oblate spheroid, when their geographical position is given.

Let s = an arc of a curve on the surface of the terrestrial spheroid.

σ = an arc of a great circle on the surface of the circumscribing sphere, of which the extremities have the same latitudes and the same difference of longitude as the two points on the surface of the spheroid.

α = an arc of the great circle of which σ is a part between its intersection with the equator and the more distant extremity of σ.

α' = an arc in like manner between its intersection and the nearest extremity of σ; consequently

$\alpha - \alpha' = \sigma$.

i = the inclination to the equator of the great circle of the celestial sphere which passes through the two given points, and $\varkappa$ = the colatitude.

$$\text{Cos}\, i = \sin \varkappa \sin \mu = \sin \varkappa' \sin \mu', \ \sin \alpha = \frac{\sin \lambda}{\sin i}, \text{ and } \sin \alpha' = \frac{\sin \lambda'}{\sin i},$$

$$s = \sigma \left\{ 1 - \frac{e^2}{4} \sin^2 i + \frac{3e^4}{16} \sin^2 i \cos^2 i - \frac{3e^4}{64} \sin^4 i. + \frac{e^4}{8} \sin^2 i \cos^2 i \frac{\sigma}{\sin \sigma} \cos \alpha \cos \alpha' \right\}$$
$$- \left\{ \frac{3e^2}{4} \sin^2 i - \frac{3e^4}{16} \sin^2 i \cos^2 i + \frac{3e^4}{16} \sin^4 i \right\} \sin \sigma \cos (\alpha + \alpha')$$
$$+ \frac{15e^4}{128} \sin^4 i \sin 2\sigma \cos (\alpha + \alpha') \quad . \quad . \quad (14)$$

Should the two points fall upon a great circle perpendicular to the equator, that is, on the meridian, $\sin i = 1$, $\cos i = 0$, $\alpha = \lambda$ and $\alpha' = \lambda'$, $\sigma = \lambda - \lambda'$, and consequently formula (14) becomes

$$s = (\lambda - \lambda') \left(1 - \frac{e^2}{4} - \frac{3e^4}{64}\right) - \left(\frac{3e^2}{4} + \frac{3e^4}{16}\right) \sin (\lambda - \lambda') \cos (\lambda + \lambda')$$
$$+ \frac{15e^4}{128} \sin 2 (\lambda - \lambda') \cos 2 (\lambda + \lambda') \quad (15)$$

If the terms involving e^4 be omitted as being almost insensible, recollecting (page 270) that $\frac{e^2}{2} = \varepsilon$, $e^2 = 2\varepsilon$, $2e^2 = 4\varepsilon$, $\frac{e^2}{4} = \frac{\varepsilon}{2}$, $\frac{3e^2}{4} = \frac{3\varepsilon}{2}$, and introducing the radius of the equator a, formula (14) becomes

$$s = a\,\sigma \sin 1''\left(1-\frac{\iota}{2}\sin^2 i\right)-\frac{3\iota}{2}\sin^2 i \sin \sigma \cos(\kappa+\kappa') \quad . \quad . \quad (16)$$

And formula (15) becomes

$$s = a \sin 1''\,(\lambda-\lambda')\left(1-\frac{\iota}{2}\right)-\frac{3\iota}{2}\sin(\lambda-\lambda')\cos(\lambda+\lambda') \quad . \quad . \quad (17)$$

To compute the azimuths or m and m',

Let $x = \cos\frac{\lambda+\lambda'}{2}\sin\frac{\lambda-\lambda'}{2}$, and $y = \sin\frac{\lambda+\lambda'}{2}\cos\frac{\lambda-\lambda'}{2}$

$$\text{Log}\sin(\mu-m) = \log\left(\cos\lambda \sin m\, 4\iota\frac{x}{\sin\sigma}\right)+\text{M}2\,\iota y \sin\lambda \quad . \quad (18)$$

$$\text{Log}\sin(m'-\mu') = \log\left(\cos\lambda' \sin m'\, 4\iota\frac{x}{\sin\sigma}\right)+\text{M}2\,\iota y \sin\lambda' \quad . \quad (19)$$

These two serve to compute μ and μ' when m and m' are given. The same formulæ will likewise determine m and m' when μ and μ' are given by first substituting $\sin\mu$ for $\sin m$, and $\sin\mu'$ for $\sin m'$ to find approximate values of m and m'. These being used in a second operation will bring out the required quantities with sufficient accuracy.

Ex. 1.—In an extensive series of operations lately performed by the French engineers to connect the island of Corsica with the Continent,

$\lambda = 43°\ 48'\ 53''.41$, colat. or $\kappa = 46°\ 11'\ 6''.59$

$\lambda' = 43\ \ 16\ \ 32\ .61$, colat. or $\kappa' = 46\ \ 43\ \ 27\ .39$

$\kappa+\kappa' = 92\ \ 54\ \ 33\ .98$

Difference of longitude, . $\frac{\kappa+\kappa}{2} = 46\ \ 27\ \ 16\ .99$

Or, $\omega = 0°\ 37'\ 58''.85$ $\kappa'-\kappa = 0\ \ 32\ \ 20\ .80$

$\frac{\omega}{2} = 0\ \ 18\ \ 59\ .425$ $\frac{\kappa'-\kappa}{2} = 0\ \ 16\ \ 10\ .40$

By the analogies of Napier, Theorem VIII., cor. page 93.

1. As sin	46°	27′	16″.99		. . .	9.860236
Is to sin	0	16	10 .40		. . .	7.672524
So is cot	0	18	59 .425		. . .	12.257734
To tan	49	35	56 .8	(1)	. .	10.070022
2. As cos	46°	27′	16″.99		. . .	9.838174
Is to cos	0	16	10 .40		. .	9.999995
So is cot	0	18	59 .425		. .	12.257734
To tan	89	46	55 .0	(2)	. .	12.419555
	49	35	56 .8	(1)		
$\mu =$	139	22	51 .8			
$\mu' =$	40	10	58 .2			

3.	$\kappa = 46^\circ\ 11'\ 6''.59$	sin	9.858285
	$\mu = 139\ \ 22\ \ 51\ .80$	sin	9.813598
	$i =$	cos	9.671883
		sin	9.945858
4.	$\lambda = 43^\circ\ 48'\ 53''.41$	sin	9.840313
	$i =$	sin	9.945858
	$\alpha = 51\ \ 39\ \ 5\ .5$	sin	9.894455
	$\lambda' = 43\ \ 16\ \ 32\ .61$	sin	9.836014
	$i =$	sin	9.945858
	$\alpha' = 50\ \ 56\ \ 37\ .0$	sin	9.890156
	$\alpha - \alpha' = \sigma = 0\ \ 42\ \ 28\ .5$		
	$\alpha + \alpha' = 102\ \ 35\ \ 42\ .5$		

5. $\sigma = 0^\circ\ 42'\ 28''.5 = 2548''.5$ log 3.406285 $\frac{s}{2}$ log . . 7.210185

1″ log sin . . 4.685575 $\sin^2 i$ log . 9.891716

a log T. LXIII. . 7.320586 -0.0012645 No 7.101901

$+1.0$

9.999450 log $+0.9987355$

(1) 258164 feet log	5.411896	a log	7.320586
		$\frac{3\,s}{2}$ log	7.687306
		$\sin^2 i$ log	9.891716
		σ sin	8.091849
		$\alpha + \alpha' = 102^\circ\ 35'\ 42''.5$ cos	$-$9.338577
(2) 214		(change sign) log	$-$2.330034

$s = 258378$ feet, or about 20 feet less than the result given by Puissant and Ivory, chiefly on account of a small difference in the value of s.

Ex. 2. From formulæ (18) and (19) are found

$m = 139^\circ\ 17'\ 4''$

$m' = 40\ \ 16\ \ 46$

$$\lambda = 43^\circ\ 48'\ 53''.41$$
$$\lambda' = 43\ \ 16\ \ 32\ .61$$
$$\lambda - \lambda' = 0\ \ 32\ \ 20\ .80$$
$$\frac{\lambda - \lambda'}{2} = 0\ \ 16\ \ 10\ .40$$
$$\lambda + \lambda' = 87\ \ 5\ \ 26\ .02$$
$$\frac{\lambda + \lambda'}{2} = 43\ \ 32'\ 43\ .01$$

Whence by formula (8,) omitting the last term,

$\frac{\lambda - \lambda'}{2} =$	$0^\circ\ 16'\ 10''.4$	cos	9.999995
$\frac{\lambda + \lambda'}{2} =$	43 32 43 .0	cosec	0.161826
$\frac{m + m'}{2} =$	89 46 55 .0	cot	7.580447
$\frac{\omega}{2}$	0 18 59 .43	tan	7.742268
	2		

ω = 0 37 58 .86 = the true difference of longitude, without using the last part of the formula, which ought to be deleted.

ON THE MEASUREMENT OF AN ARC OF THE MERIDIAN.

THE operations necessary to be performed to complete a survey of this kind are,

1. The measurement of a base towards one extremity of the arc, on a plane as nearly level as possible, in the manner directed in page 32. This must be executed with all the care which the present state of the sciences can accomplish, especially by correcting the length of the base measured for the expansions and contractions to which the chains or rods employed are liable from changes of temperature.

2. The bearing or angle which this base, or some other of the sides of the series of triangles connected with this base, makes with the meridian. The position of the meridian may be determined with a good theodolite or astronomical circle, but more correctly by one of the best transit instruments, and this operation ought to be performed at least twice,—once at each extremity of the arc measured. The angle which some of the sides of the adjacent triangles makes with the meridian, must then be accurately determined.

3. The angles, at various points carefully selected, forming a series of triangles as nearly equilateral as possible throughout the whole, are to be measured with great care by a good theodolite or repeating circle.

4. Towards the termination of the series of triangles, the measurement of a base of verification performed with the same care as the first base at the commencement of the series, to be compared with the same determined trigonometrically from the first base, which, if all the operations be executed carefully, should agree nearly. If a small difference be found in these, the sides of the triangles may be computed backwards to the first base from the second, and the mean of each of the sides thus found should be taken. In cases of great nicety the computations should be performed at least two different ways, in order to detect any error that may have been committed in either.

5. The latitudes of the extremities of the arc, or of two points adjacent and reduced to them, must be determined by the astronomical or repeating circle, from numerous observations on the same stars at the same time nearly, if convenient; so that any small error in the mean places of the stars, or in the necessary reductions, may thus be rendered insensible.

6. To ensure accuracy as far as possible, a secondary series of triangles should be deduced from different combinations of the angles, which, if both are correct, will give the same length of the arc measured nearly. This second series is generally indicated by dots in the plan of the triangles, to distinguish them from the primary series which is formed by lines.

7. In geodetical operations the angles of each triangle must be properly prepared as directed in section IV., pages 190 and 191; and also one side, called the base (already described), accurately measured in feet or toises, as they were in England and France in the trigonometrical survey and measurement of an arc of the meridian of Paris respectively.

8. The next step, if Legendre's method be employed, is to deduct the third part of the spherical excess from each of the angles, as formerly shown in page 276, § 7, in the first example.

If Delambre's method, by spherical trigonometry, be adopted, which, to show the mode of operation, will be employed in the second example, the seconds in this base, considered as an arc of a great circle on the earth's surface, must be determined, and thence its sine inferred.

For this purpose let t be the number of toises in the base; t'' these converted into seconds; d the toises in a degree upon the earth's surface at the given latitude; and d'' the number of seconds in a degree, equal to $60' \times 60'' = 3600''$; then $t : d :: t'' : d''$, whence,

$$t'' = \frac{d''t}{d} = 3600'' \frac{t}{d} \quad . \quad . \quad \text{(A)}$$

Or, $\log t'' = \text{Const. } \log 3.556303 + \log t - \log d \quad . \quad \text{(B)}$

Now, let a be the measured base, and b and c the sides to be computed, then,

$$\sin b = \sin a \frac{\sin B}{\sin A}, \ \sin c = \sin a \frac{\sin C}{\sin A}$$

To perform this computation it is necessary to have $\sin a$ expressed in feet or toises, that $\sin b$ and $\sin c$ may be determined in the same measure.

9. Let a be any small arc in toises (as these formulæ will be illustrated in toises), and r the earth's radius also in toises, then, by formulæ (32) and (33), page 14, Introduction, for small arcs,

$$\frac{\sin a}{r} = \frac{a}{r} - \frac{a^3}{6r^3} + \&c. \qquad \frac{\cos a}{r} = 1 - \frac{a^2}{2r^2} + \&c.$$

$$\sin a = a - \frac{a^3}{6r^2} + \&c. \qquad \cos a = r - \frac{a^2}{2r}$$

$$\sin a = a\left(1 - \frac{a^2}{6r^2}\right) \ldots (1) \qquad \cos a = r\left(1 - \frac{a^2}{2r^2}\right) \text{ and}$$

$$\left(\frac{\cos a}{r}\right)^{\frac{1}{3}} = \left(1 - \frac{a^2}{2r^2}\right)^{\frac{1}{3}} = 1 - \frac{a^2}{6r^2} \ldots (2)$$

Whence, by substituting for $1 - \frac{a^2}{6r^2}$ its equivalent $\frac{\cos a}{r}$ in formula (1), it becomes,

$$\sin a = a\left(1 - \frac{a^2}{6r^2}\right) = a\left(\frac{\cos a}{r}\right)^{\frac{1}{3}} \text{ in toises,} \quad . \quad . \quad . \quad \text{(C)}$$

From this it follows that to radius unity,
$\text{Log} \sin a = \log a + \frac{1}{3} \log \cos a = \log a - \frac{1}{3} \log \sec a$* $\quad . \quad$ (D)

Example.—Let $a = 14088.2858$ toises, and $d = 57020$ toises, then by formula B,

---

* On similar principles the rules for finding the degrees, &c. in small arcs, pages 231 and 232 of Introduction, are founded, and $2 \sin \frac{1}{2} a = a - \frac{1}{3} \cdot \frac{1}{8} a^3 = \gamma$ the chord.

Const. log of 3600″	. . .	3.556303
a = 14088.2858 toises, log	. .	4.148858
d = 57020 toises, *a. c. l.*	. .	5.243973
a'' = 889″48 = 14′ 49″.48	. . log a''	2.949134

By Formula D,

a = 14088.2858 log	. .	4.1488584
a'' = 14′ 49″.48	$\frac{1}{3}$ log cos . .	9.9999986
Log sin a in toises	. . .	4.1488570

The method of finding the length of a in toises, from log sin a in toises, or the converse of the preceding, is readily derived from formula (D).

Log sin a in toises	. . .	4.1488570
— $\frac{1}{3}$ log cos a'', or, + $\frac{1}{3}$ log sec a''	.	+ 14
a = 14088.2858	. log a in toises* .	4.1488584

10. The method of finding the latitude is shown by example 3, page 105, or by example 8, page 112, and in the explanation of Table XXV., while the direction of the meridian may be obtained in the manner there indicated also, though that by the transit instrument is preferable. Whence the angle between the meridian, and a side of one of the triangles, called the bearing, may be measured, and, consequently, that of all the rest may be found. In this way the latitude and direction of the meridian of the Observatory of Edinburgh, the northern extremity of the following arc, were determined, as shown by the examples, pages 105, 112, 240, &c., and the mean results of each series are as follows :—

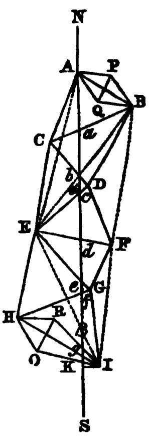
I. Latitude.

1. By the Author . . .	55° 57′ 20″.00 N.
2.	19 .30
3.	18 .60
4.	21 .60
5.	22 .00
6. By Captain Basil Hall . .	15 .00
7.	19 .00
Mean of the whole	55 57 19 .36 N.

* See page 300, first triangle, second series.

II. Bearings.

BAN	.	117° 8′ 2″.5	CAN	.	163° 32′ 1″.2
		1 .4			0 .3
		2 .0			1 .0
		1 .5			2 .0
		2 .0			0 .8
		1 .2			0 .1
		2 .0			0 .4
Mean .	.	117 8 1 .8	. .		163 32 0 .8



In the figure, NS is the meridian passing through A, Edinburgh Observatory, PQ the reduced measured base, AB, AC, two sides of one of the primary triangles, whose inclinations to the meridian are BAN and CAN.

The base PQ, by measurement at the temperature of 62° Fahrenheit, was 65488.645 feet, making an angle of 20′.4 with the horizon, at the height of 156 feet above the mean level of the sea. Hence, by rules page 193, and formula (3), page 267, we have

Const logs .	. 2.6801	. .	2.6264
$B = 65488.645$ log	4.8162	. .	4.8162
$A = 156$ feet, log	2.1931, $\theta = 20'.4$, 2 log	.	2.6193
Feet.			
$B-b = -0.490$	log 9.6894, $B-b' = -1.153$ log		0.0619
$B-b' = -1.153$			

$B-b-b' =$ 1.643 = correction of base

$B =$	. . .	65488.645 feet
Correction	. . .	— 1.643
Correct base at the level of the sea	.	65487.002

11. From the measurement and reduction of angles, according to the method of Legendre, we obtain the following results:—

No	Triangle.	Angles.	Opposite Sides.	
		° ′ ″		
1	BAN	117 8 1.8		
	BAa	62 51 58.2		
2	CAN	163 32 0.8		
	CAa	16 27 59.2		
3	PAQ	54 1 3.7	PQ	65487.0
	APQ	58 30 10.9	AQ	69004.9
	AQP	67 28 45.4	AP	74756.7
4	PBQ	51 31 1.5	PQ	65487.0
	QPB	64 0 1.2	QB	75191.6
	PQB	64 28 57.3	PB	75497.6
5	APB	122 30 12.1	AB	131734.6
	PAB	28 54 11.9	PB	75497.6
	PBA	28 35 36.0	PA	74756.7
6	AQB	131 57 42.7	AB	131734.7
	QAB	25 6 51.9	QB	75191.6
	QBA	22 55 25.4	PB	75497.6
		Mean.	AB	131734.65
7	ACB	49 9 1.5	AB	131734.6
	ABC	51 31 1.1	AC	136325.9
	CAB	79 19 57.4	CB	171143.6
8	CDB	71 12 1.2	CB	171143.6
	DCB	69 27 56.2	DB	169301.5
	CBD	39 20 2.6	CD	114591.1
9	DEC	39 16 31.8	CD	114591.1
	CDE	91 17 46.2	CE	180968.0
	DCE	49 25 42.0	DE	137497.2
10	DFE	57 12 34.8	DE	137497.2
	DEF	51 22 36.7	DF	127783.5
	EDF	71 24 48.5	EF	155028.4
11	EGF	77 39 21.4	EF	155028.4
	GEF	43 39 36.3	FG	109561.1
	GFE	58 41 2.3	EG	135577.2
12	GHE	49 39 58.0	GE	135577.2
	HGE	74 10 3.9	EH	171109.0
	HEG	56 9 58.1	HG	147737.1
13	GIH	55 48 56.6	HG	147737.1
	IHG	56 1 0.4	GI	148088.4
	HGI	68 10 3.0	IH	165782.2
14	OHR	80 6 40.2	RO	97024.3
	HOR	48 16 5.6	HR	73498.3
	HRO	51 37 14.2	OH	77206.1

No	Triangle.	Angles.	Opposite Sides.	
		° ′ ″		
15	RIO	48 19 41.8	RO	97024.2
	IRO	65 12 33.8	OI	117921.8
	ROI	66 27 44.2	IR	119083.8
16	HOI	114 43 50.3	HI	165782.2
	OHI	40 14 43.6	IO	117921.8
	OIH	25 1 26.1	OH	97024.2
17	HRI	116 49 47.7	HI	165782.2
	RHI	39 51 56.2	RI	119083.8
	RIH	23 18 16.1	HR	73498.3
18	AaB	65 37 0.7	AB	131734.6
	ABa	51 31 1.1	Aa	113219.4
	aAB	62 51 58.2	aB	128717.4
19	Cba	44 55 3.1	Ca	42426.2
	aCb	69 27 56.2	ab	56268.5
	Cab	65 37 0.7	bC	54726.9
20	bcD	43 47 10.7	bD	59864.2
	cDb	91 17 46.2	cb	86490.5
	Dbc	44 55 3.1	Dc	61085.5
21	Eec	41 10 36.3	Ec	76411.7
	eEc	95 2 13.0	ce	115611.3
	Ece	43 47 10.7	eE	80309.7
22	efG	64 39 19.8	eG	55267.5
	eGf	74 10 3.9	ef	58833.6
	Gef	41 10 36.3	fG	40262.2
23	Hgf	59 19 39.8	Hf	107474.9
	gHf	56 1 0.4	fg	103614.0
	Hfg	64 39 19.8	Hg	112929.4
24	KgI	59 19 39.8	KI	45458.6
	KIg	30 40 20.2	gK	26961.6
	gKI	90 0 0.0	Ig	52852.8
	Arcs.	Length.	Arcs.	Length.
	Aa	113219.4	Aa	113219.4
	ab	56268.5	Ab	169487.9
	bc	86490.5	Ac	255978.4
	ce	115611.3	Ae	371589.7
	ef	58833.6	Af	430423.3
	fg	103614.0	Ag	534037.3
	gK	26961.6	AK	560998.9
	AK	560998.9		

Latitude of A	λ	55° 57′ 19″.36 N.
of K	λ′	54 25 8 .86 N.
Mean latitude	l'	55 11 14 .11 N.
Diff. of lat.	λ — λ′	=1 32 10 .50
		=1.53625

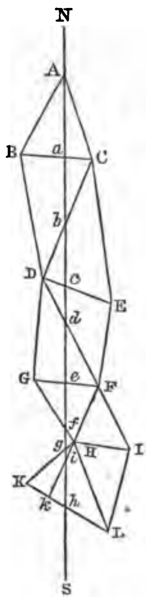
Whence $\frac{\text{A K}}{\lambda-\lambda'} = \frac{560998.85}{1.53625}$ = 365173.8 feet, the length of a degree of the meridian, at the mean latitude 55° 11′ 14″.11 N.

12. The secondary series of triangles, formed by the dotted lines, as ABE, BEF, and EFI, are now to be computed, and the partial arcs Aα, αβ, and βK; and their sum AK should, if all the operations are correct, be the same as before nearly, which forms a strong check on the ultimate conclusions.

No	Triangle.	Angles.	Opposite Sides.		No	Triangle	Angles.	Opposite Sides.	
		° ′ ″					° ′ ″		
	ACE	168 2 39.7	AE	315602.7		ABE	83 0 48.9	AE	315602.7
1	CAE	6 49 18.8	CE	180968.0	5	EAB	72 30 38.6	BE	303265.1
	CEA	5 18 1.5	AC	136325.9		AEB	24 28 32.5	AB	131734.6
	BDE	162 29 47.4	BE	303265.1		BEF	61 2 34.5	FB	265476.4
2	DEB	9 39 57.8	BD	169301.5	6	EBF	30 43 38.5	EF	155028.4
	DBE	7 50 14.8	DE	137497.2		EFB	88 13 47.0	BE	303265.1
	EGI	142 20 6.9	EI	268510.8		EFI	81 47 38.9	EI	268510.7
3	IEG	19 41 39.1	GI	148088.4	7	FEI	63 21 15.4	FI	242476.3
	GIE	17 58 14.0	GE	135577.2		EIF	34 51 5.7	EF	155028.4
4	ABC	51 31 1.1	DEB	9°39′57″.8		AαB	34 7 12.9	AB	131734.6
	CBD	39 20 2.6	DEF	51 22 36 .7	8	ABα	83 0 48.9	Aα	233106.0
Sum	ABD	90 51 3.7	FEG	43 39 36 .3		BAα	62 51 58.2	Bα	209003.1
	DBE	7 50 14.8	GEI	19 41 39 .1		Eβα	21 28 57.2	Eα	94262.0
Diff.	ABα	83 0 48.9	αEβ	124 23 49 .9	9	βEα	124 23 49.9	αβ	212385.6
	BAα	62 51 58.2	Eαβ	34 7 12 .9		Eαβ	34 7 12.9	Eβ	144380.0
Sum		145 52 47.1	Sum	158 31 2 .8		KβI	21 28 57.2	KI	45458.9
		180 0 0		180 0 0	10	βKI	90 0 0.0	βI	124130.7
Diff.	AαB	34 7 12.9	Eβα	21 28 57 .2		KIβ	68 31 2.8	βK	115507.2
	Arcs.		Arcs.						
	Aα	233106.0	Aα	233106.0		From former Series		AK	560998.9
	αβ	212385.6	Aβ	445491.6		Present . .		AK	560998.8
	βK	115507.2	AK	560998.8		Mean . . .		AK	560998.85
	AK	560998.8	And the close agreement shows the accuracy of both.						

This example is partly hypothetical, and was drawn up by the Author chiefly for the exercise of his pupils, to familiarize them with such operations.

PART OF THE MERIDIAN OF PARIS.



13. In the primary series of triangles in the figure forming a part of the meridian of Paris, the deviation of the sides AB, AC, of the triangle ABC from the meridian NS, and the latitude A of Dunkirk, were determined as before. The lengths of these sides are found by computations depending on a measured base, and the angles BAC, ABC, ACB, &c., as in last article, from which the partial arcs A*a*, *ab*, *bc*, &c., are found, and their sum is equal to the whole arc A*h*. The same is determined from the secondary series of triangles forming a check on each other and on the former, and the computations are performed by spherical trigonometry, according to the method of Delambre, in order to show the mode of operation.

In the figure NS is the meridian of Paris.

A is	Dunkerque	G is	Bonnieres
B	Watten	H	Beauquêne
C	Cassel	I	Mailli
D	Fiefs	K	Vignacourt
E	Mesnil	L	Villers Bretonneux
F	Sauti		

First Series of Secondary Triangles.

No	Triangle.	Angles.	Sines of Angles.	Sines of opposite Sides.
1	aAB	25° 19′ 42″.14	9.631246	3.754112
	aBA	74 28 45 .28	9.983867	4.106733
	AaB	80 11 33 .27	9.993606	4.116472
		Dunkerque 0 .69		Aa=12786.00t
2	Cab	80 11 33 .27	9.993606	4.075153
	aCb	79 48 35 .35	9.993095	4.074642
	abC	19 59 51 .85	9.534005	3.615551
		Cassel 0 .47	ab=11875.25t	Ab=24661.25t
3	Dbc	19 59 51 .85	9.534005	3.353452
	bDc	91 11 19 .40	9.999907	3.819354
	bcD	68 48 48 .88	9.969607	3.789054
		Fiefs 0 .13	bc= 6597.12t	Ac=31258.37t
4	cDd	42 59 49 .63	9.833760	3.548233
	Dcd	111 11 11 .12	9.969607	3.684080
	Ddc	25 48 59 .32	9.638978	3.353452
		Fiefs 0 .07	cd= 3533.73t	Ad=34792.10t
5	Fde	25 48 59 .32	9.638978	3.768036
	dFe	54 45 8 .66	9.912044	4.041102
	Fed	99 25 52 .62	9.994090	4.123147
		Sauti 0 .60	de=10992.65t	Ae=45784.75t
6	Gef	99 25 52 .62	9.994090	3.958054
	eGf	51 56 49 .41	9.896218	3.860183
	Gfe	28 37 18 .27	9.680358	3.644323
		Bonnieres 0 .30	ef= 7247.42t	Af=53032.17t
7	Hfg	28 37 18 .27	9.680358	2.857227
	fHg	91 54 0 .07	9.999761	3.176630
	Hgf	59 28 41 .66	9.935223	3.112092
		Beauquêne 0 .00	fg= 1501.86t	Ag=54534.03t
8	Kgh	59 28 41 .66	9.935223	3.905308
	gKh	65 14 50 .05	9.958145	3.928224
	Khg	55 16 28 .83	9.914815	3.884894
		Vignacourt 0 .54	gh= 8476.65t	Ah=63010.68t

Second Series, or Triangles of Verification.

No	Triangle.	Angles.	Sines of Angles.	Sines of opposite Sides.
1	ACb	143 13 41 .52	9.777158	4.392011
	CAb	16 46 27 .59	9.460301	4.075153
	AbC	19 59 51 .85	9.534005	4.148857
		Cassel 0 .96		Ab=24661.23t
2	Dbd	19 59 51 .85	9.534005	3.684080
	bDd	134 11 9 .03	9.855569	4.005645
	bdD	25 48 59 .33	9.638978	3.789054
		Fiefs 0 .21	bd=10130.85t	Ad=34792.08t
3	Fdi	25 48 59 .53	9.638978	4.007523
	dFi	119 22 0 .65	9.940266	4.306811
	Fid	34 49 1 .15	9.756603	4.123147
		Sauti 1 .33	di=20268.10t	Ai=55060.18t
4	kih	34 49 1 .15	9.756603	3.656997
	ikh	89 54 30 .30	9.999999	3.900393
	ihk	55 16 28 .83	9.911815	3.815209
		Vignacourt 0 .28	ih= 7950.48t	Ah=63010.66t

From the first series . . .	A h = 63010.68 toises
From the second . . .	63010.66
Mean of the two . . .	63010.67

Since these agree so nearly, this circumstance is a strong proof that all the operations have been correctly performed.

15. The following is one of the series of observations taken by Borda's repeating circle, for determining the latitude of Dunkirk (A), the northern extremity of the French arc.

On the 7th of March 1796, the following observations were made on β Ursæ minoris, near its upper culmination, when its declination or δ was 74° 59′ 8″.49 N. Mean observed zenith distance, corrected for refraction, or $z' = 23° 57' 27''.36$, and the approximate latitude λ' estimated at 51° 2′ N.

	h. m. s.
*'s R. Ascension	14 51 30
Clock slow Sid. Time	+ 8 37
* upon Mer. by Clock	15 0 7

		diff. m. s.	″	″
1st Observation .	14 50 58	diff. 9 9, m.	164.37, n.	0.07
2. . . .	52 9	7 58	124.61	0.04
3. . . .	53 37	6 30	82.95	0.02
4. . . .	55 9	4 58	48.43	0.01
5. . . .	56 32	3 35	25.21	0.00
6. . . .	58 6	2 1	7.99	0.00
7. . . .	59 13	0 54	1.59	0.00
8. . . .	15 0 34	0 27	0.40	0.00
9. . . .	1 58	1 51	6.72	0.00
10. . . .	3 2	2 55	16.70	0.00
11. . . .	4 25	4 18	36.30	0.00
12. . . .	5 35	5 28	58.68	0.01
13. . . .	6 55	6 48	90.79	0.02
14. . . .	8 45	8 38	146.33	0.05
			14 m.=811.07,	14n.=0.22

λ' 51° 2′ 0″.0 .	cosine	9.798560		
δ 74 59 8 .5 .	cosine	9.413400		
z 23 57 8 .5 .	cosecant	0.391499	cotangent .	0.3524
	Sum	9.603459	× 2 = .	9.2069
14 m = 811″.07	log .	2.909058,	14 n = 0″.22 log	9.3424
14 $a. c. l.$.	.	8.953872	. .	8.9539
c = — 23″.248 .	log	1.366389,	c' = + 0″.006 log	7.7556

Hence,	z' . .	23° 57′ 27″.360
	c . .	— . . 23 .248
	c' . .	+ . . 0 .006
	z . .	23 57 4 .118
	δ . .	74 59 8 .490
	λ the latitude of Dunkirk	51 2 4 .372 N.

By proceeding in this manner till a sufficient number of observations are made, the true latitude of the place of observation will be obtained with great accuracy, and similarly the inclination of one or more of the sides of the series of the triangles to the meridian.

Latitude of Dunkirk, from num. obs. or .	λ 51° 2′ 9″.2 N.
Lat. of node h near Vignacourt, or .	λ' 49 55 54 .0 N.
Difference, or $\lambda - \lambda'$ . .	1 6 15 .2
	= 1.10422

$$l = \frac{\lambda + \lambda}{2} = 50° \ 29' \ 1''.6 \text{ N.}$$

Whence $\dfrac{Ah}{\lambda - \lambda'} = \dfrac{63010.67}{1.10422} =$ . . . . . 57063.5 toises.

In English feet 364896.5.

Hence the compression may be found from a comparison of these two degrees by formula (2), page 194.

$l' =$ 55° 11′ 14″.11	constant log .	9.522879
$l =$ 50 29 1 .60		
$l' + l =$ 105 40 15 .71	cosecant . .	0.016451
$l' - l =$ 4 42 12 .51	cosecant . .	1.086192
$d' =$ 365175.8 feet		
$d =$ 364896.5 feet	*a. c. l.* . .	4.437830
$d' - d =$ 279.3 feet,	log . .	2.446071
$\varepsilon =$ 0.003232	log . .	7.509423
$= \dfrac{1}{309.4}$		

16. Besides these two methods just explained, another has been used by Roy, &c. It consists in reducing the angles of the small spherical triangles to those contained by their chords, and then calculating the sides by the rules of plane trigonometry.

To reduce the measured angles of small spherical triangles to those formed by the chords of their sides, let x be the small correction to be subtracted from C the contained angle: then, if a and b be the containing sides, and r the earth's mean radius in the same measure,

$$x = \frac{R''}{16r^2}(a+b)^2 \tan \tfrac{1}{2} C - \frac{R''}{16r^2}(a \backsim b)^2 \cot \tfrac{1}{2} C \quad . \quad . \quad \text{(A)}$$

The spherical excess, $e = \dfrac{R''s}{r^2}$ . . (B)

s being the area of the triangle.

It is also clear that the sum of the reductions to the chords is equal to the spherical excess.

In the twenty-third triangle of the trigonometrical survey formed by the stations Dunnose, Butser Hill, and Dean Hill, the angle at Dunnose being 55° 43′ 7″, that at Butser Hill 67° 12′ 22″, and that at Dean Hill 48° 4′ 32″.25. The distance of Dunnose from Butser Hill is 140580.4, and from Dunnose to Dean Hill 183496.2, and from Dean Hill to Butser 156122.1 feet.

Dean Hill (A) . .	48° 4′ 32″.25,	$a =$ 140580.4 feet
Butser Hill (B) . . .	76 12 22 ,	$b =$ 183496.2
Dunnose (C) . .	55 43 7 ,	$c =$ 156122.1

Contained angle C 55° 43′ 7″	sine	9.917128
Sides {183496.2	log	5.263627
Sides {140580.4	log	5.147928
Const. log page 193		0.373615
$e = 5''.04$ = spherical excess	log	0.702295

To compute the reduction to the chords.

$b = 183496$
$c = 156122$

$b + c = 339618$ log $\times 2 =$	1.061988		
$b - c = 27374$ log $\times 2 =$			8.874676
$\frac{R''}{16r^2}$ log	9.470525		9.470525
$\frac{1}{2}$ A $= 24° 2' 16''$ tan	9.649353	cotangent	0.350647
1st part $+ 1''.5220$ log	0.181866	2d pt. $-0''.0496$ log	8.695848
2d part $-0\ .0496$			

Cor. A $= +1\ .4724 = x$

$a = 140580$
$c = 156122$

$a + c = 296702$ log $\times 2 =$	0.945518		
$c - a = 15542$ log $\times 2 =$			8.383012
$\frac{R''}{16r^2}$ = log	9.470525		9.470525
$\frac{1}{2}$ B $= 38° 6' 11''$ tan	9.894424	cot	0.105576
1st part $= +2''.0440$ log	0.310467	2d pt. $-0''.0091$ log	7.959113
2d part $= -0\ .0091$			

Cor. B $= +2\ .0349 = x'$

$a = 140580$
$b = 183496$

$a + b = 324076$ log $\times 2 =$	1.021292		
$b - a = 42916$ log $\times 2 =$			9.265240
$\frac{R''}{16r^2}$ log	9.470525		9.470525
$\frac{1}{2}$ C $= 27° 51' 33''.5$ tan	9.723098	cot	0.276902
1st part $= +1''.6403$ log	0.214915	2d pt. $-0''.1029$ log	9.012667
2d part $= -0\ .1029$			

Cor. C $= +1\ .5374 = x''$

Red. A $= 1''.4724$
B $= 2\ .0349$
C $= 1\ .5374$

Sum of cor. $= e = 5\ .0447$ as it ought to be.

Hence the whole may be arranged so that the computations may be performed three different ways.

	Obs. Angles.	Red.	Cor. Spher. Angles.	Angles of Chords.	Mean Angles.
A	48° 4′ 32″.25	— 1″.47	48° 4′ 33″.52	48° 4′ 32″.05	48° 4′ 31″.84
B	76 12 22.00	— 2.03	76 12 23.26	76 12 21.23	76 12 21.58
C	55 43 7.00	— 1.54	55 43 8.26	55 43 6.72	55 43 6.58
Sum	180 0 1.25	— 5.04	180 0 5.04	180 0 0.00	180 0 0.00
	180 0 5.04	$\frac{5.04}{3} = 1.68 = \frac{1}{3}e$ which applied to the spherical angles, will, on Legendre's plan, give the mean angles in the last column, or — 1″.68 + 1″.26 = — 0″.42 = red. of obs. to mean angles.			
	E = — 3.79				
	$\frac{E}{3}$ = — 1.26				

If the second method by the angles of the chords be used, the measured base must be reduced from the arc measured to the chord, by formula (3), page 269.

The measured base on Hounslow Heath,

By Roy, in 1784, with glass rods, . .	27404.0137 feet
By Mudge, in 1791, with a hundred-feet chain, .	27404.2449
Mean at 62° Fahrenheit, or A = . .	27404.1293 feet

A	$= 27404.1293 \log \times 3 =$	.	3.31345
$24\,R^2$	*a. c. l.*	. .	3.98001
∴ E	$= -$ 0.0020 log	. .	7.29346

Chord $= \gamma =$ 27404.1273 sensibly the same as before. Wherefore, setting out from this base, if the angles reduced to the chords be used, the chords of the different sides of the whole series of triangles will be obtained by computation. It appears, therefore, that the method of Legendre is the most easy and simple of the three; while, at the same time, it is sufficiently accurate for any purpose, as Puissant has shown, that, in the greatest triangle yet measured, connecting the coast of Spain with the island of Iviça, the error is almost insensible.

17. When, instead of the theodolite, the repeating circle of Borda is used, as by the French astronomers Mechain and Delambre, to observe the angle contained, by distant signals not situated in the horizontal plane, they must be reduced to the horizon by appropriate formulæ, or tables derived from them. In this case the zenith distances, or altitudes of the signals, must also be observed; an operation which renders the observations by the repeating circle very laborious. This formula is similar to formula (A), page 302.

Let x be the correction, and the altitude of the signal A, and β that of B, seen from C, then

$$x = \frac{1}{4R''}(\alpha + \beta)^2 \tan \tfrac{1}{2} C - \frac{1}{4R''}(\alpha \backsim \beta)^2 \cot \tfrac{1}{2} C \quad . \quad . \text{(C)}$$

In the trigonometrical survey of this country, by Roy, Mudge, and Colby, this reduction is not required, since the reduced angles are given by the theodolite; an example may therefore be taken from Delambre, given in the first vol. of the *Base Metrique*.

Observed angle C $= 51° \ 9' \ 29''.744$ at Violan.

Aubassin	$z = 91 \ 32 \ 45$	$\alpha = -1° \ 32' \ 45''$
Bastide	$z' = 91 \ 7 \ 10$	$\beta = -1 \ 7 \ 10$

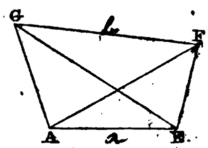
$\alpha = -1° \ 32' \ 45''$
$\beta = -1 \ 7 \ 10$

$\alpha+\beta = -2 \ 39 \ 55 = 9595''$ log $\times 2 =$		7.964090	
$\alpha-\beta = - \ 25 \ 35 = 1535$ log $\times 2 =$			6.372216
$\frac{1}{4R''}$ log	4.083515		4.083515
$\frac{1}{2}$ C $= 25° \ 34' \ 45''$ nearly tan	9.680039	cot	0.319961
Part 1st $= +53''.413$ log	1.727644	p. 2d log	0.775692
2d $= -5 \ .967$			$-5''.967$
$x = +47 \ .446$			$47''.446$
C $=$			$51° \ 9' \ 29 \ .744$
C $+ x$ the reduced angle			$51 \ 10 \ 17 \ .190$
C $+ x$ by direct calculation			$51 \ 10 \ 17 \ .186$

So that the difference is insensible.

Problem I.

To determine the distance between two inaccessible points, when the angles between a given base and each of the points at both extremities of the base are given.



Let AE be the given base a, and GF the required distance b, GAE $= \alpha$, FAE $= \beta$, AEF $= \gamma$, AEG $= \delta$.

Whence AGE $= \theta$, and AFE $= \varphi$: hence,

As sin AGE : sin GAE : : AE : GE $=$ AE sin GAE cosec AGE,
As sin AFE : sin EAF : : AE : EF $=$ AE sin EAF cosec AFE,
But Euclid II. and 12, $GF^2 = GE^2 + EF^2 - 2\,GE \,.\, EF \cos GEF$, which, by substitution, becomes

$$GF^2 = AE^2 \sin^2 GAE \operatorname{cosec}^2 AGE + AE^2 \sin^2 EAF \operatorname{cosec}^2 AFE - 2\,AE^2 \sin GAE \operatorname{cosec} AGE \sin EAF \operatorname{cosec} AFE \cos (AEF - AEG)$$

Hence,

$$b = a\,\{(\sin\alpha \operatorname{cosec}\theta)^2 + (\sin\beta \operatorname{cosec}\varphi)^2 - 2\sin\alpha\operatorname{cosec}\theta\sin\beta\operatorname{cosec}\varphi\cos(\gamma-\delta)\}^{\frac{1}{2}} \quad \ldots \quad (1.)$$

and conversely,

$$a = b \div \{(\sin\alpha\operatorname{cosec}\theta)^2 + (\sin\beta\operatorname{cosec}\varphi)^2 - 2\sin\alpha\operatorname{cosec}\theta\sin\beta\operatorname{cosec}\varphi\,(\gamma-\delta)\}^{\frac{1}{2}} \quad \ldots \quad (2.)$$

Similarly,

$$b = a\,\{(\sin\delta\operatorname{cosec}\theta)^2 + (\sin\gamma\operatorname{cosec}\varphi)^2 - 2\sin\delta\operatorname{cosec}\theta\sin\gamma\operatorname{cosec}\varphi\cos(\alpha-\beta)\}^{\frac{1}{2}} \quad \ldots \quad (3.)$$

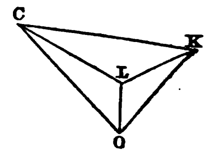
and conversely,

$$a = b \div \{(\sin\delta\operatorname{cosec}\theta)^2 + (\sin\gamma\operatorname{cosec}\varphi)^2 - 2\sin\delta\operatorname{cosec}\theta\sin\gamma\operatorname{cosec}\varphi\operatorname{cosec}(\alpha-\beta)\}^{\frac{1}{2}} \quad \ldots \quad (4.)$$

Problem II.

To determine the same thing, if the given base have a perpendicular instead of a parallel direction to the required distance.

Let OL $= a$, LOK $= \alpha$, LOC $= \gamma$, CKL $= \theta$, OKC $= \psi$, OLK $= \varkappa$: CK $= b$, LKO $= \beta$, LCO $= \delta$, KCL $= \varphi$, OCK $= \omega$, OLC $= \varrho$.



By a similar method of investigation,

$$b = a\left\{(\sin\alpha \operatorname{cosec}\beta)^2 + (\sin\gamma \operatorname{cosec}\delta)^2 - 2\sin\alpha \operatorname{cosec}\beta \sin\gamma \operatorname{cosec}\delta \cos(\theta+\varphi)\right\}^{\frac{1}{2}} \quad . \; . \; . \quad (5.)$$

$$a = b \div \left\{(\sin\alpha \operatorname{cosec}\beta)^2 + (\sin\gamma \operatorname{cosec}\delta)^2 - 2\sin\alpha \operatorname{cosec}\beta \sin\gamma \operatorname{cosec}\delta \cos(\theta+\varphi)\right\}^{\frac{1}{2}} \quad . \; . \; . \quad (6.)$$

Similarly,

$$b = a\left\{(\sin\varkappa \operatorname{cosec}\beta)^2 + (\sin\varrho \operatorname{cosec}\delta)^2 - 2\sin\varkappa \operatorname{cosec}\beta \sin\varrho \operatorname{cosec}\delta \cos(\psi+\omega)\right\}^{\frac{1}{2}} \quad . \; . \; . \quad (7.)$$

$$a = b \div \left\{(\sin\varkappa \operatorname{cosec}\beta)^2 + (\sin\varrho \operatorname{cosec}\delta) - 2\sin\varkappa \operatorname{cosec}\beta \sin\varrho \operatorname{cosec}\delta \cos(\psi+\omega)\right\}^{\frac{1}{2}} \quad . \; . \; . \quad (8.)$$

Example illustrative of Formula I. and II.

Let $\alpha = 139^\circ\ 15'\ 45''$ Whence $\alpha - \beta = 85^\circ\ 45'\ 22''$ $\theta = 8^\circ\ 55'\ 15''$
$\beta = 53\ 30\ 23$ $\gamma - \delta = 82\ 35\ 55$ $\varphi = 12\ 4\ 42$
$\gamma = 114\ 24\ 55$ $a =$ 6244,16 feet.
$\delta = 31\ 49\ 0$ required b.

$\alpha = 139^\circ\ 15'\ 45''$ sin 9.814643 constant logarithm, or log 2. . . . 0.301030
$\theta = 8\ 55\ 15$ cosec 0.809473 $\gamma - \delta = 82^\circ\ 35'\ 55''$ cosine . . . 9.109982

0.624116 . . . . 0.624116
2 $\beta = 53^\circ\ 30'\ 23''$ sine 9.905215

No. 1, + 17.71054 log 1.248232 $\varphi = 12\ 4\ 42$ cosec 0.679337

0.584552 . 0.584552

2 No.—0.619680

No. 2, + 14.76064 . . . . log + 1.169104 . —4.16562
No. 3,— 4.16562 . . . . :

Sum, 28.30556 log . 1.451872

half, . 0.725936
$a =$ 6244.16 feet, log . 3.795474

$b =$ 33220.8 feet, log . 4.521410

Formula 3 would give the same result.

It is obvious, that if b had been given, it is only necessary to subtract half the log of the sum from the log b to obtain the log a.

NOTE 1.—As the Nautical Almanac has been considerably changed in form and contents since the preceding part of this work was written and printed, it will be necessary to make a few slight alterations on some of the astronomical operations. The greater part of the articles in the Nautical Almanac are now given to *mean* time, with variations to a given time,—such as a day, an hour, or ten minutes; and to this circumstance, if the calculator pay proper attention, he will readily interpolate for any intermediate time. In computing Occultations and Eclipses, as in pages 149, &c. because the moon's R. A. is given for every hour in *time*, the moon's *second* R. A. may be taken for 1^h after the first estimated time instead of 3^h, and then using the constant log 9.52288 in the computation of the proportional part by P. L. In case the first difference be very small, as some fraction of a second, the whole may be conceived to be multiplied by 60, which is readily done by estimating the minutes hours, the seconds minutes, and multiplying the decimal by 60.

For the same reason, it would be advantageous to convert arc (5) in the same manner as arc (1) into time, by using the constant log 1.17609, and adding the result to the star's estimated R. A. in time, as in page 152.

Thus P. L. arc (5) would become 2.22423, to which the corresponding arc is $1^m\ 4^s.44$.

This added to $16^h\ 16^m\ 45^s.4$ gives $16^h\ 17^m\ 49^s.84$ for the corrected R. A. of the point observed.

	h.	m.	s.		h.	m.	s.	
C. R. A. P.	16	17	49.8					
1st R. A.	16	17	50.0	1st diff.	0	0	0.2	
2d R. A.	16	20	1.7	2d diff.	0	2	11.7	
				Or, 1st .	0	0	12.0	P. L. 2.95424
				2d .	2	11	42.0	P. L. 0.13569
				P.P. for 3^h			16.5	P. L. 2.81856
				for 1^h		=	5.5	
Estimated time					18	4	0.0	
Greenwich time					18	3	54.5	
Bahia time					15	30	0.3	
Longitude in time					2	33	54.2	= 38° 28′ 30″ W.

In operations where differences are given for 1° or 1^h, diurnal logs may be employed by using the const. log 8.61979, or proportional logs by the const. log 9.52288.

EXAMPLES.—Required the R. A. of the Moon on January 8, 1834, at $8^h\ 45^m$ mean time in longitude 60° east?

	h.	m.		h.	h.	m.	s.
Time	8	45	Moon's R. A. at	4 =	18	12	2.75
Long. in T.	4	0 E.	at	5 =	18	14	22.92
Red. Time	4	45	Increase in	1 =		2	20.17

By Diurnal Logs.		By Prop. Logs.	
Const. log .	8.61979	Const. log .	9.52288
Time 0h 45m 0s D. L.	1.50515	0h 45m 0s P. L.	0.60206
Incr. 0 2 20.17 D. L.	1.01171	0 2 20.17 P. L.	1.88677
P. P. 0 1 45.13 D. L.	1.13665	0 1 45.13 P. L.	2.01171
		Or	
		Const. log .	9.52288
		0h 45m 0s. 0 P. L.	0.60206
		2 20 10. 2 P. L.	0.10862
		1 45 7. 3 P. L.	0.23356
		÷ 60 = 0 1 45.12	

Note 2.—If the rule, page 142, be employed to compute the equation to equal altitudes, and the difference of the sun's declination for one hour be taken from the Nautical Almanac, the constant log is 6.744727 in place of 5.364517; or if such tables as those by Bailly, Riddle, and those in the Author's new Supplementary Tables, be used, the hourly variation may be employed instead of twice the daily variation, by adding the const. log 1.681241, in the practice of which four places of decimals will be sufficient.

To compute the contained arc, or c, page 194, that is, to convert feet measured on the earth's surface into arcs, let d be the distance in feet, R″ an arc equal to radius in seconds, r the earth's radius in feet, and c'' the contained arc in seconds.

$$c'' = R'' \times r \times d = \text{const. log } 7.994535 + \log d.$$

Example.—Let $d = 61777$ feet, then

Const. log . . .	7.994535
$d = 61777$ log . . .	4.790827
$c'' = 610''.5 = 10' \, 10''.5$ log . .	2.785362

To find m and n correctly, when the time from noon or the meridian exceeds the limits of Table XXVIII. See additions and corrections, page xv.

To the constant log 5.615455 add the log of the time from Table LXXV. the sum will be the log of m. To the same const. log add twice the log of the time from the same table, the sum will be the log of n. Thus,

Const. log .	5.615455	. .	5.615455
30m log Table LXXV. .	7.631197	× 2 .	5.262394
$m = 1764''.62$ log .	3.246652	$n = 7''.55$ log	0.877849

Formula (15), page 197, would be more convenient if developed thus,

$$o = m\left(1 + \frac{p - m}{p}\right)\sin^2 a + \left(\frac{p - m}{p}\right)^2 \sin^4 a + \&c.$$

$$o = 60819 \times 1.00248 = 60970 \text{ fathoms.}$$

5

MATHEMATICAL TABLES.

TABLE I.

THE MILES AND PARTS OF A MILE IN A DEGREE OF LONGITUDE AT EVERY DEGREE OF LATITUDE.

D. L.	Miles.	D. L.	Miles.	D. L.	Miles.	D. L.	Miles.	D. L.	Miles.	D. L.	Miles.
1	59.99	16	57.67	31	51.43	46	41.68	61	29.09	76	14.52
2	59.96	17	57.38	32	50.88	47	40.92	62	28.17	77	13.50
3	59.92	18	57.06	33	50.32	48	40.15	63	27.24	78	12.47
4	59.85	19	56.73	34	49.74	49	39.36	64	26.30	79	11.45
5	59.77	20	56.38	35	49.15	50	38.57	65	25.36	80	10.42
6	59.67	21	56.01	36	48.54	51	37.76	66	24.40	81	9.39
7	59.55	22	55.63	37	47.92	52	36.94	67	23.44	82	8.35
8	59.42	23	55.23	38	47.28	53	36.11	68	22.48	83	7.31
9	59.26	24	54.81	39	46.63	54	35.27	69	21.50	84	6.27
10	59.08	25	54.38	40	45.96	55	34.41	70	20.52	85	5.23
11	58.89	26	53.93	41	45.28	56	33.55	71	19.53	86	4.19
12	58.68	27	53.46	42	44.59	57	32.68	72	18.54	87	3.14
13	58.46	28	52.97	43	43.88	58	31.80	73	17.54	88	2.09
14	58.22	29	52.47	44	43.16	59	30.90	74	16.54	89	1.05
15	57.95	30	51.96	45	42.43	60	30.00	75	15.53	90	0.00

TABLE II.

LOGARITHMS OF NUMBERS.

No. 1——100						Log. 0.000000——2.000000			
No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

A Table of Logarithms of Numbers from 1 to 100,000.

P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.
	100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
41	1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
82	2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
124	3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
165	4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
206	5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
247	6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
288	7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
330	8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
371	9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
	110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
38	1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
75	2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
113	3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
150	4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
188	5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
226	6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
263	7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
301	8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
338	9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
	120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
35	1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
69	2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
104	3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
138	4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
173	5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
208	6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
242	7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
277	8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
311	9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
	130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
32	1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
64	2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
96	3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
128	4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
160	5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
193	6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
225	7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
257	8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
289	9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
	140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
30	1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
60	2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
90	3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
120	4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
149	5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
179	6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
209	7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
239	8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
269	9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
	150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
28	1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
56	2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
84	3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
112	4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
139	5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
167	6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
195	7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
223	8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
251	9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272
P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.

P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.
	160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206550	271
26	1	6826	7096	7365	•7634	7904	8173	8441	8710	8979	9247	269
52	2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
79	3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
105	4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
131	5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
157	6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
183	7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
209	8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
236	9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
	170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	255
25	1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
50	2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
74	3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
99	4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
124	5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
149	6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
174	7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
198	8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
223	9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
	180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
23	1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
47	2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
70	3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
94	4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
117	5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
140	6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
164	7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
187	8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
211	9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
	190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
22	1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
44	2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
67	3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
89	4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
111	5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
133	6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
155	7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
178	8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
200	9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
	200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
21	1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
42	2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
63	3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
84	4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
105	5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
127	6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
148	7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
169	8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
190	9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
	210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
20	1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
40	2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
61	3	8380	8583	8787	8991	9194	9398	9601	9805	330008	330211	203
81	4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
101	5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
121	6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
141	7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
162	8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
182	9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198
P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.

P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.
	220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
19	1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
39	2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
58	3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
77	4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
96	5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
116	6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
135	7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
154	8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
174	9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
	230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
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37	2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
55	3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
74	4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
92	5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
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129	7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
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166	9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
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71	4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
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	250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
17	1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
34	2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
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66	4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
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	300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
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43	3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
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	310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
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69	5	8311	8448	8586	8724	8862	8999	9137	9275	9412	9550	138
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25	2	4026	4153	4280	4407	4534	4661	4787	4914	5041	5167	127
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	350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
12	1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
24	2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
37	3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
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	360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
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	370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
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23	2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
35	3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
46	4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
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77	7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
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75	7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
86	8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
96	9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
	410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
10	1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
21	2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
31	3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
42	4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
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63	6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
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84	8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
94	9	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
	420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
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41	4	7366	7468	7571	7673	7775	7878	7980	8082	8185	8287	102
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70	7	640481	640581	640680	640779	640879	640978	1077	1177	1276	1375	99
80	8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
90	9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
	440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
10	1	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
19	2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
29	3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
39	4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
48	5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
58	6	9335	9432	9530	9627	9724	9821	9919	650016	650113	650210	97
68	7	650308	650405	650502	650599	650696	650793	650890	0987	1084	1181	97
78	8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
87	9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
	450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
9	1	4177	4273	4369	4465	4562	4658	4754	4850	4946	5042	96
19	2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
28	3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
38	4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
47	5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
57	6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
66	7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
76	8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
85	9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95
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9	1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
19	2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
28	3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
37	4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
46	5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
56	6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
65	7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
74	8	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
84	9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
	470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
9	1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
18	2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
27	3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
36	4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
45	5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
55	6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
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73	8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
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	480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
9	1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
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27	3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
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45	5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
54	6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
63	7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
72	8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
81	9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
	490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
9	1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
18	2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
26	3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
35	4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
44	5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
53	6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
62	7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
70	8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
79	9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
	500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
9	1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
17	2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
26	3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
34	4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
43	5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
52	6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
60	7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
69	8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
77	9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
	510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
8	1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
17	2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
25	3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
34	4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
42	5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
50	6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
59	7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
67	8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
76	9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84
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8	1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
17	2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
25	3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
33	4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
41	5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
50	6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
58	7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
66	8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
75	9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
	530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
8	1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
16	2	5912	5993	6075	6156	6238	6320	6401	6483	6564	6646	82
24	3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
32	4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
40	5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
49	6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
57	7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
65	8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
73	9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
	540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
8	1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
16	2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
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40	5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
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56	7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
64	8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
72	9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
	550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
8	1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
16	2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
23	3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
31	4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
39	5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
47	6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
55	7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
62	8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
70	9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
	560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
8	1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
15	2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
23	3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
31	4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
38	5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
46	6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
54	7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
62	8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
69	9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
	570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
7	1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
15	2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
22	3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
30	4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
37	5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
45	6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
52	7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
60	8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
67	9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
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	580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
7	1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
15	2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
22	3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
30	4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
37	5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
44	6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
52	7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
59	8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
67	9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
	590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
7	1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
15	2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
22	3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
29	4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
36	5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
44	6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
51	7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
58	8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
66	9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
	600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
7	1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
14	2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
22	3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
29	4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
36	5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
43	6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
50	7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
58	8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
65	9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
	610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
7	1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
14	2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
21	3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
28	4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
35	5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
43	6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
50	7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
57	8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
64	9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
	620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
7	1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
14	2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
21	3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
28	4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
35	5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
42	6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
49	7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
56	8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
63	9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
	630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
7	1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
14	2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
20	3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
27	4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
34	5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
41	6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
48	7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
54	8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
61	9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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	640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
7	1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
13	2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
20	3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
27	4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
33	5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
40	6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
47	7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
54	8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
60	9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
	650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
7	1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
13	2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
20	3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
26	4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
33	5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
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53	8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
59	9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
	660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
6	1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
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19	3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
26	4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
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39	6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
45	7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
52	8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
58	9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
	670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
6	1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
13	2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
19	3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
26	4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
32	5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
38	6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
45	7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
51	8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
58	9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
	680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
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25	4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
31	5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
38	6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
44	7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
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57	9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
	690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
6	1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
13	2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
19	3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
25	4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
31	5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
38	6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
44	7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
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57	9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62
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6	1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
12	2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
19	3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
25	4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
31	5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
37	6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
43	7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
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56	9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
	710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
6	1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
12	2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
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24	4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
30	5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
37	6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
43	7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
49	8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
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	720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
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12	2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
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24	4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
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54	9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
	730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
6	1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
12	2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
18	3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
24	4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
29	5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
35	6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
41	7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
47	8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
53	9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
	740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
6	1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
12	2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
17	3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
23	4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
29	5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
35	6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
41	7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
46	8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
52	9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
	750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
6	1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
11	2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
17	3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
23	4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
28	5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
34	6	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
40	7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
46	8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
51	9	880242	880299	880356	880413	880471	880528	0585	0642	0699	0756	57
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6	1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
11	2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
17	3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
23	4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
28	5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
34	6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
40	7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
46	8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
51	9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
	770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
6	1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
11	2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
17	3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
22	4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
28	5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
34	6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
39	7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
45	8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
50	9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
	780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
6	1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
11	2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
16	3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
22	4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
27	5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
33	6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
38	7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
44	8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
49	9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
	790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
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22	4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
27	5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
33	6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
38	7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
44	8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
49	9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
	800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
5	1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
11	2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
16	3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
22	4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
27	5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
32	6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
38	7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
43	8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
49	9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
	810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
5	1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
11	2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
16	3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
21	4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
26	5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
32	6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
37	7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
42	8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
48	9	3284	3337	3390	3443	3496	3549	3602	3655	3708	3761	53
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5	1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
11	2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
16	3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
21	4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
26	5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
32	6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
37	7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
42	8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
48	9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
	830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
5	1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
10	2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
16	3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
21	4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
26	5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
31	6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
36	7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
42	8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
47	9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
	840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
5	1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
10	2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
15	3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
20	4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
25	5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
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36	7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
41	8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
46	9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
	850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
5	1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
10	2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
15	3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
20	4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
25	5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
31	6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
36	7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
41	8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
46	9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
	860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
5	1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
10	2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
15	3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
20	4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
25	5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
30	6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
35	7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
40	8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
45	9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
	870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
5	1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
10	2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
15	3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
20	4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
25	5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
30	6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
35	7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
40	8	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
45	9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.

P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.
	880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
5	1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
10	2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
15	3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
20	4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
24	5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
29	6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
34	7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
39	8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
44	9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
	890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
5	1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
10	2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
15	3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
20	4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
24	5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
29	6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
34	7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
39	8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
44	9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
	900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
5	1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
10	2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
14	3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
19	4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
24	5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
29	6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
34	7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
38	8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
43	9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
	910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
5	1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
9	2	9995	960042	960090	960138	960185	960233	960281	960328	960376	960423	48
14	3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
19	4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
23	5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
28	6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
33	7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
38	8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
42	9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
	920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
5	1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
9	2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
14	3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
19	4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
23	5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
28	6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
33	7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
38	8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
42	9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
	930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
5	1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
9	2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
14	3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
18	4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
23	5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
28	6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
32	7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
37	8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
41	9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
P. P.	N.	0	1	2	3	4	5	6	7	8	9	D

P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.
	940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
5	1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
9	2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
14	3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
18	4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
23	5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
28	6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
32	7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
37	8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
41	9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
	950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
4	1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
9	2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
13	3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
18	4	9548	9591	9639	9685	9730	9776	9821	9867	9912	9958	46
22	5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
27	6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
31	7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
36	8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
40	9	1819	1864	1909	1954	2000	2045	2090	2135	2161	2226	45
	960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
4	1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
9	2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
13	3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
18	4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
22	5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
27	6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
31	7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
36	8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
40	9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
	970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
4	1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
9	2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
13	3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
18	4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
22	5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
27	6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
31	7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
36	8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
40	9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
	980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
4	1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
9	2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
13	3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
18	4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
22	5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
26	6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
31	7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
35	8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
40	9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
	990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
4	1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
9	2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
13	3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
18	4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
22	5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
26	6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
31	7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
35	8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
40	9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
P. P.	N.	0	1	2	3	4	5	6	7	8	9	D.

TABLE III.

THE ANGLES WHICH EVERY POINT AND QUARTER POINT OF THE COMPASS MAKES WITH THE MERIDIAN.

North		Points.	° ′ ″	Points.	South.	
		0 ¼	2 48 45	0 ¼		
		0 ½	5 37 30	0 ½		
		0 ¾	8 26 15	0 ¾		
N. b. E.	N. b. W.	1	11 15 0	1	S. b. E.	S. b. W.
		1 ¼	14 3 45	1 ¼		
		1 ½	16 52 30	1 ½		
		1 ¾	19 41 15	1 ¾		
N.N.E.	N.N.W.	2	22 30 0	2	S.S.E.	S.S.W.
		2 ¼	25 18 45	2 ¼		
		2 ½	28 7 30	2 ½		
		2 ¾	30 56 15	2 ¾		
N.E. b. N.	N.W. b. N.	3	33 45 0	3	S.E. b. S.	S.W. b. S.
		3 ¼	36 33 45	3 ¼		
		3 ½	39 22 30	3 ½		
		3 ¾	42 11 15	3 ¾		
N.E.	N.W.	4	45 0 0	4	S.E.	S.W.
		4 ¼	47 48 45	4 ¼		
		4 ½	50 37 30	4 ½		
		4 ¾	53 26 15	4 ¾		
N.E. b. E.	N.W. b. W.	5	56 15 0	5	S.E. b. E.	S.W. b. W.
		5 ¼	59 3 45	5 ¼		
		5 ½	61 52 30	5 ½		
		5 ¾	64 41 15	5 ¾		
E.N.E.	W.N.W.	6	67 30 0	6	E.S.E.	W.S.W
		6 ¼	70 18 45	6 ¼		
		6 ½	73 7 30	6 ½		
		6 ¾	75 56 15	6 ¾		
E. b. N.	W. b. N.	7	78 45 0	7	E. b. S.	W. b. S.
		7 ¼	81 33 45	7 ¼		
		7 ½	84 22 30	7 ½		
		7 ¾	87 11 15	7 ¾		
East.	West.	8	90 0 0	8	East.	West.

TABLE IV.

LOGARITHMIC SINES, TANGENTS, AND SECANTS, TO EVERY POINT AND QUARTER POINT OF THE COMPASS.

Points.	Sine.	Cosine.	Tangent.	Cotang.	Secant.	Cosec.	Points.
0	0.000000	10.000000	0.000000	Infinite.	10.000000	Infinite.	8
0 ¼	8.690796	9.999477	8.691319	11.308681	10.000523	11.309204	7 ¾
0 ½	8.991302	9.997904	8.993398	11.006602	10.002096	11.008698	7 ½
0 ¾	9.166520	9.995274	9.171247	10.828753	10.004726	10.833480	7 ¼
1	9.290236	9.991574	9.298662	10.701338	10.008426	10.709764	7
1 ¼	9.385571	9.986786	9.398785	10.601215	10.013214	10.614429	6 ¾
1 ½	9.462824	9.980885	9.481939	10.518061	10.019115	10.537176	6 ½
1 ¾	9.527488	9.973841	9.553647	10.446353	10.026159	10.472512	6 [illegible]
2	9.582840	9.965615	9.617224	10.382776	10.034385	10.417160	6
2 ¼	9.630992	9.956163	9.674829	10.325171	10.043837	10.369008	5 ¾
2 ½	9.673387	9.945430	9.727957	10.272043	10.054570	10.326613	5 ½
2 ¾	9.711050	9.933350	9.777700	10.222300	10.066650	10.288950	5 ¼
3	9.744739	9.919846	9.824893	10.175107	10.080154	10.255261	5
3 ¼	9.775027	9.904828	9.870199	10.129801	10.095172	10.224973	4 ¾
3 ½	9.802359	9.888185	9.914173	10.085827	10.111815	10.197641	4 ½
3 ¾	9.827084	9.869790	9.957295	10.042705	10.130210	10.172916	4 ¼
4	9.849485	9.849485	10.000000	10.000000	10.150515	10.150515	4
	Cosine.	Sine.	Cotang.	Tangent.	Cosec.	Secant.	

0 Hour, or 0 Degree.

m.	s.	′	Sine.	Cosec.	Tang.	D. S. T.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
0	0	0	0.000000	Infinite.	0.000000	....	Infinite.	10.000000		10.000000	60	60	0
	4	1	6.463726	13.536274	6.463726	501717	13.536274	000000	00	000000	59		56
	8	2	764756	235244	764756	293484	235244	000000	00	000000	58		52
	12	3	940847	059153	940847	208231	059153	000000	00	000000	57		48
	16	4	7.065786	12.934214	7.065786	161517	12.934214	000000	00	000000	56		44
	20	5	162696	837304	162696	131969	837304	000000	00	000000	55		40
	24	6	241877	758123	241878	111577	758122	000001	01	9.999999	54		36
	28	7	308824	691176	308825	99653	691175	000001	01	999999	53		32
	32	8	366816	633184	366817	85254	633183	000001	01	999999	52		28
	36	9	417968	582032	417970	76263	582030	000001	01	999999	51		24
	40	10	463725	536275	463727	68988	536273	000002	01	999998	50		20
	44	11	505118	494882	505120	62981	494880	000002	01	999998	49		16
	48	12	542906	457094	542909	57934	457091	000003	01	999997	48		12
	52	13	577668	422332	577672	53642	422328	000003	01	999997	47		8
	56	14	609853	390147	609857	49939	390143	000004	01	999996	46		4
1	0	15	7.639816	12.360184	7.639820	46715	12.360180	10.000004	01	9.999996	45	59	0
	4	16	667845	332156	667849	43882	332151	000005	01	999995	44		56
	8	17	694173	305827	694179	41373	305821	000005	01	999995	43		52
	12	18	718997	281003	719003	39136	280997	000006	01	999994	42		48
	16	19	742477	257523	742484	37128	257516	000007	01	999993	41		44
	20	20	764754	235246	764761	35136	235239	000007	01	999993	40		40
	24	21	785943	214057	785951	33673	214049	000008	01	999992	39		36
	28	22	806146	193854	806155	32176	193845	000009	01	999991	38		32
	32	23	825451	174549	825460	30806	174540	000010	01	999990	37		28
	36	24	843934	156066	843944	29548	156056	000011	02	999989	36		24
	40	25	861662	138338	861674	28389	138326	000012	02	999988	35		20
	44	26	878695	121305	878708	27318	121292	000012	02	999988	34		16
	48	27	895085	104915	895099	26324	104901	000013	02	999987	33		12
	52	28	910879	089121	910894	25400	089106	000014	02	999986	32		8
	56	29	926119	073881	926134	24539	073866	000015	02	999985	31		4
2	0	30	7.940842	12.059158	7.940858	23734	12.059142	10.000017	02	9.999983	30	58	0
	4	31	955082	044918	955100	22981	044900	000018	02	999982	29		56
	8	32	968870	031130	968889	22274	031111	000019	02	999981	28		52
	12	33	982233	017767	982253	21609	017747	000020	02	999980	27		48
	16	34	995198	004802	995219	20982	004781	000021	02	999979	26		44
	20	35	8.007787	11.992213	8.007809	20391	11.992191	000023	02	999977	25		40
	24	36	020021	979979	020045	19832	979955	000024	02	999976	24		36
	28	37	031919	968081	031945	19304	968055	000025	02	999975	23		32
	32	38	043501	956499	043527	18802	956473	000027	02	999973	22		28
	36	39	054781	945219	054809	18326	945191	000028	02	999972	21		24
	40	40	065776	934224	065806	17873	934194	000029	02	999971	20		20
	44	41	076500	923500	076531	17443	923469	000031	02	999969	19		16
	48	42	086965	913035	086997	17033	913003	000032	02	999968	18		12
	52	43	097183	902817	097217	16640	902783	000034	02	999966	17		8
	56	44	107167	892833	107202	16267	892797	000036	03	999964	16		4
3	0	45	8.116926	11.883074	8.116963	15909	11.883037	10.000037	03	9.999963	15	57	0
	4	46	126471	873529	126510	15567	873490	000039	03	999961	14		56
	8	47	135810	864190	135851	15240	864149	000041	03	999959	13		52
	12	48	144953	855047	144996	14926	855004	000042	03	999958	12		48
	16	49	153907	846093	153952	14624	846048	000044	03	999956	11		44
	20	50	162681	837319	162727	14334	837273	000046	03	999954	10		40
	24	51	171280	828720	171328	14056	828672	000048	03	999952	9		36
	28	52	179713	820287	179763	13788	820237	000050	03	999950	8		32
	32	53	187985	812015	188036	13530	811964	000052	03	999948	7		28
	[illegible]	54	196102	80[illegible]898	196156	13282	803844	000054	03	999946	6		24
	40	55	204070	795930	204126	13043	795874	000056	03	999944	5		20
	44	56	211895	788105	211953	12812	788047	000058	04	999942	4		16
	48	57	219581	780419	219641	12509	780359	000060	04	999940	3		12
	52	58	227134	772866	227195	12374	772805	000062	04	999938	2		8
	56	59	234557	765443	234621	12166	765379	000064	04	999936	1		4
4	0	60	241855	758145	241921	11965	758079	000066	04	999934	0	56	0
m.	s.	′	Cosine.	Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 89 Degrees.

P. P. to s or ″											P. P. to s or ″
	1s	15″	3560	1s	15″	3560	1s	15″	0		
	2	30	7120	2	30	7120	2	30	0		
	3	45	10680	3	45	10681	3	45	1		

			0 Hour,			or				1 Degree.			
m.	s.	′	Sine.	Cosec.	Tang.	D. S. T.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
4	0	0	8.241855	11.758145	8.241921	11965	11.758079	10.000066	04	9.999934	60	56	0
	4	1	249033	750967	249102	11770	750898	000068	04	999932	59		56
	8	2	256094	743906	256165	11582	743835	000071	04	999929	58		52
	12	3	263042	736958	263115	11400	736885	000073	04	999927	57		48
	16	4	269881	730119	269956	11223	730044	000075	04	999925	56		44
	20	5	276614	723386	276691	11052	723309	000078	04	999922	55		40
	24	6	283243	716757	283323	10885	716677	000080	04	999920	54		36
	28	7	289773	710227	289856	10724	710144	000082	04	999918	53		32
	32	8	296207	703793	296292	10568	703708	000085	04	999915	52		28
	36	9	302546	697454	302634	10416	697366	000087	04	999913	51		24
	40	10	308794	691206	308884	10268	691116	000090	04	999910	50		20
	44	11	314954	685046	315046	10124	684954	000093	04	999907	49		16
	48	12	321027	678973	321122	9984	678878	000095	04	999905	48		12
	52	13	327016	672984	327114	9849	672886	000098	04	999902	47		8
	56	14	332924	667076	333025	9716	666975	000101	05	999899	46		4
5	0	15	8.338753	11.661247	8.338856	9588	11.661144	10.000103	05	9.999897	45	55	0
	4	16	344504	655496	344610	9463	655390	000106	05	999894	44		56
	8	17	350181	649819	350289	9340	649711	000109	05	999891	43		52
	12	18	355783	644217	355895	9222	644105	000112	05	999888	42		48
	16	19	361315	638685	361430	9106	638570	000115	05	999885	41		44
	20	20	366777	633223	366895	8993	633105	000118	05	999882	40		40
	24	21	372171	627829	372292	8883	627708	000121	05	999879	39		36
	28	22	377499	622501	377622	8775	622378	000124	05	999876	38		32
	32	23	382762	617238	382889	8670	617111	000127	05	999873	37		28
	36	24	387962	612038	388092	8567	611908	000130	05	999870	36		24
	40	25	393101	606899	393234	8467	606766	000133	05	999867	35		20
	44	26	398179	601821	398315	8369	601685	000136	05	999864	34		16
	48	27	403199	596801	403338	8274	596662	000139	05	999861	33		12
	52	28	408161	591839	408304	8180	591696	000142	05	999858	32		8
	56	29	413068	586932	413213	8089	586787	000146	05	999854	31		4
6	0	30	8.417919	11.582081	8.418068	8000	11.581932	10.000149	06	9.999851	30	54	0
	4	31	422717	577283	422869	7912	577131	000152	06	999848	29		56
	8	32	427462	572538	427618	7826	572382	000156	06	999844	28		52
	12	33	432156	567844	432315	7743	567685	000159	06	999841	27		48
	16	34	436800	563200	436962	7660	563038	000162	06	999838	26		44
	20	35	441394	558606	441560	7580	558440	000166	06	999834	25		40
	24	36	445941	554059	446110	7502	553890	000169	06	999831	24		36
	28	37	450440	549560	450613	7425	549387	000173	06	999827	23		32
	32	38	454893	545107	455070	7349	544930	000177	06	999823	22		28
	36	39	459301	540699	459481	7276	540519	000180	06	999820	21		24
	40	40	463665	536335	463849	7203	536151	000184	06	999816	20		20
	44	41	467985	532015	468172	7132	531828	000188	06	999812	19		16
	48	42	472263	527737	472454	7063	527546	000191	06	999809	18		12
	52	43	476498	523502	476693	6995	523307	000195	06	999805	17		8
	56	44	480693	519307	480892	6928	519108	000199	06	999801	16		4
7	0	45	8.484848	11.515152	8.485050	6862	11.514950	10.000203	07	9.999797	15	53	0
	4	46	488963	511037	489170	6798	510830	000207	07	999793	14		56
	8	47	493040	506960	493250	6735	506750	000210	07	999790	13		52
	12	48	497078	502922	497293	6673	502707	000214	07	999786	12		48
	16	49	501080	498920	501298	6612	498702	000218	07	999782	11		44
	20	50	505045	494955	505267	6552	494733	000222	07	999778	10		40
	24	51	508974	491026	509200	6493	490800	000226	07	999774	9		36
	28	52	512867	487133	513098	6435	486902	000231	07	999769	8		32
	32	53	516726	483274	516961	6379	483039	000235	07	999765	7		28
	36	54	520551	479449	520790	6323	479210	000239	07	999761	6		24
	40	55	524343	475657	524586	6268	475414	000243	07	999757	5		20
	44	56	528102	471898	528349	6215	471651	000247	07	999753	4		16
	48	57	531828	468172	532080	6162	467920	000252	07	999748	3		12
	52	58	535523	464477	535779	6110	464221	000256	07	999744	2		8
	56	59	539186	460814	539447	6059	460553	000260	07	999740	1		4
8	0	60	542819	457181	543084	6008	456916	000265	07	999735	0	52	0
m.	s.	′	Cosine.	Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			5 Hours,			or				88 Degrees.			

P. P. to s or ″	1s	15″	1200	1s	15″	1200	1s	15″	1	P. P. to s or ″
	2	30	2400	2	30	2400	2	30	2	
	3	45	3600	3	45	3600	3	45	3	

0 Hour, or 2 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D	Cosine.	′	m.	s.
8	0	0	8.542819	6004	11.457181	8.543084	6012	11.456916	10.000265	07	9.999735	60	52	0
	4	1	546422	5955	453578	546691	5962	453309	000269	07	999731	59		56
	8	2	549995	5906	450005	550268	5914	449732	000274	07	999726	58		52
	12	3	553539	5858	446461	553817	5866	446183	000278	08	999722	57		48
	16	4	557054	5811	442946	557336	5819	442664	000283	08	999717	56		44
	20	5	560540	5765	439460	560828	5773	439172	000287	08	999713	55		40
	24	6	563999	5719	436001	564291	5727	435709	000292	08	999708	54		36
	28	7	567431	5674	432569	567727	5682	432273	000296	08	999704	53		32
	32	8	570836	5630	429164	571137	5638	428863	000301	08	999699	52		28
	36	9	574214	5587	425786	574520	5595	425480	000306	08	999694	51		24
	40	10	577566	5544	422434	577877	5552	422123	000311	08	999689	50		20
	44	11	580892	5502	419108	581208	5510	418792	000315	08	999685	49		16
	48	12	584193	5460	415807	584514	5468	415486	000320	08	999680	48		12
	52	13	587469	5419	412531	587795	5427	412205	000325	08	999675	47		8
	56	14	590721	5379	409279	591051	5387	408949	000330	08	999670	46		4
9	0	15	8.593948	5339	11.406052	8.594283	5347	11.405717	10.000335	08	9.999665	45	51	0
	4	16	597152	5300	402848	597492	5308	402508	000340	08	999660	44		56
	8	17	600332	5261	399668	600677	5270	399323	000345	08	999655	43		52
	12	18	603489	5223	396511	603839	5232	396161	000350	08	999650	42		48
	16	19	606623	5186	393377	606978	5194	393022	000355	09	999645	41		44
	20	20	609734	5149	390266	610094	5158	389906	000360	09	999640	40		40
	24	21	612823	5112	387177	613189	5121	386811	000365	09	999635	39		36
	28	22	615891	5076	384109	616262	5085	383738	000371	09	999629	38		32
	32	23	618937	5041	381063	619313	5050	380687	000376	09	999624	37		28
	36	24	621962	5006	378038	622343	5015	377657	000381	09	999619	36		24
	40	25	624965	4972	375035	625352	4981	374648	000386	09	999614	35		20
	44	26	627948	4938	372052	628340	4947	371660	000392	09	999608	34		16
	48	27	630911	4904	369089	631308	4913	368692	000397	09	999603	33		12
	52	28	633854	4871	366146	634256	4880	365744	000403	09	999597	32		8
	56	29	636776	4839	363224	637184	4848	362816	000408	09	999592	31		4
10	0	30	8.639680	4806	11.360320	8.640093	4816	11.359907	10.000414	09	9.999586	30	50	0
	4	31	642563	4775	357437	642982	4784	357018	000419	09	999581	29		56
	8	32	645428	4743	354572	645853	4753	354147	000425	09	999575	28		52
	12	33	648274	4712	351726	648704	4722	351296	000430	09	999570	27		48
	16	34	651102	4682	348898	651537	4691	348463	000436	09	999564	26		44
	20	35	653911	4652	346089	654352	4661	345648	000442	10	999558	25		40
	24	36	656702	4622	343298	657149	4631	342851	000447	10	999553	24		36
	28	37	659475	4592	340525	659928	4602	340072	000453	10	999547	23		32
	32	38	662230	4563	337770	662689	4573	337311	000459	10	999541	22		28
	36	39	664968	4535	335032	665433	4544	334567	000465	10	999535	21		24
	40	40	667689	4506	332311	668160	4516	331840	000471	10	999529	20		20
	44	41	670393	4479	329607	670870	4488	329130	000476	10	999524	19		16
	48	42	673080	4451	326920	673563	4461	326437	000482	10	999518	18		12
	52	43	675751	4424	324249	676239	4434	323761	000488	10	999512	17		8
	56	44	678405	4397	321595	678900	4417	321100	000494	10	999506	16		4
11	0	45	8.681043	4370	11.318957	8.681544	4380	11.318456	10.000500	10	9.999500	15	49	0
	4	46	683665	4344	316335	684172	4354	315828	000507	10	999493	14		56
	8	47	686272	4318	313728	686784	4328	313216	000513	10	999487	13		52
	12	48	688863	4292	311137	689381	4303	310619	000519	10	999481	12		48
	16	49	691438	4267	308562	691963	4277	308037	000525	10	999475	11		44
	20	50	693998	4242	306002	694529	4252	305471	000531	10	999469	10		40
	24	51	696543	4217	303457	697081	4228	302919	000537	11	999463	9		36
	28	52	699073	4192	300927	699617	4203	300383	000544	11	999456	8		32
	32	53	701589	4168	298411	702139	4179	297861	000550	11	999450	7		28
	36	54	704090	4144	295910	704646	4155	295354	000557	11	999443	6		24
	40	55	706577	4121	293423	707140	4132	292860	000563	11	999437	5		20
	44	56	709049	4097	290951	709618	4108	290382	000569	11	999431	4		16
	48	57	711507	4074	288493	712083	4085	287917	000576	11	999424	3		12
	52	58	713952	4051	286048	714534	4062	285465	000582	11	999418	2		8
	56	59	716383	4029	283617	716972	4040	283028	000589	11	999411	1		4
12	0	60	718800	4006	281200	719396	4017	280604	000596	11	999404	0	48	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 87 Degrees.

P. P. to s or ″	1s	15″	721	1s	15″	722	1s	15″	1	P. P. to s or ″
	2	30	1442	2	30	1445	2	30	3	
	3	45	2163	3	45	2167	3	45	4	

0 Hour,							or			3 Degrees.				
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
12	0	0	8.718800	4006	11.281200	8.719396	4017	11.280604	10.000596	11	9.999404	60	48	0
	4	1	721204	3984	278796	721806	3995	278194	000602	11	999398	59		56
	8	2	723595	3962	276405	724204	3974	275796	000609	11	999391	58		52
	12	3	725972	3941	274028	726588	3952	273412	000616	11	999384	57		48
	16	4	728337	3919	271663	728959	3930	271041	000622	11	999378	56		44
	20	5	730688	3898	269312	731317	3909	268683	000629	11	999371	55		40
	24	6	733027	3877	266973	733663	3889	266337	000636	12	999364	54		36
	28	7	735354	3857	264646	735996	3868	264004	000643	12	999357	53		32
	32	8	737667	3836	262333	738317	3848	261683	000650	12	999350	52		28
	36	9	739969	3816	260031	740626	3827	259374	000657	12	999343	51		24
	40	10	742259	3796	257741	742922	3807	257078	000664	12	999336	50		20
	44	11	744536	3776	255464	745207	3787	254793	000671	12	999329	49		16
	48	12	746802	3756	253198	747479	3768	252521	000678	12	999322	48		12
	52	13	749055	3737	250945	749740	3749	250260	000685	12	999315	47		8
	56	14	751297	3717	248703	751989	3729	248011	000692	12	999308	46		4
13	0	15	8.753528	3698	11.246472	8.754227	3710	11.245773	10.000699	12	9.999301	45	47	0
	4	16	755747	3678	244253	756453	3692	243547	000706	12	999294	44		56
	8	17	757955	3661	242045	758668	3673	241332	000714	12	999286	43		52
	12	18	760151	3642	239849	760872	3655	239128	000721	12	999279	42		48
	16	19	762337	3624	237663	763065	3636	236935	000728	12	999272	41		44
	20	20	764511	3606	235489	765246	3618	234754	000735	12	999265	40		40
	24	21	766675	3588	233325	767417	3600	232583	000743	12	999257	39		36
	28	22	768828	3570	231172	769578	3583	230422	000750	13	999250	38		32
	32	23	770970	3553	229030	771727	3565	228273	000758	13	999242	37		28
	36	24	773101	3535	226899	773866	3548	226134	000765	13	999235	36		24
	40	25	775223	3518	224777	775995	3531	224005	000773	13	999227	35		20
	44	26	777333	3501	222667	778114	3514	221886	000780	13	999220	34		16
	48	27	779434	3484	220566	780222	3497	219778	000788	13	999212	33		12
	52	28	781524	3467	218476	782320	3480	217680	000795	13	999205	32		8
	56	29	783605	3451	216395	784408	3464	215592	000803	13	999197	31		4
14	0	30	8.785675	3434	11.214325	8.786486	3447	11.213514	10.000811	13	9.999189	30	46	0
	4	31	787736	3418	212264	788554	3432	211446	000819	13	999181	29		56
	8	32	789787	3402	210213	790613	3415	209387	000826	13	999174	28		52
	12	33	791828	3386	208172	792662	3399	207338	000834	13	999166	27		48
	16	34	793859	3370	206141	794701	3383	205299	000842	13	999158	26		44
	20	35	795881	3354	204119	796731	3368	203269	000850	13	999150	25		40
	24	36	797894	3339	202106	798752	3352	201248	000858	13	999142	24		36
	28	37	799897	3323	200103	800763	3337	199237	000866	13	999134	23		32
	32	38	801892	3308	198108	802765	3322	197235	000874	13	999126	22		28
	36	39	803876	3293	196124	804758	3307	195242	000882	13	999118	21		24
	40	40	805852	3278	194148	806742	3292	193258	000890	13	999110	20		20
	44	41	807819	3263	192181	808717	3278	191283	000898	13	999102	19		16
	48	42	809777	3249	190223	810683	3262	189317	000906	14	999094	18		12
	52	43	811726	3234	188274	812641	3248	187359	000914	14	999086	17		8
	56	44	813667	3219	186333	814589	3233	185411	000923	14	999077	16		4
15	0	45	8.815599	3205	11.184401	8.816529	3219	11.183471	10.000931	14	9.999069	15	45	0
	4	46	817522	3191	182478	818461	3205	181539	000939	14	999061	14		56
	8	47	819436	3177	180564	820384	3191	179616	000947	14	999053	13		52
	12	48	821343	3163	178657	822298	3177	177702	000956	14	999044	12		48
	16	49	823240	3149	176760	824205	3163	175795	000964	14	999036	11		44
	20	50	825130	3135	174870	826103	3150	173897	000973	14	999027	10		40
	24	51	827011	3122	172989	827992	3136	172008	000981	14	999019	9		36
	28	52	828884	3108	171116	829874	3123	170126	000990	14	999010	8		32
	32	53	830749	3095	169251	831748	3110	168252	000998	14	999002	7		28
	36	54	832607	3082	167393	833613	3096	166387	001007	14	998993	6		24
	40	55	834456	3069	165544	835471	3083	164529	001016	14	998984	5		20
	44	56	836297	3056	163703	837321	3070	162679	001024	14	998976	4		16
	48	57	838130	3043	161870	839163	3057	160837	001033	15	998967	3		12
	52	58	839956	3030	160044	840998	3045	159002	001042	15	998958	2		8
	56	59	841774	3017	158226	842825	3032	157175	001050	15	998950	1		4
16	0	60	843585	3005	156415	844644	3019	155356	001059	15	998941	0	44	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
5 Hours,							or			86 Degrees.				

P. P. to s or ″	1^s	15″	515	1^s	15″	517	1^s	15″	2	P. P. to s or ″
	2	30	1030	2	30	1034	2	30	4	
	3	45	1544	3	45	1551	3	45	6	

0 Hour, or 4 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
16	0	0	8.843585	3005	11.156415	8.844644	3019	11.155356	10.001059	15	9.998941	60	44	0
	4	1	845387	2992	154613	846455	3007	153545	001068	15	998932	59		56
	8	2	847183	2980	152817	848260	2995	151740	001077	15	998923	58		52
	12	3	848971	2967	151029	850057	2982	149943	001086	15	998914	57		48
	16	4	850751	2955	149249	851846	2970	148154	001095	15	998905	56		44
	20	5	852525	2943	147475	853628	2958	146372	001104	15	998896	55		40
	24	6	854291	2931	145709	855403	2946	144597	001113	15	998887	54		36
	28	7	856049	2919	143951	857171	2935	142829	001122	15	998878	53		32
	32	8	857801	2908	142199	858932	2923	141068	001131	15	998869	52		28
	36	9	859546	2896	140454	860686	2911	139314	001140	15	998860	51		24
	40	10	861283	2884	138717	862433	2900	137567	001149	15	998851	50		20
	44	11	863014	2873	136986	864173	2888	135827	001159	15	998841	49		16
	48	12	864738	2861	135262	865906	2877	134094	001168	15	998832	48		12
	52	13	866455	2850	133545	867632	2866	132368	001177	16	998823	47		8
	56	14	868165	2839	131835	869351	2854	130649	001187	16	998813	46		4
17	0	15	8.869868	2828	11.130132	8.871064	2843	11.128936	10.001196	16	9.998804	45	43	0
	4	16	871565	2817	128435	872770	2832	127230	001205	16	998795	44		56
	8	17	873255	2806	126745	874469	2821	125531	001215	16	998785	43		52
	12	18	874938	2795	125062	876162	2811	123838	001224	16	998776	42		48
	16	19	876615	2784	123385	877849	2800	122151	001234	16	998766	41		44
	20	20	878285	2773	121715	879529	2789	120471	001243	16	998757	40		40
	24	21	879949	2763	120051	881202	2779	118798	001253	16	998747	39		36
	28	22	881607	2752	118393	882869	2768	117131	001262	16	998738	38		32
	32	23	883258	2742	116742	884530	2758	115470	001272	16	998728	37		28
	36	24	884903	2731	115097	886185	2747	113815	001282	16	998718	36		24
	40	25	886542	2721	113458	887833	2737	112167	001292	16	998708	35		20
	44	26	888174	2711	111826	889476	2727	110524	001301	16	998699	34		16
	48	27	889801	2700	110199	891112	2717	108888	001311	16	998689	33		12
	52	28	891421	2690	108579	892742	2707	107258	001321	16	998679	32		8
	56	29	893035	2680	106965	894366	2697	105634	001331	17	998669	31		4
18	0	30	8.894643	2670	11.105357	8.895984	2687	11.104016	10.001341	17	9.998659	30	42	0
	4	31	896246	2660	103754	897596	2677	102404	001351	17	998649	29		56
	8	32	897842	2651	102158	899203	2667	100797	001361	17	998639	28		52
	12	33	899432	2641	100568	900803	2658	099197	001371	17	998629	27		48
	16	34	901017	2631	098983	902398	2648	097602	001381	17	998619	26		44
	20	35	902596	2622	097404	903987	2638	096013	001391	17	998609	25		40
	24	36	904169	2612	095831	905570	2629	094430	001401	17	998599	24		36
	28	37	905736	2603	094264	907147	2620	092853	001411	17	998589	23		32
	32	38	907297	2593	092703	908719	2610	091281	001422	17	998578	22		28
	36	39	908853	2584	091147	910285	2601	089715	001432	17	998568	21		24
	40	40	910404	2575	089596	911846	2592	088154	001442	17	998558	20		20
	44	41	911949	2566	088051	913401	2583	086599	001452	17	998548	19		16
	48	42	913488	2556	086512	914951	2574	085049	001463	17	998537	18		12
	52	43	915022	2547	084978	916495	2565	083505	001473	17	998527	17		8
	56	44	916550	2538	083450	918034	2556	081966	001484	18	998516	16		4
19	0	45	8.918073	2529	11.081927	8.919568	2547	11.080432	10.001494	18	9.998506	15	41	0
	4	46	919591	2520	080409	921096	2538	078904	001505	18	998495	14		56
	8	47	921103	2512	078897	922619	2530	077381	001515	18	998485	13		52
	12	48	922610	2503	077390	924136	2521	075864	001526	18	998474	12		48
	16	49	924112	2494	075888	925649	2512	074351	001536	18	998464	11		44
	20	50	925609	2486	074391	927156	2503	072844	001547	18	998453	10		40
	24	51	927100	2477	072900	928658	2495	071342	001558	18	998442	9		36
	28	52	928587	2469	071413	930155	2486	069845	001569	18	998431	8		32
	32	53	930068	2460	069932	931647	2478	068353	001579	18	998421	7		28
	36	54	931544	2452	068456	933134	2470	066866	001590	18	998410	6		24
	40	55	933015	2443	066985	934616	2461	065384	001601	18	998399	5		20
	44	56	934481	2435	065519	936093	2453	063907	001612	18	998388	4		16
	48	57	935942	2427	064058	937565	2445	062435	001623	18	998377	3		12
	52	58	937398	2419	062602	939032	2437	060968	001634	18	998366	2		8
	56	59	938850	2411	061150	940494	2430	059506	001645	18	998355	1		4
20	0	60	940296	2403	059704	941952	2421	058048	001656	18	998344	0	40	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 85 Degrees.

P. P. to s or ″										P. P. to s or ″
	1s	15″	401	1s	15″	403	1s	15″	3	
	2	30	801	2	30	806	2	30	5	
	3	45	1202	3	45	1209	3	45	8	

0 Hour,						or				5 Degrees.				
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
20	0	0	8.940296	2403	11.059704	8.941952	2421	11.058048	10.001656	19	9.998344	60	40	0
	4	1	941738	2394	058262	943404	2413	056596	001667	19	998333	59		56
	8	2	943174	2387	056826	944852	2405	055148	001678	19	998322	58		52
	12	3	944606	2379	055394	946295	2397	053705	001689	19	998311	57		48
	16	4	946034	2371	053966	947734	2390	052266	001700	19	998300	56		44
	20	5	947456	2363	052544	949168	2382	050832	001711	19	998289	55		40
	24	6	948874	2355	051126	950597	2374	049403	001723	19	998277	54		36
	28	7	950287	2348	049713	952021	2366	047979	001734	19	998266	53		32
	32	8	951696	2340	048304	953441	2359	046559	001745	19	998255	52		28
	36	9	953100	2332	046900	954856	2351	045144	001757	19	998243	51		24
	40	10	954499	2325	045501	956267	2344	043733	001768	19	998232	50		20
	44	11	955894	2317	044106	957674	2337	042326	001780	19	998220	49		16
	48	12	957284	2310	042716	959075	2329	040925	001791	19	998209	48		12
	52	13	958670	2302	041330	960473	2322	039527	001803	19	998197	47		8
	56	14	960052	2295	039948	961866	2314	038134	001814	19	998186	46		4
21	0	15	8.961429	2288	11.038571	8.963255	2307	11.036745	10.001826	19	9.998174	45	39	0
	4	16	962801	2280	037199	964639	2300	035361	001837	19	998163	44		56
	8	17	964170	2273	035830	966019	2293	033981	001849	19	998151	43		52
	12	18	965534	2266	034466	967394	2286	032606	001861	20	998139	42		48
	16	19	966893	2259	033107	968766	2279	031234	001872	20	998128	41		44
	20	20	968249	2252	031751	970133	2271	029867	001884	20	998116	40		40
	24	21	969600	2245	030400	971496	2265	028504	001896	20	998104	39		36
	28	22	970947	2238	029053	972855	2257	027145	001908	20	998092	38		32
	32	23	972289	2231	027711	974209	2251	025791	001920	20	998080	37		28
	36	24	973628	2224	026372	975560	2244	024440	001932	20	998068	36		24
	40	25	974962	2217	025038	976906	2237	023094	001944	20	998056	35		20
	44	26	976293	2210	023707	978248	2230	021752	001956	20	998044	34		16
	48	27	977619	2203	022381	979586	2223	020414	001968	20	998032	33		12
	52	28	978941	2197	021059	980921	2217	019079	001980	20	998020	32		8
	56	29	980259	2190	019741	982251	2210	017749	001992	20	998008	31		4
22	0	30	8.981573	2183	11.018427	8.983577	2204	11.016423	10.002004	20	9.997996	30	38	0
	4	31	982883	2177	017117	984899	2197	015101	002016	20	997984	29		56
	8	32	984189	2170	015811	986217	2191	013783	002028	20	997972	28		52
	12	33	985491	2163	014509	987532	2184	012468	002041	20	997959	27		48
	16	34	986789	2157	013211	988842	2178	011158	002053	20	997947	26		44
	20	35	988083	2150	011917	990149	2171	009851	002065	21	997935	25		40
	24	36	989374	2144	010626	991451	2165	008549	002078	21	997922	24		36
	28	37	990660	2138	009340	992750	2158	007250	002090	21	997910	23		32
	32	38	991943	2131	008057	994045	2152	005955	002103	21	997897	22		28
	36	39	993222	2125	006778	995337	2146	004663	002115	21	997885	21		24
	40	40	994497	2119	005503	996624	2140	003376	002128	21	997872	20		20
	44	41	995768	2112	004232	997908	2134	002092	002140	21	997860	19		16
	48	42	997036	2106	002964	999188	2127	11·000812	002153	21	997847	18		12
	52	43	998299	2100	001701	9.000465	2121	10.999535	002165	21	997835	17		8
	56	44	999560	2094	000441	001738	2115	998262	002178	21	997822	16		4
23	0	45	9.000816	2088	10.999184	9.003007	2109	10.996993	10.002191	21	9.997809	15	37	0
	4	46	002069	2082	997931	004272	2103	995728	002203	21	997797	14		56
	8	47	003318	2076	996682	005534	2097	994466	002216	21	997784	13		52
	12	48	004563	2070	995437	006792	2091	993208	002229	21	997771	12		48
	16	49	005805	2064	994195	008047	2085	991953	002242	21	997758	11		44
	20	50	007044	2058	992956	009298	2080	990702	002255	21	997745	10		40
	24	51	008278	2052	991722	010546	2074	989454	002268	21	997732	9		36
	28	52	009510	2046	990490	011790	2068	988210	002281	21	997719	8		32
	32	53	010737	2040	989263	013031	2062	986969	002294	21	997706	7		28
	36	54	011962	2034	988038	014268	2056	985732	002307	22	997693	6		24
	40	55	013182	2029	986818	015502	2051	984498	002320	22	997680	5		20
	44	56	014400	2023	985600	016732	2045	983268	002333	22	997667	4		16
	48	57	015613	2017	984387	017959	2040	982041	002346	22	997654	3		12
	52	58	016824	2012	983176	019183	2033	980817	002359	22	997641	2		8
	56	59	018031	2006	981969	020403	2028	979597	002372	22	997628	1		4
24	0	60	019235	2000	980765	021620	2023	978380	002386	22	997614	0	36	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
5 Hours,						or				84 Degrees.				

P. P. to s or ″	1s	15″	327	1s	15″	330	1s	15″	3	P. P. to s or ″
	2	30	655	2	30	661	2	30	6	
	3	45	982	3	45	992	3	45	9	

0 Hour, or 6 Degrees.

m.	s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	'	m.	s.
24	0	0	9.019235	2000	10.980765	9.021620	2023	10.978380	10.002386	22	9.997614	60	36	0
	4	1	020435	1995	979565	022834	2017	977166	002399	22	997601	59		56
	8	2	021632	1989	978368	024044	2011	975956	002412	22	997588	58		52
	12	3	022825	1984	977175	025251	2006	974749	002426	22	997574	57		48
	16	4	024016	1978	975984	026455	2000	973545	002439	22	997561	56		44
	20	5	025203	1973	974797	027655	1995	972345	002453	22	997547	55		40
	24	6	026386	1967	973614	028852	1990	971148	002466	23	997534	54		36
	28	7	027567	1962	972433	030046	1985	969954	002480	23	997520	53		32
	32	8	028744	1957	971256	031237	1979	968763	002493	23	997507	52		28
	36	9	029918	1951	970082	032425	1974	967575	002507	23	997493	51		24
	40	10	031089	1947	968911	033609	1969	966391	002520	23	997480	50		20
	44	11	032257	1941	967743	034791	1964	965209	002534	23	997466	49		16
	48	12	033421	1936	966579	035969	1958	964031	002548	23	997452	48		12
	52	13	034582	1930	965418	037144	1953	962856	002561	23	997439	47		8
	56	14	035741	1925	964259	038316	1948	961684	002575	23	997425	46		4
25	0	15	9.036896	1920	10.963104	9.039485	1943	10.960515	10.002589	23	9.997411	45	35	0
	4	16	038048	1915	961952	040651	1938	959349	002603	23	997397	44		56
	8	17	039197	1910	960803	041813	1933	958187	002617	23	997383	43		52
	12	18	040342	1905	959658	042973	1928	957027	002631	23	997369	42		48
	16	19	041485	1899	958515	044130	1923	955870	002645	23	997355	41		44
	20	20	042625	1895	957375	045284	1918	954716	002659	23	997341	40		40
	24	21	043762	1889	956238	046434	1913	953566	002673	24	997327	39		36
	28	22	044895	1884	955105	047582	1908	952418	002687	24	997313	38		32
	32	23	046026	1879	953974	048727	1903	951273	002701	24	997299	37		28
	36	24	047154	1875	952846	049869	1898	950131	002715	24	997285	36		24
	40	25	048279	1870	951721	051008	1893	948992	002729	24	997271	35		20
	44	26	049400	1865	950600	052144	1889	947856	002743	24	997257	34		16
	48	27	050519	1860	949481	053277	1884	946723	002758	24	997242	33		12
	52	28	051635	1855	948365	054407	1879	945593	002772	24	997228	32		8
	56	29	052749	1850	947251	055535	1874	944465	002786	24	997214	31		4
26	0	30	9.053859	1845	10.946141	9.056659	1870	10.943341	10.002801	24	9.997199	30	34	0
	4	31	054966	1841	945034	057781	1865	942219	002815	24	997185	29		56
	8	32	056071	1836	943929	058900	1860	941100	002830	24	997170	28		52
	12	33	057172	1831	942828	060016	1855	939984	002844	24	997156	27		48
	16	34	058271	1827	941729	061130	1851	938870	002859	24	997141	26		44
	20	35	059367	1822	940633	062240	1846	937760	002873	24	997127	25		40
	24	36	060460	1817	939540	063348	1842	936652	002888	24	997112	24		36
	28	37	061551	1813	938449	064453	1837	935547	002902	24	997098	23		32
	32	38	062639	1808	937361	065556	1833	934444	002917	25	997083	22		28
	36	39	063724	1804	936276	066655	1828	933345	002932	25	997068	21		24
	40	40	064806	1799	935194	067752	1824	932248	002947	25	997053	20		20
	44	41	065885	1794	934115	068846	1819	931154	002961	25	997039	19		16
	48	42	066962	1790	933038	069938	1815	930062	002976	25	997024	18		12
	52	43	068036	1786	931964	071027	1810	928973	002991	25	997009	17		8
	56	44	069107	1781	930893	072113	1806	927887	003006	25	996994	16		4
27	0	45	9.070176	1777	10.929824	9.073197	1802	10.926803	10.003021	25	9.996979	15	33	0
	4	46	071242	1772	928758	074278	1797	925722	003036	25	996964	14		56
	8	47	072306	1768	927694	075356	1793	924644	003051	25	996949	13		52
	12	48	073366	1763	926634	076432	1789	923568	003066	25	996934	12		48
	16	49	074424	1759	925576	077505	1784	922495	003081	25	996919	11		44
	20	50	075480	1755	924520	078576	1780	921424	003096	25	996904	10		40
	24	51	076533	1750	923467	079644	1776	920356	003111	25	996889	9		36
	28	52	077583	1746	922417	080710	1772	919290	003126	25	996874	8		32
	32	53	078631	1742	921369	081773	1767	918227	003142	25	996858	7		28
	36	54	079676	1738	920324	082833	1763	917167	003157	25	996843	6		24
	40	55	080719	1733	919281	083891	1759	916109	003172	25	996828	5		20
	44	56	081759	1729	918241	084947	1755	915053	003188	26	996812	4		16
	48	57	082797	1725	917203	086000	1751	914000	003203	26	996797	3		12
	52	58	083832	1721	916168	087050	1747	912950	003218	26	996782	2		8
	56	59	084864	1717	915136	088098	1743	911902	003234	26	996766	1		4
28	0	60	085894	1713	914106	089144	1738	910856	003249	26	996751	0	32	0
m.	s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	'	m.	s.

5 Hours, or 83 Degrees.

P. P. to s or "	1s	15"	277	1s	15"	280	1s	15"	3	P. P. to s or "
	2	30	554	2	30	561	2	30	7	
	3	45	831	3	45	841	3	45	11	

0 Hour, or 7 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
28	0	0	9.085894	1713	10.914106	9.089144	1738	10.910856	10.003249	26	9.996751	60	32	0
	4	1	086922	1709	913078	090187	1735	909813	003265	26	996735	59		56
	8	2	087947	1704	912053	091228	1730	908772	003280	26	996720	58		52
	12	3	088970	1700	911030	092266	1727	907734	003296	26	996704	57		48
	16	4	089990	1696	910010	093302	1722	906698	003312	26	996688	56		44
	20	5	091008	1692	908992	094336	1719	905664	003327	26	996673	55		40
	24	6	092024	1688	907976	095367	1715	904633	003343	26	996657	54		36
	28	7	093037	1684	906963	096395	1711	903605	003359	26	996641	53		32
	32	8	094047	1680	905953	097422	1707	902578	003375	26	996625	52		28
	36	9	095056	1676	904944	098446	1703	901554	003390	26	996610	51		24
	40	10	096062	1673	903938	099468	1699	900532	003406	26	996594	50		20
	44	11	097065	1668	902935	100487	1695	899513	003422	27	996578	49		16
	48	12	098066	1665	901934	101504	1691	898496	003438	27	996562	48		12
	52	13	099065	1661	900935	102519	1687	897481	003454	27	996546	47		8
	56	14	100062	1657	899938	103532	1684	896468	003470	27	996530	46		4
29	0	15	9.101056	1653	10.898944	9.104542	1680	10.895458	10.003486	27	9.996514	45	31	0
	4	16	102048	1649	897952	105550	1676	894450	003502	27	996498	44		56
	8	17	103037	1645	896963	106556	1672	893444	003518	27	996482	43		52
	12	18	104025	1642	895975	107559	1669	892441	003535	27	996465	42		48
	16	19	105010	1638	894990	108560	1665	891440	003551	27	996449	41		44
	20	20	105992	1634	894008	109559	1661	890441	003567	27	996433	40		40
	24	21	106973	1630	893027	110556	1658	889444	003583	27	996417	39		36
	28	22	107951	1627	892049	111551	1654	888449	003600	27	996400	38		32
	32	23	108927	1623	891073	112543	1650	887457	003616	27	99638[illegible]	37		28
	36	24	109901	1619	890099	113533	1647	886467	003632	27	996368	36		24
	40	25	110873	1616	889127	114521	1643	885479	003649	27	996351	35		20
	44	26	111842	1612	888158	115507	1639	884493	003665	27	996335	34		16
	48	27	112809	1608	887191	116491	1636	883509	003682	27	996318	33		12
	52	28	113774	1605	886226	117472	1632	882528	003698	28	996302	32		8
	56	29	114737	1601	885263	118452	1629	881548	003715	28	996285	31		4
30	0	30	9.115698	1597	10.884302	9.119429	1625	10.880571	10.003731	28	9.996269	30	30	0
	4	31	116656	1594	883344	120404	1622	879596	003748	28	996252	29		56
	8	32	117613	1590	882387	121377	1618	878623	003765	28	996235	28		52
	12	33	118567	1587	881433	122348	1615	877652	003781	28	996219	27		48
	16	34	119519	1583	880481	123317	1611	876683	003798	28	996202	26		44
	20	35	120469	1580	879531	124284	1608	875716	003815	28	996185	25		40
	24	36	121417	1576	878583	125249	1604	874751	003832	28	996168	24		36
	28	37	122362	1573	877638	126211	1601	873789	003849	28	996151	23		32
	32	38	123306	1569	876694	127172	1597	872828	003866	28	996134	22		28
	36	39	124248	1566	875752	128130	1594	871870	003883	28	996117	21		24
	40	40	125187	1562	874813	129087	1591	870913	003900	28	996100	20		20
	44	41	126125	1559	873875	130041	1587	869959	003917	29	996083	19		16
	48	42	127060	1556	872940	130994	1584	869006	003934	29	996066	18		12
	52	43	127993	1552	872007	131944	1581	868056	003951	29	996049	17		8
	56	44	128925	1549	871075	132893	1577	867107	003968	29	996032	16		4
31	0	45	9.129854	1545	10.870146	9.133839	1574	10.866161	10.003985	29	9.996015	15	29	0
	4	46	130781	1542	869219	134784	1571	865216	004002	29	995998	14		56
	8	47	131706	1539	868294	135726	1567	864274	004020	29	995980	13		52
	12	48	132630	1535	867370	136667	1564	863333	004037	29	995963	12		48
	16	49	133551	1532	866449	137605	1561	862395	004054	29	995946	11		44
	20	50	134470	1529	865530	138542	1558	861458	004072	29	995928	10		40
	24	51	135387	1525	864613	139476	1555	860524	004089	29	995911	9		36
	28	52	136303	1522	863697	140409	1551	859591	004106	29	995894	8		32
	32	53	137216	1519	862784	141340	1548	858660	004124	29	995876	7		28
	36	54	138128	1516	861872	142269	1545	857731	004141	29	995859	6		24
	40	55	139037	1512	860963	143196	1542	856804	004159	29	995841	5		20
	44	56	139944	1509	860056	144121	1539	855879	004177	29	995823	4		16
	48	57	140850	1506	859150	145044	1535	854956	004194	29	995806	3		12
	52	58	141754	1503	858246	145966	1532	854034	004212	29	995788	2		8
	56	59	142655	1500	857345	146885	1529	853115	004229	29	995771	1		4
32	0	60	143555	1496	856445	147803	1526	852197	004247	29	995753	0	28	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 82 Degrees.

P. P. to s or ″	1s	15″	240	1s	15″	244	1s	15″	4	P. P. to s or ″
	2	30	479	2	30	487	2	30	8	
	3	45	719	3	45	731	3	45	13	

0 Hour, or 8 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
32	0	0	9.143555	1496	10.856445	9.147803	1526	10.852197	10.004247	30	9.995753	60	28	0
	4	1	144453	1493	855547	148718	1523	851282	004265	30	995735	59		56
	8	2	145349	1490	854651	149632	1520	850368	004283	30	995717	58		52
	12	3	146243	1487	853757	150544	1517	849456	004301	30	995699	57		48
	16	4	147136	1484	852864	151454	1514	848546	004319	30	995681	56		44
	20	5	148026	1481	851974	152363	1511	847637	004336	30	995664	55		40
	24	6	148915	1478	851085	153269	1508	846731	004354	30	995646	54		36
	28	7	149802	1475	850198	154174	1505	845826	004372	30	995628	53		32
	32	8	150686	1472	849314	155077	1502	844923	004390	30	995610	52		28
	36	9	151569	1469	848431	155978	1499	844022	004409	30	995591	51		24
	40	10	152451	1466	847549	156877	1496	843123	004427	30	995573	50		20
	44	11	153330	1463	846670	157775	1493	842225	004445	30	995555	49		16
	48	12	154208	1460	845792	158671	1480	841329	004463	30	995537	48		12
	52	13	155083	1457	844917	159565	1487	840435	004481	30	995519	47		8
	56	14	155957	1454	844043	160457	1484	839543	004499	31	995501	46		4
33	0	15	9.156830	1451	10.843170	9.161347	1481	10.838653	10.004518	31	9.995482	45	27	0
	4	16	157700	1448	842300	162236	1478	837764	004536	31	995464	44		56
	8	17	158569	1445	841431	163123	1475	836877	004554	31	995446	43		52
	12	18	159435	1442	840565	164008	1473	835992	004573	31	995427	42		48
	16	19	160301	1439	839699	164892	1470	835108	004591	31	995409	41		44
	20	20	161164	1436	838836	165774	1467	834226	004610	31	995390	40		40
	24	21	162025	1433	837975	166654	1464	833346	004628	31	995372	39		36
	28	22	162885	1430	837115	167532	1461	832468	004647	31	995353	38		32
	32	23	163743	1427	836257	168409	1458	831591	004666	31	995334	37		28
	36	24	164600	1424	835400	169284	1455	830716	004684	31	995316	36		24
	40	25	165454	1422	834546	170157	1453	829843	004703	31	995297	35		20
	44	26	166307	1419	833693	171029	1450	828971	004722	31	995278	34		16
	48	27	167159	1416	832841	171899	1447	828101	004740	31	995260	33		12
	52	28	168008	1413	831992	172767	1444	827233	004759	32	995241	32		8
	56	29	168856	1410	831144	173634	1442	826366	004778	32	995222	31		4
34	0	30	9.169702	1407	10.830298	9.174499	1439	10.825501	10.004797	32	9.995203	30	26	0
	4	31	170547	1405	829453	175362	1436	824638	004816	32	995184	29		56
	8	32	171389	1402	828611	176224	1433	823776	004835	32	995165	28		52
	12	33	172230	1399	827770	177084	1431	822916	004854	32	995146	27		48
	16	34	173070	1396	826930	177942	1428	822058	004873	32	995127	26		44
	20	35	173908	1394	826092	178799	1425	821201	004892	32	995108	25		40
	24	36	174744	1391	825256	179655	1423	820345	004911	32	995089	24		36
	28	37	175578	1388	824422	180508	1420	819492	004930	32	995070	23		32
	32	38	176411	1386	823589	181360	1417	818640	004949	32	995051	22		28
	36	39	177242	1383	822758	182211	1415	817789	004968	32	995032	21		24
	40	40	178072	1380	821928	183059	1412	816941	004987	32	995013	20		20
	44	41	178900	1377	821100	183907	1409	816093	005007	32	994993	19		16
	48	42	179726	1374	820274	184752	1407	815248	005026	32	994974	18		12
	52	43	180551	1372	819449	185597	1404	814403	005045	32	994955	17		8
	56	44	181374	1369	818626	186439	1402	813561	005065	32	994935	16		4
35	0	45	9.182196	1367	10.817804	9.187280	1399	10.812720	10.005084	33	9.994916	15	25	0
	4	46	183016	1364	816984	188120	1396	811880	005104	33	994896	14		56
	8	47	183834	1361	816166	188958	1393	811042	005123	33	994877	13		52
	12	48	184651	1359	815349	189794	1391	810206	005143	33	994857	12		48
	16	49	185466	1356	814534	190629	1389	809371	005162	33	994838	11		44
	20	50	186280	1353	813720	191462	1386	808538	005182	33	994818	10		40
	24	51	187092	1351	812908	192294	1384	807706	005202	33	994798	9		36
	28	52	187903	1348	812097	193124	1381	806876	005221	33	994779	8		32
	32	53	188712	1346	811288	193953	1379	806047	005241	33	994759	7		28
	36	54	189519	1343	810481	194780	1376	805220	005261	33	994739	6		24
	40	55	190325	1341	809675	195606	1374	804394	005281	33	994719	5		20
	44	56	191130	1338	808870	196430	1371	803570	005300	33	994700	4		16
	48	57	191933	1336	808067	197253	1369	802747	005320	33	994680	3		12
	52	58	192734	1333	807266	198074	1366	801926	005340	33	994660	2		8
	56	59	193534	1330	806466	198894	1364	801106	005360	33	994640	1		4
36	0	60	194332	1328	805668	199713	1361	800287	005380	33	994620	0	24	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 81 Degrees.

P. P. to s or ″	1s	15″	211	1s	15″	216	1s	15″	5	P. P. to s or ″
	2	30	422	2	30	432	2	30	10	
	3	45	633	3	45	648	3	45	14	

0 Hour, or 9 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
36	0	0	9.194332	1328	10.805668	9.199713	1361	10.800287	10.005380	33	9.994620	60	24	0
	4	1	195129	1326	804871	200529	1359	799471	005400	33	994600	59		56
	8	2	195925	1323	804075	201345	1356	798655	005420	33	994580	58		52
	12	3	196719	1321	803281	202159	1354	797841	005440	34	994560	57		48
	16	4	197511	1318	802489	202971	1352	797029	005460	34	994540	56		44
	20	5	198302	1316	801698	203782	1349	796218	005481	34	994519	55		40
	24	6	199091	1313	800909	204592	1347	795408	005501	34	994499	54		36
	28	7	199879	1311	800121	205400	1345	794600	005521	34	994479	53		32
	32	8	200666	1308	799334	206207	1342	793793	005541	34	994459	52		28
	36	9	201451	1306	798549	207013	1340	792987	005562	34	994438	51		24
	40	10	202234	1304	797766	207817	1338	792183	005582	34	994418	50		20
	44	11	203017	1301	796983	208619	1335	791381	005602	34	994398	49		16
	48	12	203797	1299	796203	209420	1333	790580	005623	34	994377	48		12
	52	13	204577	1296	795423	210220	1331	789780	005643	34	994357	47		8
	56	14	205354	1294	794646	211018	1328	788982	005664	34	994336	46		4
37	0	15	9.206131	1292	10.793869	9.211815	1326	10.788185	10.005684	34	9.994316	45	23	0
	4	16	206906	1289	793094	212611	1324	787389	005705	34	994295	44		56
	8	17	207679	1287	792321	213405	1321	786595	005726	35	994274	43		52
	12	18	208452	1285	791548	214198	1319	785802	005746	35	994254	42		48
	16	19	209222	1282	790778	214989	1317	785011	005767	35	994233	41		44
	20	20	209992	1280	790008	215780	1315	784220	005788	35	994212	40		40
	24	21	210760	1278	789240	216568	1312	783432	005809	35	994191	39		36
	28	22	211526	1275	788474	217356	1310	782644	005829	35	994171	38		32
	32	23	212291	1273	787709	218142	1308	781858	005850	35	994150	37		28
	36	24	213055	1271	786945	218926	1305	781074	005871	35	994129	36		24
	40	25	213818	1268	786182	219710	1303	780290	005892	35	994108	35		20
	44	26	214579	1266	785421	220492	1301	779508	005913	35	994087	34		16
	48	27	215338	1264	784662	221272	1299	778728	005934	35	994066	33		12
	52	28	216097	1261	783903	222052	1297	777948	005955	35	994045	32		8
	56	29	216854	1259	783146	222830	1294	777170	005976	35	994024	31		4
38	0	30	9.217609	1257	10.782391	9.223607	1292	10.776393	10.005997	35	9.994003	30	22	0
	4	31	218363	1255	781637	224382	1290	775618	006018	35	993982	29		56
	8	32	219116	1253	780884	225156	1288	774844	006040	35	993960	28		52
	12	33	219868	1250	780132	225929	1286	774071	006061	35	993939	27		48
	16	34	220618	1248	779382	226700	1284	773300	006082	35	993918	26		44
	20	35	221367	1246	778633	227471	1281	772529	006103	36	993897	25		40
	24	36	222115	1244	777885	228239	1279	771761	006125	36	993875	24		36
	28	37	222861	1242	777139	229007	1277	770993	006146	36	993854	23		32
	32	38	223606	1239	776394	229773	1275	770227	006168	36	993832	22		28
	36	39	224349	1237	775651	230539	1273	769461	006189	36	993811	21		24
	40	40	225092	1235	774908	231302	1271	768698	006211	36	993789	20		20
	44	41	225833	1233	774167	232065	1269	767935	006232	36	993768	19		16
	48	42	226573	1231	773427	232826	1267	767174	006254	36	993746	18		12
	52	43	227311	1228	772689	233586	1265	766414	006275	36	993725	17		8
	56	44	228048	1226	771952	234345	1262	765655	006297	36	993703	16		4
39	0	45	9.228784	1224	10.771216	9.235103	1260	10.764897	10.006319	36	9.993681	15	21	0
	4	46	229518	1222	770482	235859	1258	764141	006340	36	993660	14		56
	8	47	230252	1220	769748	236614	1256	763386	006362	36	993638	13		52
	12	48	230984	1218	769016	237368	1254	762632	006384	36	993616	12		48
	16	49	231714	1216	768286	238120	1252	761880	006406	37	993594	11		44
	20	50	232444	1214	767556	238872	1250	761128	006428	37	993572	10		40
	24	51	233172	1212	766828	239622	1248	760378	006450	37	993550	9		36
	28	52	233899	1209	766101	240371	1246	759629	006472	37	993528	8		32
	32	53	234625	1207	765375	241118	1244	758882	006494	37	993506	7		28
	36	54	235349	1205	764651	241865	1242	758135	006516	37	993484	6		24
	40	55	236073	1203	763927	242610	1240	757390	006538	37	993462	5		20
	44	56	236795	1201	763205	243354	1238	756646	006560	37	993440	4		16
	48	57	237515	1199	762485	244097	1236	755903	006582	37	993418	3		12
	52	58	[illegible]	1197	761765	244839	1234	755161	006604	37	993396	2		8
	56	59	238953	1195	761047	245579	1232	754421	006626	37	993374	1		4
40	0	60	239670	1193	760330	246319	1230	753681	006649	37	993351	0	20	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 80 Degrees.

P. P. to s or ″	1s	15″	189	1s	15″	194	1s	15″	5	P. P. to s or ″
	2	30	377	2	30	388	2	30	10	
	3	45	566	3	45	581	3	45	16	

			0 Hour,				or				10 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
40	0	0	9.239670	1193	10.760330	9.246319	1230	10.753681	10.006649	37	9.993351	60	20	0
	4	1	240386	1191	759614	247057	1228	752943	006671	37	993329	59		56
	8	2	241101	1189	758899	247794	1226	752206	006693	37	993307	58		52
	12	3	241814	1187	758186	248530	1224	751470	006715	37	993285	57		48
	16	4	242526	1185	757474	249264	1222	750736	006738	37	993262	56		44
	20	5	243237	1183	756763	249998	1220	750002	006760	37	993240	55		40
	24	6	243947	1181	756053	250730	1218	749270	006783	38	993217	54		36
	28	7	244656	1179	755344	251461	1217	748539	006805	38	993195	53		32
	32	8	245363	1177	754637	252191	1215	747809	006828	38	993172	52		28
	36	9	246069	1175	753931	252920	1213	747080	006851	38	993149	51		24
	40	10	246775	1173	753225	253648	1211	746352	006873	38	993127	50		20
	44	11	247478	1171	752522	254374	1209	745626	006896	38	993104	49		16
	48	12	248181	1169	751819	255100	1207	744900	006919	38	993081	48		12
	52	13	248883	1167	751117	255824	1205	744176	006941	38	993059	47		8
	56	14	249583	1165	750417	256547	1203	743453	006964	38	993036	46		4
41	0	15	9.250282	1163	10.749718	9.257269	1201	10.742731	10.006987	38	9.993013	45	19	0
	4	16	250980	1161	749020	257990	1200	742010	007010	38	992990	44		56
	8	17	251677	1159	748323	258710	1198	741290	007033	38	992967	43		52
	12	18	252373	1158	747627	259429	1196	740571	007056	38	992944	42		48
	16	19	253067	1156	746933	260146	1194	739854	007079	38	992921	41		44
	20	20	253761	1154	746239	260863	1192	739137	007102	38	992898	40		40
	24	21	254453	1152	745547	261578	1190	738422	007125	38	992875	39		36
	28	22	255144	1150	744856	262292	1189	737708	007148	38	992852	38		32
	32	23	255834	1148	744166	263005	1187	736995	007171	39	992829	37		28
	36	24	256523	1146	743477	263717	1185	736283	007194	39	992806	36		24
	40	25	257211	1144	742789	264428	1183	735572	007217	39	992783	35		20
	44	26	257898	1142	742102	265138	1181	734862	007241	39	992759	34		16
	48	27	258583	1141	741417	265847	1179	734153	007264	39	992736	33		12
	52	28	259268	1139	740732	266555	1178	733445	007287	39	992713	32		8
	56	29	259951	1137	740049	267261	1176	732739	007310	39	992690	31		4
42	0	30	9.260633	1135	10.739367	9.267967	1174	10.732033	10.007334	39	9.992666	30	18	0
	4	31	261314	1133	738686	268671	1172	731329	007357	39	992643	29		56
	8	32	261994	1131	738006	269375	1170	730625	007381	39	992619	28		52
	12	33	262673	1130	737327	270077	1169	729923	007404	39	992596	27		48
	16	34	263351	1128	736649	270779	1167	729221	007428	39	992572	26		44
	20	35	264027	1126	735973	271479	1165	728521	007451	39	992549	25		40
	24	36	264703	1124	735297	272178	1164	727822	007475	39	992525	24		36
	28	37	265377	1122	734623	272876	1162	727124	007499	39	992501	23		32
	32	38	266051	1120	733949	273573	1160	726427	007522	40	992478	22		28
	36	39	266723	1119	733277	274269	1158	725731	007546	40	992454	21		24
	40	40	267395	1117	732605	274964	1157	725036	007570	40	992430	20		20
	44	41	268065	1115	731935	275658	1155	724342	007594	40	992406	19		16
	48	42	268734	1113	731266	276351	1153	723649	007618	40	992382	18		12
	52	43	269402	1111	730598	277043	1151	722957	007641	40	992359	17		8
	56	44	270069	1110	729931	277734	1150	722266	007665	40	992335	16		4
43	0	45	9.270735	1108	10.729265	9.278424	1148	10.721576	10.007689	40	9.992311	15	17	0
	4	46	271400	1106	728600	279113	1147	720887	007713	40	992287	14		56
	8	47	272064	1105	727936	279801	1145	720199	007737	40	992263	13		52
	12	48	272726	1103	727274	280488	1143	719512	007761	40	992239	12		48
	16	49	273388	1101	726612	281174	1141	718826	007786	40	992214	11		44
	20	50	274049	1099	725951	281858	1140	718142	007810	40	992190	10		40
	24	51	274708	1098	725292	282542	1138	717458	007834	40	992166	9		36
	28	52	275367	1096	724633	283225	1136	716775	007858	40	992142	8		32
	32	53	276024	1094	723976	283907	1135	716093	007882	41	992118	7		28
	36	54	276681	1092	723319	284588	1133	715412	007907	41	992093	6		24
	40	55	277337	1091	722663	285268	1131	714732	007931	41	992069	5		20
	44	56	277991	1089	722009	285947	1130	714053	007956	41	992044	4		16
	48	57	278645	1087	721355	286624	1128	713376	007980	41	992020	3		12
	52	58	279297	1086	720703	287301	1126	712699	008004	41	991996	2		8
	56	59	279948	1084	720052	287977	1125	712023	008029	41	991971	1		4
44	0	60	280599	1082	719401	288652	1123	711348	008053	41	991947	0	16	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			5 Hours,				or				79 Degrees.			

P. P. to s or ″	1s	15″	170	1s	15″	176	1s	15″	6	P. P. to s or ″.
	2	30	340	2	30	352	2	30	12	
	3	45	511	3	45	528	3	45	18	

0 Hour, or 11 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
44	0	0	9.280599	1082	10.719401	9.288652	1123	10.711348	10.008053	41	9.991947	60	16	0
	4	1	281248	1081	718752	289326	1122	710674	008078	41	991922	59		56
	8	2	281897	1079	718103	289999	1120	710001	008103	41	991897	58		52
	12	3	282544	1077	717456	290671	1118	709329	008127	41	991873	57		48
	16	4	283190	1076	716810	291342	1117	708658	008152	41	991848	56		44
	20	5	283836	1074	716164	292013	1115	707987	008177	41	991823	55		40
	24	6	284480	1072	715520	292682	1114	707318	008201	41	991799	54		36
	28	7	285124	1071	714876	293350	1112	706650	008226	42	991774	53		32
	32	8	285766	1069	714234	294017	1111	705983	008251	42	991749	52		28
	36	9	286408	1067	713592	294684	1109	705316	008276	42	991724	51		24
	40	10	287048	1066	712952	295349	1107	704651	008301	42	991699	50		20
	44	11	287687	1064	712313	296013	1106	703987	008326	42	991674	49		16
	48	12	288326	1063	711674	296677	1104	703323	008351	42	991649	48		12
	52	13	288964	1061	711036	297339	1103	702661	008376	42	991624	47		8
	56	14	289600	1059	710400	298001	1101	701999	008401	42	991599	46		4
45	0	15	9.290236	1058	10.709764	9.298662	1100	10.701338	10.008426	42	9.991574	45	15	0
	4	16	290870	1056	709130	299322	1098	700678	008451	42	991549	44		56
	8	17	291504	1054	708496	299980	1096	700020	008476	42	991524	43		52
	12	18	292137	1053	707863	300638	1095	699362	008502	42	991498	42		48
	16	19	292768	1051	707232	301295	1093	698705	008527	42	991473	41		44
	20	20	293399	1050	706601	301951	1092	698049	008552	42	991448	40		40
	24	21	294029	1048	705971	302607	1090	697393	008578	42	991422	39		36
	28	22	294658	1046	705342	303261	1089	696739	008603	42	991397	38		32
	32	23	295286	1045	704714	303914	1087	696086	008628	43	991372	37		28
	36	24	295913	1043	704087	304567	1086	695433	008654	43	991346	36		24
	40	25	296539	1042	703461	305218	1084	694782	008679	43	991321	35		20
	44	26	297164	1040	702836	305869	1083	694131	008705	43	991295	34		16
	48	27	297788	1039	702212	306519	1081	693481	008730	43	991270	33		12
	52	28	298412	1037	701588	307168	1080	692832	008756	43	991244	32		8
	56	29	299034	1036	700966	307815	1078	692185	008782	43	991218	31		4
46	0	30	9.299655	1034	10.700345	9.308463	1077	10.691537	10.008807	43	9.991193	30	14	0
	4	31	300276	1032	699724	309109	1075	690891	008833	43	991167	29		56
	8	32	300895	1031	699105	309754	1074	690246	008859	43	991141	28		52
	12	33	301514	1029	698486	310398	1073	689602	008885	43	991115	27		48
	16	34	302132	1028	697868	311042	1071	688958	008910	43	991090	26		44
	20	35	302748	1027	697252	311685	1070	688315	008936	43	991064	25		40
	24	36	303364	1025	696636	312327	1068	687673	008962	43	991038	24		36
	28	37	303979	1023	696021	312967	1067	687033	008988	43	991012	23		32
	32	38	304593	1022	695407	313608	1065	686392	009014	43	990986	22		28
	36	39	305207	1020	694793	314247	1064	685753	009040	43	990960	21		24
	40	40	305819	1019	694181	314885	1062	685115	009066	44	990934	20		20
	44	41	306430	1017	693570	315523	1061	684477	009092	44	990908	19		16
	48	42	307041	1016	692959	316159	1060	683841	009118	44	990882	18		12
	52	43	307650	1014	692350	316795	1058	683205	009145	44	99085[illegible]	17		8
	56	44	308259	1013	691741	317430	1057	682570	009171	44	990829	16		4
47	0	45	9.308867	1011	10.691133	9.318064	1055	10.681936	10.009197	44	9.990803	15	13	0
	4	46	309474	1010	690526	318697	1054	681303	009223	44	990777	14		56
	8	47	310080	1008	689920	319329	1053	680671	009250	44	990750	13		52
	12	48	310685	1007	689315	319961	1051	680039	009276	44	990724	12		48
	16	49	311289	1006	688711	320592	1050	679408	009303	44	990697	11		44
	20	50	311893	1004	688107	321222	1048	678778	009329	44	990671	10		40
	24	51	312495	1003	687505	321851	1047	678149	009356	44	990644	9		36
	28	52	313097	1001	686903	322479	1045	677521	009382	44	990618	8		32
	32	53	313698	1000	686302	323106	1044	676894	009409	44	990591	7		28
	36	54	314297	998	685703	323733	1043	676267	009435	44	990565	6		24
	40	55	314897	997	685104	324358	1041	675642	009462	44	990538	5		20
	44	56	315495	996	684505	324983	1040	675017	009489	45	990511	4		16
	48	57	316092	994	688908	325607	1039	674393	009515	45	990485	3		12
	52	58	316689	993	688311	326231	1037	673769	009542	45	990458	2		8
	56	59	317284	991	682716	326853	1036	673147	009569	45	990431	1		4
48	0	60	317879	990	682121	327475	1035	672525	009596	45	990404	0	12	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 78 Degrees.

P. P. to s or ″	1ˢ	15″	155	1ˢ	15″	162	1ˢ	15″	6	P. P. to s or ″
	2	30	310	2	30	323	2	30	13	
	3	45	466	3	45	485	3	45	19	

	0 Hour,						or				12 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
48	0	0	9.317879	990	10.682121	9.327474	1035	10.672526	10.009596	45	9.990404	60	12	0
	4	1	318473	988	681527	328095	1033	671905	009622	45	990378	59		56
	8	2	319066	987	680934	328715	1032	671285	009649	45	990351	58		52
	12	3	319658	986	680342	329334	1030	670666	009676	45	990324	57		48
	16	4	320249	984	679751	329953	1029	670047	009703	45	990297	56		44
	20	5	320840	983	679160	330570	1028	669430	009730	45	990270	55		40
	24	6	321430	982	678570	331187	1026	668813	009757	45	990243	54		36
	28	7	322019	980	677981	331803	1025	668197	009785	45	990215	53		32
	32	8	322607	979	677393	332418	1024	667582	009812	45	990188	52		28
	36	9	323194	977	676806	333033	1023	666967	009839	45	990161	51		24
	40	10	323780	976	676220	333646	1021	666354	009866	45	990134	50		20
	44	11	324366	975	675634	334259	1020	665741	009893	46	990107	49		16
	48	12	324950	973	675050	334871	1019	665129	009921	46	990079	48		12
	52	13	325534	972	674466	335482	1017	664518	009948	46	990052	47		8
	56	14	326117	970	673883	336093	1016	663907	009975	46	990025	46		4
49	0	15	9.326700	969	10.673300	9.336702	1015	10.663298	10.010003	46	9.989997	45	11	0
	4	16	327281	968	672719	337311	1013	662689	010030	46	989970	44		56
	8	17	327862	966	672138	337919	1012	662081	010058	46	989942	43		52
	12	18	328442	965	671558	338527	1011	661473	010085	46	989915	42		48
	16	19	329021	964	670979	339133	1010	660867	010113	46	989887	41		44
	20	20	329599	962	670401	339739	1008	660261	010140	46	989860	40		40
	24	21	330176	961	669824	340344	1007	659656	010168	46	989832	39		36
	28	22	330753	960	669247	340948	1006	659052	010196	46	989804	38		32
	32	23	331329	958	668671	341552	1004	658448	010223	46	989777	37		28
	36	24	331903	957	668097	342155	1003	657845	010251	47	989749	36		24
	40	25	332478	956	667522	342757	1002	657243	010279	47	989721	35		20
	44	26	333051	954	666949	343358	1000	656642	010307	47	989693	34		16
	48	27	333624	953	666376	343958	999	656042	010335	47	989665	33		12
	52	28	334195	952	665805	344558	998	655442	010363	47	989637	32		8
	56	29	334766	950	665234	345157	997	654843	010391	47	989609	31		4
50	0	30	9.335337	949	10.664663	9.345755	996	10.654245	10.010418	47	9.989582	30	10	0
	4	31	335906	948	664094	346353	994	653647	010447	47	989553	29		56
	8	32	336475	946	663525	346949	993	653051	010475	47	989525	28		52
	12	33	337043	945	662957	347545	992	652455	010503	47	989497	27		48
	16	34	337610	944	662390	348141	991	651859	010531	47	989469	26		44
	20	35	338176	943	661824	348735	990	651265	010559	47	989441	25		40
	24	36	338742	941	661258	349329	988	650671	010587	47	989413	24		36
	28	37	339307	940	660693	349922	987	650078	010615	47	989385	23		32
	32	38	339871	939	660129	350514	986	649486	010644	47	989356	22		28
	36	39	340434	937	659566	351106	985	648894	010672	47	989328	21		24
	40	40	340996	936	659004	351697	983	648303	010700	47	989300	20		20
	44	41	341558	935	658442	352287	982	647713	010729	47	989271	19		16
	48	42	342119	934	657881	352876	981	647124	010757	47	989243	18		12
	52	43	342679	932	657321	353465	980	646535	010786	47	989214	17		8
	56	44	343239	931	656761	354053	979	645947	010814	47	989186	16		4
51	0	45	9.343797	930	10.656203	9.354640	977	10.645360	10.010843	47	9.989157	15	9	0
	4	46	344355	929	655645	355227	976	644773	010872	48	989128	14		56
	8	47	344912	927	655088	355813	975	644187	010900	48	989100	13		52
	12	48	345469	926	654531	356398	974	643602	010929	48	989071	12		48
	16	49	346024	925	653976	356982	973	643018	010958	48	989042	11		44
	20	50	346579	924	653421	357566	971	642434	010986	48	989014	10		40
	24	51	347134	922	652866	358149	970	641851	011015	48	988985	9		36
	28	52	347687	921	652313	358731	969	641269	011044	48	988956	8		32
	32	53	348240	920	651760	359313	968	640687	011073	48	988927	7		28
	36	54	348792	919	651208	359893	967	640107	011102	48	988898	6		24
	40	55	349343	917	650657	360474	966	639526	011131	48	988869	5		20
	44	56	349893	916	650107	361053	965	638947	011160	48	988840	4		16
	48	57	350443	915	649557	361632	963	638368	011189	49	988811	3		12
	52	58	350992	914	649008	362210	962	637790	011218	49	98878[illegible]	2		8
	56	59	351540	913	648460	362787	961	637213	011247	49	98875[illegible]	1		4
52	0	60	352088	911	647912	363364	960	636636	011276	49	988724	0	8	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
	5 Hours,						or				77 Degrees.			

P. P. to s or ″	1s	15″	142	1s	15″	149	1s	15″	7	P. P. to s or ″
	2	30	285	2	30	299	2	30	14	
	3	45	427	3	45	448	3	45	22	

0 Hour, or 13 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
52	0	0	9.352088	911	10.647912	9.363364	960	10.636636	10.011276	49	9.988724	60	8	0
	4	1	352635	910	647365	363940	959	636060	011305	49	988695	59		56
	8	2	353181	909	646819	364515	958	635485	011334	49	988666	58		52
	12	3	353726	908	646274	365090	957	634910	011364	49	988636	57		48
	16	4	354271	907	645729	365664	955	634336	011393	49	988607	56		44
	20	5	354815	905	645185	366237	954	633763	011422	49	988578	55		40
	24	6	355358	904	644642	366810	953	633190	011452	49	988548	54		36
	28	7	355901	903	644099	367382	952	632618	011481	49	988519	53		32
	32	8	356443	902	643557	367953	951	632047	011511	49	988489	52		28
	36	9	356984	901	643016	368524	950	631476	011540	49	988460	51		24
	40	10	357524	899	642476	369094	949	630906	011570	49	988430	50		20
	44	11	358064	898	641936	369663	948	630337	011599	49	988401	49		16
	48	12	358603	897	641397	370232	946	629768	011629	49	988371	48		12
	52	13	359141	896	640859	370799	945	629201	011658	49	988342	47		8
	56	14	359678	895	640322	371367	944	628633	011688	50	988312	46		4
53	0	15	9.360215	893	10.639785	9.371933	943	10.628067	10.011718	50	9.988282	45	7	0
	4	16	360752	892	639249	372499	942	627501	011748	50	988252	44		56
	8	17	361287	891	638713	373064	941	626936	011777	50	988223	43		52
	12	18	361822	890	638178	373629	940	626371	011807	50	988193	42		48
	16	19	362356	889	637644	374193	939	625807	011837	50	988163	41		44
	20	20	362889	888	637111	374756	938	625244	011867	50	988133	40		40
	24	21	363422	887	636578	375319	937	624681	011897	50	988103	39		36
	28	22	363954	885	636046	375881	935	624119	011927	50	988073	38		32
	32	23	364485	884	635515	376442	934	623558	011957	50	988043	37		28
	36	24	365016	883	634984	377003	933	622997	011987	50	988013	36		24
	40	25	365546	882	634454	377563	932	622437	012017	50	987983	35		20
	44	26	366075	881	633925	378122	931	621878	012047	50	987953	34		16
	48	27	366604	880	633396	378681	930	621319	012078	50	987922	33		12
	52	28	367131	879	632869	379239	929	620761	012108	50	987892	32		8
	56	29	367659	877	632341	379797	928	620203	012138	50	987862	31		4
54	0	30	9.368185	876	10.631815	9.380354	927	10.619646	10.012168	51	9.987832	30	6	0
	4	31	368711	875	631289	380910	926	619090	012199	51	987801	29		56
	8	32	369236	874	630764	381466	925	618534	012229	51	987771	28		52
	12	33	369761	873	630239	382020	924	617980	012260	51	987740	27		48
	16	34	370285	872	629715	382575	923	617425	012290	51	987710	26		44
	20	35	370808	871	629192	383129	922	616871	012321	51	987679	25		40
	24	36	371330	870	628670	383682	921	616318	012351	51	987649	24		36
	28	37	371852	869	628148	384234	920	615766	012382	51	987618	23		32
	32	38	372373	867	627627	384786	919	615214	012412	51	987588	22		28
	36	39	372894	866	627106	385337	918	614663	012443	51	987557	21		24
	40	40	373414	865	626586	385888	917	614112	012474	51	987526	20		20
	44	41	373933	864	626067	386438	915	613562	012505	51	987496	19		16
	48	42	374452	863	625548	386987	914	613013	012535	51	987465	18		12
	52	43	374970	862	625030	387536	913	612464	012566	51	987434	17		8
	56	44	375487	861	624513	388084	912	611916	012597	52	987403	16		4
55	0	45	9.376003	860	10.623997	9.388631	911	10.611369	10.012628	52	9.987372	15	5	0
	4	46	376519	859	623481	389178	910	610822	012659	52	987341	14		56
	8	47	377035	858	622965	389724	909	610276	012690	52	987310	13		52
	12	48	377549	857	622451	390270	908	609730	012721	52	987279	12		48
	16	49	378063	856	621937	390815	907	609185	012752	52	987248	11		44
	20	50	378577	854	621423	391360	906	608640	012783	52	987217	10		40
	24	51	379089	853	620911	391903	905	608097	012814	52	987186	9		36
	28	52	379601	852	620399	392447	904	607553	012845	52	987155	8		32
	32	53	380113	851	619887	392989	903	607011	012876	52	987124	7		28
	36	54	380624	850	619376	393531	902	606469	012908	52	987092	6		24
	40	55	381134	849	618866	394073	901	605927	012939	52	987061	5		20
	44	56	381643	848	618357	394614	900	605386	012970	52	987030	4		16
	48	57	382152	847	617848	395154	899	604846	013002	52	986998	3		12
	52	58	382661	846	617339	395694	898	604306	013033	52	986967	2		8
	56	59	383168	845	616832	396233	897	603767	013064	52	986936	1		4
56	0	60	383675	844	616325	396771	896	603229	013096	52	986904	0	4	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

5 Hours, or 76 Degrees.

P. P. to s or ″										
	1s	15″	131	1s	15″	139	1s	15″	8	
	2	30	263	2	30	278	2	30	15	
	3	45	394	3	45	417	3	45	23	P. P. to s or ″

0 Hour,							or					14 Degrees.		
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
56	0	0	9.383675	844	10.616325	9.396771	896	10.603229	10.013096	52	9.986904	60	4	0
	4	1	384182	843	615818	397309	896	602691	013127	53	986873	59		56
	8	2	384687	842	615313	397846	895	602154	013159	53	986841	58		52
	12	3	385192	841	614808	398383	894	601617	013191	53	986809	57		48
	16	4	385697	840	614303	398919	893	601081	013222	53	986778	56		44
	20	5	386201	839	613799	399455	892	600545	013254	53	986746	55		40
	24	6	386704	838	613296	399990	891	600010	013286	53	986714	54		36
	28	7	387207	837	612793	400524	890	599476	013317	53	986683	53		32
	32	8	387709	836	612291	401058	889	598942	013349	53	986651	52		28
	36	9	388210	835	611790	401591	888	598409	013381	53	986619	51		24
	40	10	388711	834	611289	402124	887	597876	013413	53	986587	50		20
	44	11	389211	833	610789	402656	886	597344	013445	53	986555	49		16
	48	12	389711	832	610289	403187	885	596813	013477	53	986523	48		12
	52	13	390210	831	609790	403718	884	596282	013509	53	986491	47		8
	56	14	390708	830	609292	404249	883	595751	013541	53	986459	46		4
57	0	15	9.391206	828	10.608794	9.404778	882	10.595222	10.013573	53	9.986427	45	3	0
	4	16	391703	827	608297	405308	881	594692	013605	53	986395	44		56
	8	17	392199	826	607801	405836	880	594164	013637	54	986363	43		52
	12	18	392695	825	607305	406364	879	593636	013669	54	986331	42		48
	16	19	393191	824	606809	406892	878	593108	013701	54	986299	41		44
	20	20	393685	823	606315	407419	877	592581	013734	54	986266	40		40
	24	21	394179	822	605821	407945	876	592055	013766	54	986234	39		36
	28	22	394673	821	605327	408471	875	591529	013798	54	986202	38		32
	32	23	395166	820	604834	408997	874	591003	013831	54	986169	37		28
	36	24	395658	819	604342	409521	874	590479	013863	54	986137	36		24
	40	25	396150	818	603850	410045	873	589955	013896	54	986104	35		20
	44	26	396641	817	603359	410569	872	589431	013928	54	986072	34		16
	48	27	397132	817	602868	411092	871	588908	013961	54	986039	33		12
	52	28	397621	816	602379	411615	870	588385	013993	54	986007	32		8
	56	29	398111	815	601889	412137	869	587863	014026	54	985974	31		4
58	0	30	9.398600	814	10.601400	9.412658	868	10.587342	10.014058	54	9.985942	30	2	0
	4	31	399088	813	600912	413179	867	586821	014091	55	985909	29		56
	8	32	399575	812	600425	413699	866	586301	014124	55	985876	28		52
	12	33	400062	811	599938	414219	865	585781	014157	55	985843	27		48
	16	34	400549	810	599451	414738	864	585262	014189	55	985811	26		44
	20	35	401035	809	598965	415257	864	584743	014222	55	985778	25		40
	24	36	401520	808	598480	415775	863	584225	014255	55	985745	24		36
	28	37	402005	807	597995	416293	862	583707	014288	55	985712	23		32
	32	38	402489	806	597511	416810	861	583190	014321	55	985679	22		28
	36	39	402972	805	597028	417326	860	582674	014354	55	985646	21		24
	40	40	403455	804	596545	417842	859	582158	014387	55	985613	20		20
	44	41	403938	803	596062	418358	858	581642	014420	55	985580	19		16
	48	42	404420	802	595580	418873	857	581127	014453	55	985547	18		12
	52	43	404901	801	595099	419387	856	580613	014486	55	985514	17		8
	56	44	405382	800	594618	419901	855	580099	014520	55	985480	16		4
59	0	45	9.405862	799	10.594138	9.420415	855	10.579585	10.014553	55	9.985447	15	1	0
	4	46	406341	798	593659	420927	854	579073	014586	56	985414	14		56
	8	47	406820	797	593180	421440	853	578560	014619	56	985381	13		52
	12	48	407299	796	592701	421952	852	578048	014653	56	985347	12		48
	16	49	407777	795	592223	422463	851	577537	014686	56	985314	11		44
	20	50	408254	794	591746	422974	850	577026	014720	56	985280	10		40
	24	51	408731	794	591269	423484	849	576516	014753	56	985247	9		36
	28	52	409207	793	590793	423993	848	576007	014787	56	985213	8		32
	32	53	409682	792	590318	424503	848	575497	014820	56	985180	7		28
	36	54	410157	791	589843	425011	847	574989	014854	56	985146	6		24
	40	55	410632	790	589368	425519	846	574481	014887	56	985113	5		20
	44	56	411106	789	588894	426027	845	573973	014921	56	985079	4		16
	48	57	411579	788	588421	426534	844	573466	014955	56	985045	3		12
	52	58	412052	787	587948	427041	843	572959	014989	56	985011	2		8
	56	59	412524	786	587476	427547	843	572453	015022	56	984978	1		4
60	0	60	412996	785	587004	428052	842	571948	015056	56	984944	0	0	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
5 Hours,							or					75 Degrees.		

P. P. to s or ″	1s	15″	122	1s	15″	130	1s	15″	8	P. P. to s or ″
	2	30	244	2	30	260	2	30	16	
	3	45	366	3	45	391	3	45	25	

1 Hour,						or			15 Degrees.					
m.	s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	'	m.	s.
0	0	0	9.412996	785	10.587004	9.428052	842	10.571948	10.015056	57	9.984944	60	60	0
	4	1	413467	784	586533	428557	841	571443	015090	57	984910	59		56
	8	2	413938	783	586062	429062	840	570938	015124	57	984876	58		52
	12	3	414408	783	585592	429566	839	570434	015158	57	984842	57		48
	16	4	414878	782	585122	430070	838	569930	015192	57	984808	56		44
	20	5	415347	781	584653	430573	838	569427	015226	57	984774	55		40
	24	6	415815	780	584185	431075	837	568925	015260	57	984740	54		36
	28	7	416283	779	583717	431577	836	568423	015294	57	984706	53		32
	32	8	416751	778	583249	432079	835	567921	015328	57	984672	52		28
	36	9	417217	777	582783	432580	834	567420	015362	57	984638	51		24
	40	10	417684	776	582316	433080	833	566920	015397	57	984603	50		20
	44	11	418150	775	581850	433580	832	566420	015431	57	984569	49		16
	48	12	418615	774	581385	434080	832	565920	015465	57	984535	48		12
	52	13	419079	773	580921	434579	831	565421	015500	57	984500	47		8
	56	14	419544	773	580456	435078	830	564922	015534	57	984466	46		4
1	0	15	9.420007	772	10.579993	9.435576	829	10.564424	10.015568	58	9.984432	45	59	0
	4	16	420470	771	579530	436073	828	563927	015603	58	984397	44		56
	8	17	420933	770	579067	436570	828	563430	015637	58	984363	43		52
	12	18	421395	769	578605	437067	827	562933	015672	58	984328	42		48
	16	19	421857	768	578143	437563	826	562437	015706	58	984294	41		44
	20	20	422318	767	577682	438059	825	561941	015741	58	984259	40		40
	24	21	422778	767	577222	438554	824	561446	015776	58	984224	39		36
	28	22	423238	766	576762	439048	823	560952	015810	58	984190	38		32
	32	23	423697	765	576303	439543	823	560457	015845	58	984155	37		28
	36	24	424156	764	575844	440036	822	559964	015880	58	984120	36		24
	40	25	424615	763	575385	440529	821	559471	015915	58	984085	35		20
	44	26	425073	762	574927	441022	820	558978	015950	58	984050	34		16
	48	27	425530	761	574470	441514	819	558486	015985	58	984015	33		12
	52	28	425987	760	574013	442006	819	557994	016019	58	983981	32		8
	56	29	426443	760	573557	442497	818	557503	016054	58	983946	31		4
2	0	30	9.426899	759	10.573101	9.442988	817	10.557012	10.016089	58	9.983911	30	58	0
	4	31	427354	758	572646	443479	816	556521	016125	58	983875	29		56
	8	32	427809	757	572191	443968	816	556032	016160	59	983840	28		52
	12	33	428263	756	571737	444458	815	555542	016195	59	983805	27		48
	16	34	428717	755	571283	444947	814	555053	016230	59	983770	26		44
	20	35	429170	754	570830	445435	813	554565	016265	59	983735	25		40
	24	36	429623	753	570377	445923	812	554077	016300	59	983700	24		36
	28	37	430075	752	569925	446411	812	553589	016336	59	983664	23		32
	32	38	430527	752	569473	446898	811	553102	016371	59	983629	22		28
	36	39	430978	751	569022	447384	810	552616	016406	59	983594	21		24
	40	40	431429	750	568571	447870	809	552130	016442	59	983558	20		20
	44	41	431879	749	568121	448356	809	551644	016477	59	983523	19		16
	48	42	432329	749	567671	448841	808	551159	016513	59	983487	18		12
	52	43	432778	748	567222	449326	807	550674	016548	59	983452	17		8
	56	44	433226	747	566774	449810	806	550190	016584	59	983416	16		4
3	0	45	9.433675	746	10.566325	9.450294	806	10.549706	10.016619	59	9.983381	15	57	0
	4	46	434122	745	565878	450777	805	549223	016655	59	983345	14		56
	8	47	434569	744	565431	451260	804	548740	016691	59	983309	13		52
	12	48	435016	744	564984	451743	803	548257	016727	60	983273	12		48
	16	49	435462	743	564538	452225	802	547775	016762	60	983238	11		44
	20	50	435908	742	564092	452706	802	547294	016798	60	983202	10		40
	24	51	436353	741	563647	453187	801	546813	016834	60	983166	9		36
	28	52	436798	740	563202	453668	800	546332	016870	60	983130	8		32
	32	53	437242	740	562758	454148	799	545852	016906	60	983094	7		28
	36	54	437686	739	562314	454628	799	545372	016942	60	983058	6		24
	40	55	438129	738	561871	455107	798	544893	016978	60	983022	5		20
	44	56	438572	737	561428	455586	797	544414	017014	60	982986	4		16
	48	57	439014	736	560986	456064	796	543936	017050	60	982950	3		12
	52	58	439456	736	560544	456542	796	543458	017086	60	982914	2		8
	56	59	439897	735	560103	457019	795	542981	017122	60	982878	1		4
4	0	60	440338	734	559662	457496	794	542504	017158	60	982842	0	56	0
m.	s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	'	m.	s.
4 Hours,						or			74 Degrees.					

P. P. to s or "	1s	15"	114	1s	15"	123	1s	15"	9	P. P. to s or "
	2	30	228	2	30	245	2	30	17	
	3	45	341	3	45	368	3	45	26	

1 Hour, or 16 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
4	0	0	9.440338	734	10.559662	9.457496	794	10.542504	10.017158	60	9.982842	60	56	0
	4	1	440778	733	559222	457973	793	542027	017195	60	982805	59		56
	8	2	441218	732	558782	458449	793	541551	017231	61	982769	58		52
	12	3	441658	731	558342	458925	792	541075	017267	61	982733	57		48
	16	4	442096	731	557904	459400	791	540600	017304	61	982696	56		44
	20	5	442535	730	557465	459875	790	540125	017340	61	982660	55		40
	24	6	442973	729	557027	460349	790	539651	017376	61	982624	54		36
	28	7	443410	728	556590	460823	789	539177	017413	61	982587	53		32
	32	8	443847	727	556153	461297	788	538703	017449	61	982551	52		28
	36	9	444284	727	555716	461770	788	538230	017486	61	982514	51		24
	40	10	444720	726	555280	462242	787	537758	017523	61	982477	50		20
	44	11	445155	725	554845	462714	786	537286	017559	61	982441	49		16
	48	12	445590	724	554410	463186	785	536814	017596	61	982404	48		12
	52	13	446025	723	553975	463658	785	536342	017633	61	982367	47		8
	56	14	446459	723	553541	464128	784	535872	017669	61	982331	46		4
5	0	15	9.446893	722	10.553107	9.464599	783	10.535401	10.017706	61	9.982294	45	55	0
	4	16	447326	721	552674	465069	783	534931	017743	61	982257	44		56
	8	17	447759	720	552241	465539	782	534461	017780	62	982220	43		52
	12	18	448191	720	551809	466008	781	533992	017817	62	982183	42		48
	16	19	448623	719	551377	466476	780	533524	017854	62	982146	41		44
	20	20	449054	718	550946	466945	780	533055	017891	62	982109	40		40
	24	21	449485	717	550515	467413	779	532587	017928	62	982072	39		36
	28	22	449915	716	550085	467880	778	532120	017965	62	982035	38		32
	32	23	450345	716	549655	468347	778	531653	018002	62	981998	37		28
	36	24	450775	715	549225	468814	777	531186	018039	62	981961	36		24
	40	25	451204	714	548796	469280	776	530720	018076	62	981924	35		20
	44	26	451632	713	548368	469746	775	530254	018114	62	981886	34		16
	48	27	452060	713	547940	470211	775	529789	018151	62	981849	33		12
	52	28	452488	712	547512	470676	774	529324	018188	62	981812	32		8
	56	29	452915	711	547085	471141	773	528859	018226	62	981774	31		4
6	0	30	9.453342	710	10.546658	9.471605	773	10.528395	10.018263	63	9.981737	30	54	0
	4	31	453768	710	546232	472068	772	527932	018300	63	981700	29		56
	8	32	454194	709	545806	472532	771	527468	018338	63	981662	28		52
	12	33	454619	708	545381	472995	771	527005	018375	63	981625	27		48
	16	34	455044	707	544956	473457	770	526543	018413	63	981587	26		44
	20	35	455469	707	544531	473919	769	526081	018451	63	981549	25		40
	24	36	455893	706	544107	474381	769	525619	018488	63	981512	24		36
	28	37	456316	705	543684	474842	768	525158	018526	63	981474	23		32
	32	38	456739	704	543261	475303	767	524697	018564	63	981436	22		28
	36	39	457162	704	542838	475763	767	524237	018601	63	981399	21		24
	40	40	457584	703	542416	476223	766	523777	018639	63	981361	20		20
	44	41	458006	702	541994	476683	765	523317	018677	63	981323	19		16
	48	42	458427	701	541573	477142	765	522858	018715	63	981285	18		12
	52	43	458848	701	541152	477601	764	522399	018753	63	981247	17		8
	56	44	459268	700	540732	478059	763	521941	018791	63	981209	16		4
7	0	45	9.459688	699	10.540312	9.478517	763	10.521483	10.018829	63	9.981171	15	53	0
	4	46	460108	698	539892	478975	762	521025	018867	64	981133	14		56
	8	47	460527	698	539473	479432	761	520568	018905	64	981095	13		52
	12	48	460946	697	539054	479889	761	520111	018943	64	981057	12		48
	16	49	461364	696	538636	480345	760	519655	018981	64	981019	11		44
	20	50	461782	695	538218	480801	759	519199	019019	64	980981	10		40
	24	51	462199	695	537801	481257	759	518743	019058	64	980942	9		36
	28	52	462616	694	537384	481712	758	518288	019096	64	980904	8		32
	32	53	463032	693	536968	482167	757	517833	019134	64	980866	7		28
	36	54	463448	693	536552	482621	757	517379	019173	64	980827	6		24
	40	55	463864	692	536136	483075	756	516925	019211	64	980789	5		20
	44	56	464279	691	535721	483529	755	516471	019250	64	980750	4		16
	48	57	464694	690	535306	483982	755	516018	019288	64	980712	3		12
	52	58	465108	690	534892	484435	754	515565	019327	64	980673	2		8
	56	59	465522	689	534478	484887	753	515113	019365	64	980635	1		4
8	0	60	465935	688	534065	485339	753	514661	019404	64	980596	0	52	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

4 Hours, or 73 Degrees.

P. P. to s or ″	1s	15″	106	1s	15″	116	1s	15″	9	P. P. to s or ″
	2	30	213	2	30	231	2	30	19	
	3	45	319	3	45	347	3	45	28	

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
			1 Hour,				or			17 Degrees.				
8	0	0	9.465935	688	10.534065	9.485339	753	10.514661	10.019404	64	9.980596	60	52	0
	4	1	466348	688	533652	485791	752	514209	019442	64	980558	59		56
	8	2	466761	687	533239	486242	751	513758	019481	65	980519	58		52
	12	3	467173	686	532827	486693	751	513307	019520	65	980480	57		48
	16	4	467585	685	532415	487143	750	512857	019558	65	980442	56		44
	20	5	467996	685	532004	487593	749	512407	019597	65	980403	55		40
	24	6	468407	684	531593	488043	749	511957	019636	65	980364	54		36
	28	7	468817	683	531183	488492	748	511508	019675	65	980325	53		32
	32	8	469227	683	530773	488941	747	511059	019714	65	980286	52		28
	36	9	469637	682	530363	489390	747	510610	019753	65	980247	51		24
	40	10	470046	681	529954	489838	746	510162	019792	65	980208	50		20
	44	11	470455	680	529545	490286	746	509714	019831	65	980169	49		16
	48	12	470863	680	529137	490733	745	509267	019870	65	980130	48		12
	52	13	471271	679	528729	491180	744	508820	019909	65	980091	47		8
	56	14	471679	678	528321	491627	744	508373	019948	65	980052	46		4
9	0	15	9.472086	678	10.527914	9.492073	743	10.507927	10.019988	65	9.980012	45	51	0
	4	16	472492	677	527508	492519	743	507481	020027	65	979973	44		56
	8	17	472898	676	527102	492965	742	507035	020066	66	979934	43		52
	12	18	473304	676	526696	493410	741	506590	020105	66	979895	42		48
	16	19	473710	675	526290	493854	740	506146	020145	66	979855	41		44
	20	20	474115	674	525885	494299	740	505701	020184	66	979816	40		40
	24	21	474519	674	525481	494743	740	505257	020224	66	979776	39		36
	28	22	474923	673	525077	495186	739	504814	020263	66	979737	38		32
	32	23	475327	672	524673	495630	738	504370	020303	66	979697	37		28
	36	24	475730	672	524270	496073	737	503927	020342	66	979658	36		24
	40	25	476133	671	523867	496515	737	503485	020382	66	979618	35		20
	44	26	476536	670	523464	496957	736	503043	020421	66	979579	34		16
	48	27	476938	669	523062	497399	736	502601	020461	66	979539	33		12
	52	28	477340	669	522660	497841	735	502159	020501	66	979499	32		8
	56	29	477741	668	522259	498282	734	501718	020541	66	979459	31		4
10	0	30	9.478142	667	10.521858	9.498722	734	10.501278	10.020580	66	9.979420	30	50	0
	4	31	478542	667	521458	499163	733	500837	020620	66	979380	29		56
	8	32	478942	666	521058	499603	733	500397	020660	66	979340	28		52
	12	33	479342	665	520658	500042	732	499958	020700	67	979300	27		48
	16	34	479741	665	520259	500481	731	499519	020740	67	979260	26		44
	20	35	480140	664	519860	500920	731	499080	020780	67	979220	25		40
	24	36	480539	663	519461	501359	730	498641	020820	67	979180	24		36
	28	37	480937	663	519063	501797	730	498203	020860	67	979140	23		32
	32	38	481334	662	518666	502235	729	497765	020900	67	979100	22		28
	36	39	481731	661	518269	502672	728	497328	020941	67	979059	21		24
	40	40	482128	661	517872	503109	728	496891	020981	67	979019	20		20
	44	41	482525	660	517475	503546	727	496454	021021	67	978979	19		16
	48	42	482921	659	517079	503982	727	496018	021061	67	978939	18		12
	52	43	483316	659	516684	504418	726	495582	021102	67	978898	17		8
	56	44	483712	658	516288	504854	725	495146	021142	67	978858	16		4
11	0	45	9.484107	657	10.515893	9.505289	725	10.494711	10.021183	67	9.978817	15	49	0
	4	46	484501	657	515499	505724	724	494276	021223	67	978777	14		56
	8	47	484895	656	515105	506159	724	493841	021263	67	978737	13		52
	12	48	485289	655	514711	506593	723	493407	021304	68	978696	12		48
	16	49	485682	655	514318	507027	722	492973	021345	68	978655	11		44
	20	50	486075	654	513925	507460	722	492540	021385	68	978615	10		40
	24	51	486467	653	513533	507893	721	492107	021426	68	978574	9		36
	28	52	486860	653	513140	508326	721	491674	021467	68	978533	8		32
	32	53	487251	652	512749	508759	720	491241	021507	68	978493	7		28
	36	54	487643	651	512357	509191	719	490809	021548	68	978452	6		24
	40	55	488034	651	511966	509622	719	490378	021589	68	978411	5		20
	44	56	488424	650	511576	510054	718	489946	021630	68	978370	4		16
	48	57	488814	650	511186	510485	718	489515	021671	68	978329	3		12
	52	58	489204	649	510796	510916	717	489084	021712	68	978288	2		8
	56	59	489593	648	510407	511346	716	488654	021753	68	978247	1		4
12	0	60	489982	648	510018	511776	716	488224	021794	68	978206	0	48	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			4 Hours,				or			72 Degrees.				

P. P. to s or ″	1s	15″	100	1s	15″	110	1s	15″	10	P. P. to s or ″
	2	30	200	2	30	220	2	30	20	
	3	45	300	3	45	331	3	45	30	

			1 Hour,				or			18 Degrees.			
m. s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
12 0	0	9.489982	648	10.510018	9.511776	716	10.488224	10.021794	68	9.978206	60	48	0
4	1	490371	648	509629	512206	716	487794	021835	68	978165	59		56
8	2	490759	647	509241	512635	715	487365	021876	68	978124	58		52
12	3	491147	646	508853	513064	714	486936	021917	69	978083	57		48
16	4	491535	646	508465	513493	714	486507	021958	69	978042	56		44
20	5	491922	645	508078	513921	713	486079	021999	69	978001	55		40
24	6	492308	644	507692	514349	713	485651	022041	69	977959	54		36
28	7	492695	644	507305	514777	712	485223	022082	69	977918	53		32
32	8	493081	643	506919	515204	712	484796	022123	69	977877	52		28
36	9	493466	642	506534	515631	711	484369	022165	69	977835	51		24
40	10	493851	642	506149	516057	710	483943	022206	69	977794	50		20
44	11	494236	641	505764	516484	710	483516	022248	69	977752	49		16
48	12	494621	641	505379	516910	709	483090	022289	69	977711	48		12
52	13	495005	640	504995	517335	709	482665	022331	69	977669	47		8
56	14	495388	639	504612	517761	708	482239	022372	69	977628	46		4
13 0	15	9.495772	639	10.504228	9.518185	708	10.481815	10.022414	69	9.977586	45	47	0
4	16	496154	638	503846	518610	707	481390	022456	70	977544	44		56
8	17	496537	637	503463	519034	706	480966	022497	70	977503	43		52
12	18	496919	637	503081	519458	706	480542	022539	70	977461	42		48
16	19	497301	636	502699	519882	705	480118	022581	70	977419	41		44
20	20	497682	636	502318	520305	705	479695	022623	70	977377	40		40
24	21	498064	635	501936	520728	704	479272	022665	70	977335	39		36
28	22	498444	634	501556	521151	704	478849	022707	70	977293	38		32
32	23	498825	634	501175	521573	703	478427	022749	70	977251	37		28
36	24	499204	633	500796	521995	703	478005	022791	70	977209	36		24
40	25	499584	632	500416	522417	702	477583	022833	70	977167	35		20
44	26	499963	632	500037	522838	702	477162	022875	70	977125	34		16
48	27	500342	631	499658	523259	701	476741	022917	70	977083	33		12
52	28	500721	631	499279	523680	701	476320	022959	70	977041	32		8
56	29	501099	630	498901	524100	700	475900	023001	70	976999	31		4
14 0	30	9.501476	629	10.498524	9.524520	699	10.475480	10.023043	70	9.976957	30	46	0
4	31	501854	629	498146	524939	699	475061	023086	70	976914	29		56
8	32	502231	628	497769	525359	698	474641	023128	71	976872	28		52
12	33	502607	628	497393	525778	698	474222	023170	71	976830	27		48
16	34	502984	627	497016	526197	697	473803	023213	71	976787	26		44
20	35	503360	626	496640	526615	697	473385	023255	71	976745	25		40
24	36	503735	626	496265	527033	696	472967	023298	71	976702	24		36
28	37	504110	625	495890	527451	696	472549	023340	71	976660	23		32
32	38	504485	625	495515	527868	695	472132	023383	71	976617	22		28
36	39	504860	624	495140	528285	695	471715	023426	71	976574	21		24
40	40	505234	623	494766	528702	694	471298	023468	71	976532	20		20
44	41	505608	623	494392	529119	693	470881	023511	71	976489	19		16
48	42	505981	622	494019	529535	693	470465	023554	71	976446	18		12
52	43	506354	622	493646	529950	692	470050	023596	71	976404	17		8
56	44	506727	621	493273	530366	692	469634	023639	71	976361	16		4
15 0	45	9.507099	620	10.492901	9.530781	691	10.469219	10.023682	71	9.976318	15	45	0
4	46	507471	620	492529	531196	691	468804	023725	71	976275	14		56
8	47	507843	619	492157	531611	690	468389	023768	72	976232	13		52
12	48	508214	619	491786	532025	690	467975	023811	72	976189	12		48
16	49	508585	618	491415	532439	689	467561	023854	72	976146	11		44
20	50	508956	618	491044	532853	689	467147	023897	72	976103	10		40
24	51	509326	617	490674	533266	688	466734	023940	72	976060	9		36
28	52	509696	616	490304	533679	688	466321	023983	72	976017	8		32
32	53	510065	616	489935	534092	687	465908	024026	72	975974	7		28
36	54	510434	615	489566	534504	687	465496	024070	72	975930	6		24
40	55	510803	615	489197	534916	686	465084	024113	72	975887	5		20
44	56	511172	614	488828	535328	686	464672	024156	72	975844	4		16
48	57	511540	613	488460	535739	685	464261	024200	72	975800	3		12
52	58	511907	613	488093	536150	685	463850	024243	72	975757	2		8
56	59	512275	612	487725	536561	684	463439	024286	72	975714	1		4
16 0	60	512642	612	487358	536972	684	463028	024330	72	975670	0	44	0
m. s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
		4 Hours,					or			71 Degrees.			

P. P. to s or ″	1s	15″	94	1s	15″	105	1s	15″	10	P. P. to s or ″
	2	30	189	2	30	210	2	30	21	
	3	45	283	3	45	315	3	45	31	

1 Hour, or 19 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
16	0	0	9.512642	612	10.487358	9.536972	684	10.463028	10.024330	73	9.975670	60	44	0
	4	1	513009	611	486991	537382	683	462618	024373	73	975627	59		56
	8	2	513375	611	486625	537792	683	462208	024417	73	975583	58		52
	12	3	513741	610	486259	538202	682	461798	024461	73	975539	57		48
	16	4	514107	609	485893	538611	682	461389	024504	73	975496	56		44
	20	5	514472	609	485528	539020	681	460980	024548	73	975452	55		40
	24	6	514837	608	485163	539429	681	460571	024592	73	975408	54		36
	28	7	515202	608	484798	539837	680	460163	024635	73	975365	53		32
	32	8	515566	607	484434	540245	680	459755	024679	73	975321	52		28
	36	9	515930	607	484070	540653	679	459347	024723	73	975277	51		24
	40	10	516294	606	483706	541061	679	458939	024767	73	975233	50		20
	44	11	516657	605	483343	541468	678	458532	024811	73	975189	49		16
	48	12	517020	605	482980	541875	678	458125	024855	73	975145	48		12
	52	13	517382	604	482618	542281	677	457719	024899	73	975101	47		8
	56	14	517745	604	482255	542688	677	457312	024943	73	975057	46		4
17	0	15	9.518107	603	10.481893	9.543094	676	10.456906	10.024987	73	9.975013	45	43	0
	4	16	518468	603	481532	543499	676	456501	025031	74	974969	44		56
	8	17	518829	602	481171	543905	675	456095	025075	74	974925	43		52
	12	18	519190	601	480810	544310	675	455690	025120	74	974880	42		48
	16	19	519551	601	480449	544715	674	455285	025164	74	974836	41		44
	20	20	519911	600	480089	545119	674	454881	025208	74	974792	40		40
	24	21	520271	600	479729	545524	673	454476	025252	74	974748	39		36
	28	22	520631	599	479369	545928	673	454072	025297	74	974703	38		32
	32	23	520990	599	479010	546331	672	453669	025341	74	974659	37		28
	36	24	521349	598	478651	546735	672	453265	025386	74	974614	36		24
	40	25	521707	598	478293	547138	671	452862	025430	74	974570	35		20
	44	26	522066	597	477934	547540	671	452460	025475	74	974525	34		16
	48	27	522424	596	477576	547943	670	452057	025519	74	974481	33		12
	52	28	522781	596	477219	548345	670	451655	025564	74	974436	32		8
	56	29	523138	595	476862	548747	669	451253	025609	74	974391	31		4
18	0	30	9.523495	595	10.476505	9.549149	669	10.450851	10.025653	75	9.974347	30	42	0
	4	31	523852	594	476148	549550	668	450450	025698	75	974302	29		56
	8	32	524208	594	475792	549951	668	450049	025743	75	974257	28		52
	12	33	524564	593	475436	550352	667	449648	025788	75	974212	27		48
	16	34	524920	593	475080	550752	667	449248	025833	75	974167	26		44
	20	35	525275	592	474725	551152	666	448848	025878	75	974122	25		40
	24	36	525630	591	474370	551552	666	448448	025923	75	974077	24		36
	28	37	525984	591	474016	551952	665	448048	025968	75	974032	23		32
	32	38	526339	590	473661	552351	665	447649	026013	75	973987	22		28
	36	39	526693	590	473307	552750	665	447250	026058	75	973942	21		24
	40	40	527046	589	472954	553149	664	446851	026103	75	973897	20		20
	44	41	527400	589	472600	553548	664	446452	026148	75	973852	19		16
	48	42	527753	588	472247	553946	663	446054	026193	75	973807	18		12
	52	43	528105	588	471895	554344	663	445656	026239	75	973761	17		8
	56	44	528458	587	471542	554741	662	445259	026284	76	973716	16		4
19	0	45	9.528810	587	10.471190	9.555139	662	10.444861	10.026329	76	9.973671	15	41	0
	4	46	529161	586	470839	555536	661	444464	026375	76	973625	14		56
	8	47	529513	586	470487	555933	661	444067	026420	76	973580	13		52
	12	48	529864	585	470136	556329	660	443671	026465	76	973535	12		48
	16	49	530215	585	469785	556725	660	443275	026511	76	973489	11		44
	20	50	530565	584	469435	557121	659	442879	026556	76	973444	10		40
	24	51	530915	584	469085	557517	659	442483	026602	76	973398	9		36
	28	52	531265	583	468735	557913	659	442087	026648	76	973352	8		32
	32	53	531614	582	468386	558308	658	441692	026693	76	973307	7		28
	36	54	531963	582	468037	558702	658	441298	026739	76	973261	6		24
	40	55	532312	581	467688	559097	657	440903	026785	76	973215	5		20
	44	56	532661	581	467339	559491	657	440509	026831	76	973169	4		16
	48	57	533009	580	466991	559885	656	440115	026876	76	973124	3		12
	52	58	533357	580	466643	560279	656	439721	026922	76	973078	2		8
	56	59	533704	579	466296	560673	655	439327	026968	77	973032	1		4
20	0	60	534052	578	465948	561066	655	438934	027014	77	972986	0	40	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

4 Hours, or 70 Degrees.

P. P. to s or ″										P. P. to s or ″
	1s	15″	89	1s	15″	100	1s	15″	11	
	2	30	178	2	30	200	2	30	22	
	3	45	268	3	45	301	3	45	34	

			1 Hour,				or			20 Degrees.				
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
20	0	0	9.534052	578	10.465948	9.561066	655	10.438934	10.027014	77	9.972986	60	40	0
	4	1	534399	577	465601	561459	654	438541	027060	77	972940	59		56
	8	2	534745	577	465255	561851	654	438149	027106	77	972894	58		52
	12	3	535092	577	464908	562244	653	437756	027152	77	972848	57		48
	16	4	535438	576	464562	562636	653	437364	027198	77	972802	56		44
	20	5	535783	576	464217	563028	653	436972	027245	77	972755	55		40
	24	6	536129	575	463871	563419	652	436581	027291	77	972709	54		36
	28	7	536474	574	463526	563811	652	436189	027337	77	972663	53		32
	32	8	536818	574	463182	564202	651	435798	027383	77	972617	52		28
	36	9	537163	573	462837	564592	651	435408	027430	77	972570	51		24
	40	10	537507	573	462493	564983	650	435017	027476	77	972524	50		20
	44	11	537851	572	462149	565373	650	434627	027522	77	972478	49		16
	48	12	538194	572	461806	565763	649	434237	027569	78	972431	48		12
	52	13	538538	571	461462	566153	649	433847	027615	78	972385	47		8
	56	14	538880	571	461120	566542	649	433458	027662	78	972338	46		4
21	0	15	9.539223	570	10.460777	9.566932	648	10.433068	10.027709	78	9.972291	45	39	0
	4	16	539565	570	460435	567320	648	432680	027755	78	972245	44		56
	8	17	539907	569	460093	567709	647	432291	027802	78	972198	43		52
	12	18	540249	569	459751	568098	647	431902	027849	78	972151	42		48
	16	19	540590	568	459410	568486	646	431514	027895	78	972105	41		44
	20	20	540931	568	459069	568873	646	431127	027942	78	972058	40		40
	24	21	541272	567	458728	569261	645	430739	027989	78	972011	39		36
	28	22	541613	567	458387	569648	645	430352	028036	78	971964	38		32
	32	23	541953	566	458047	570035	645	429965	028083	78	971917	37		28
	36	24	542293	566	457707	570422	644	429578	028130	78	971870	36		24
	40	25	542632	565	457368	570809	644	429191	028177	78	971823	35		20
	44	26	542971	565	457029	571195	643	428805	028224	78	971776	34		16
	48	27	543310	564	456690	571581	643	428419	028271	79	971729	33		12
	52	28	543649	564	456351	571967	642	428033	028318	79	971682	32		8
	56	29	543987	563	456013	572352	642	427648	028365	79	971635	31		4
22	0	30	9.544325	563	10.455675	9.572738	642	10.427262	10.028412	79	9.971588	30	38	0
	4	31	544663	562	455337	573123	641	426877	028460	79	971540	29		56
	8	32	545000	562	455000	573507	641	426493	028507	79	971493	28		52
	12	33	545338	561	454662	573892	640	426108	028554	79	971446	27		48
	16	34	545674	561	454326	574276	640	425724	028602	79	971398	26		44
	20	35	546011	560	453989	574660	639	425340	028649	79	971351	25		40
	24	36	546347	560	453653	575044	639	424956	028697	79	971303	24		36
	28	37	546683	559	453317	575427	639	424573	028744	79	971256	23		32
	32	38	547019	559	452981	575810	638	424190	028792	79	971208	22		28
	36	39	547354	558	452646	576193	638	423807	028839	79	971161	21		24
	40	40	547689	558	452311	576576	637	423424	028887	79	971113	20		20
	44	41	548024	557	451976	576959	637	423041	028934	80	971066	19		16
	48	42	548359	557	451641	577341	636	422659	028982	80	971018	18		12
	52	43	548693	556	451307	577723	636	422277	029030	80	970970	17		8
	56	44	549027	556	450973	578104	636	421896	029078	80	970922	16		4
23	0	45	9.549360	555	10.450640	9.578486	635	10.421514	10.029126	80	9.970874	15	37	0
	4	46	549693	555	450307	578867	635	421133	029173	80	970827	14		56
	8	47	550026	554	449974	579248	634	420752	029221	80	970779	13		52
	12	48	550359	554	449641	579629	634	420371	029269	80	970731	12		48
	16	49	550692	553	449308	580009	634	419991	029317	80	970683	11		44
	20	50	551024	553	448976	580389	633	419611	029365	80	970635	10		40
	24	51	551356	552	448644	580769	633	419231	029414	80	970586	9		36
	28	52	551687	552	448313	581149	632	418851	029462	80	970538	8		32
	32	53	552018	552	447982	581528	632	418472	029510	80	970490	7		28
	36	54	552349	551	447651	581907	632	418093	029558	80	970442	6		24
	40	55	552680	551	447320	582286	631	417714	029606	80	970394	5		20
	44	56	553010	550	446990	582665	631	417335	029655	81	970345	4		16
	48	57	553341	550	446659	583043	630	416957	029703	81	970297	3		12
	52	58	553670	549	446330	583422	630	416578	029751	81	970249	2		8
	56	59	554000	549	446000	583800	629	416200	029800	81	970200	1		4
24	0	60	554329	548	445671	584177	629	415823	029848	81	970152	0	36	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			4 Hours,				or			69 Degrees.				

P. P. to s or ″	1s	15″	84	1s	15″	96	1s	15″	12	P. P. to s or ″
	2	30	169	2	30	193	2	30	24	
	3	45	253	3	45	289	3	45	36	

1 Hour,							or				21 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
24	0	0	9.554329	548	10.445671	9.584177	629	10.415823	10.029848	81	9.970152	60	36	0
	4	1	554658	548	445342	584555	629	415445	029897	81	970103	59		56
	8	2	554987	547	445013	584932	628	415068	029945	81	970055	58		52
	12	3	555315	547	444685	585309	628	414691	029994	81	970006	57		48
	16	4	555643	546	444357	585686	627	414314	030043	81	969957	56		44
	20	5	555971	546	444029	586062	627	413938	030091	81	969909	55		40
	24	6	556299	545	443701	586439	627	413561	030140	81	969860	54		36
	28	7	556626	545	443374	586815	626	413185	030189	81	969811	53		32
	32	8	556953	544	443047	587190	626	412810	030238	81	969762	52		28
	36	9	557280	544	442720	587566	625	412434	030286	81	969714	51		24
	40	10	557606	543	442394	587941	625	412059	030335	81	969665	50		20
	44	11	557932	543	442068	588316	625	411684	030384	82	969616	49		16
	48	12	558258	543	441742	588691	624	411309	030433	82	969567	48		12
	52	13	558583	542	441417	589066	624	410934	030482	82	969518	47		8
	56	14	558909	542	441091	589440	623	410560	030531	82	969469	46		4
25	0	15	9.559234	541	10.440766	9.589814	623	10.410186	10.030580	82	9.969420	45	35	0
	4	16	559558	541	440442	590188	623	409812	030630	82	969370	44		56
	8	17	559883	540	440117	590562	622	409438	030679	82	969321	43		52
	12	18	560207	540	439793	590935	622	409065	030728	82	969272	42		48
	16	19	560531	539	439469	591308	622	408692	030777	82	969223	41		44
	20	20	560855	539	439145	591681	621	408319	030827	82	969173	40		40
	24	21	561178	538	438822	592054	621	407946	030876	82	969124	39		36
	28	22	561501	538	438499	592426	620	407574	030925	82	969075	38		32
	32	23	561824	537	438176	592798	620	407202	030975	82	969025	37		28
	36	24	562146	537	437854	593171	619	406829	031024	82	968976	36		24
	40	25	562468	536	437532	593542	619	406458	031074	83	968926	35		20
	44	26	562790	536	437210	593914	618	406086	031123	83	968877	34		16
	48	27	563112	536	436888	594285	618	405715	031173	83	968827	33		12
	52	28	563433	535	436567	594656	618	405344	031223	83	968777	32		8
	56	29	563755	535	436245	595027	617	404973	031272	83	968728	31		4
26	0	30	9.564075	534	10.435925	9.595398	617	10.404602	10.031322	83	9.968678	30	34	0
	4	31	564396	534	435604	595768	617	404232	031372	83	968628	29		56
	8	32	564716	533	435284	596138	616	403862	031422	83	968578	28		52
	12	33	565036	533	434964	596508	616	403492	031472	83	968528	27		48
	16	34	565356	532	434644	596878	616	403122	031521	83	968479	26		44
	20	35	565676	532	434324	597247	615	402753	031571	83	968429	25		40
	24	36	565995	531	434005	597616	615	402384	031621	83	968379	24		36
	28	37	566314	531	433686	597985	615	402015	031671	83	968329	23		32
	32	38	566632	531	433368	598354	614	401646	031722	83	968278	22		28
	36	39	566951	530	433049	598722	614	401278	031772	84	968228	21		24
	40	40	567269	530	432731	599091	613	400909	031822	84	968178	20		20
	44	41	567587	529	432413	599459	613	400541	031872	84	968128	19		16
	48	42	567904	529	432096	599827	613	400173	031922	84	968078	18		12
	52	43	568222	528	431778	600194	612	399806	031973	84	968027	17		8
	56	44	568539	528	431461	600562	612	399438	032023	84	967977	16		4
27	0	45	9.568856	528	10.431144	9.600929	611	10.399071	10.032073	84	9.967927	15	33	0
	4	46	569172	527	430828	601296	611	398704	032124	84	967876	14		56
	8	47	569488	527	430512	601662	611	398338	032174	84	967826	13		52
	12	48	569804	526	430196	602029	610	397971	032225	84	967775	12		48
	16	49	570120	526	429880	602395	610	397605	032275	84	967725	11		44
	20	50	570435	525	429565	602761	610	397239	032326	84	967674	10		40
	24	51	570751	525	429249	603127	609	396873	032376	84	967624	9		36
	28	52	571066	524	428934	603493	609	396507	032427	84	967573	8		32
	32	53	571380	524	428620	603858	609	396142	032478	85	967522	7		28
	36	54	571695	523	428305	604223	608	395777	032529	85	967471	6		24
	40	55	572009	523	427991	604588	608	395412	032579	85	967421	5		20
	44	56	572323	523	427677	604953	607	395047	032630	85	967370	4		16
	48	57	572636	522	427364	605317	607	394683	032681	85	967319	3		12
	52	58	572950	522	427050	605682	607	394318	032732	85	967268	2		8
	56	59	573263	521	426737	606046	606	393954	032783	85	967217	1		4
28	0	60	573575	521	426425	606410	606	393590	032834	85	967166	0	32	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
4 Hours,							or				68 Degrees.			

P. P. to s or ″										
	1s	15″	80	1s	15″	93	1s	15″	12	
	2	30	160	2	30	185	2	30	25	
	3	45	240	3	45	278	3	45	37	P. P. to s or ″

TABLE V. Logarithmic Sines, Tangents,

1 Hour, or 22 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
28	0	0	9.573575	521	10.426425	9.606410	606	10.393590	10.032834	85	9.967166	60	32	0
	4	1	573888	520	426112	606773	606	393227	032885	85	967115	59		56
	8	2	574200	520	425800	607137	605	392863	032936	85	967064	58		52
	12	3	574512	519	425488	607500	605	392500	032987	85	967013	57		48
	16	4	574824	519	425176	607863	604	392137	033039	85	966961	56		44
	20	5	575136	519	424864	608225	604	391775	033090	85	966910	55		40
	24	6	575447	518	424553	608588	604	391412	033141	85	966859	54		36
	28	7	575758	518	424242	608950	603	391050	033192	85	966808	53		32
	32	8	576069	517	423931	609312	603	390688	033244	86	966756	52		28
	36	9	576379	517	423621	609674	603	390326	033295	86	966705	51		24
	40	10	576689	516	423311	610036	602	389964	033347	86	966653	50		20
	44	11	576999	516	423001	610397	602	389603	033398	86	966602	49		16
	48	12	577309	516	422691	610759	602	389241	033450	86	966550	48		12
	52	13	577618	515	422382	611120	601	388880	033501	86	966499	47		8
	56	14	577927	515	422073	611480	601	388520	033553	86	966447	46		4
29	0	15	9.578236	514	10.421764	9.611841	601	10.388159	10.033605	86	9.966395	45	31	0
	4	16	578545	514	421455	612201	600	387799	033656	86	966344	44		56
	8	17	578853	513	421147	612561	600	387439	033708	86	966292	43		52
	12	18	579162	513	420838	612921	600	387079	033760	86	966240	42		48
	16	19	579470	513	420530	613281	599	386719	033812	86	966188	41		44
	20	20	579777	512	420223	613641	599	386359	033864	86	966136	40		40
	24	21	580085	512	419915	614000	598	386000	033915	87	966085	39		36
	28	22	580392	511	419608	614359	598	385641	033967	87	966033	38		32
	32	23	580699	511	419301	614718	598	385282	034019	87	965981	37		28
	36	24	581005	511	418995	615077	597	384923	034071	87	965929	36		24
	40	25	581312	510	418688	615435	597	384565	034124	87	965876	35		20
	44	26	581618	510	418382	615793	597	384207	034176	87	965824	34		16
	48	27	581924	509	418076	616151	596	383849	034228	87	965772	33		12
	52	28	582229	509	417771	616509	596	383491	034280	87	965720	32		8
	56	29	582535	509	417465	616867	596	383133	034332	87	965668	31		4
30	0	30	9.582840	508	10.417160	9.617224	595	10.382776	10.034385	87	9.965615	30	30	0
	4	31	583145	508	416855	617582	595	382418	034437	87	965563	29		56
	8	32	583449	507	416551	617939	595	382061	034489	87	965511	28		52
	12	33	583754	507	416246	618295	594	381705	034542	87	965458	27		48
	16	34	584058	506	415942	618652	594	381348	034594	87	965406	26		44
	20	35	584361	506	415639	619008	594	380992	034647	88	965353	25		40
	24	36	584665	506	415335	619364	593	380636	034699	88	965301	24		36
	28	37	584968	505	415032	619721	593	380279	034752	88	965248	23		32
	32	38	585272	505	414728	620076	593	379924	034805	88	965195	22		28
	36	39	585574	504	414426	620432	592	379568	034857	88	965143	21		24
	40	40	585877	504	414123	620787	592	379213	034910	88	965090	20		20
	44	41	586179	503	413821	621142	592	378858	034963	88	965037	19		16
	48	42	586482	503	413518	621497	591	378503	035016	88	964984	18		12
	52	43	586783	503	413217	621852	591	378148	035069	88	964931	17		8
	56	44	587085	502	412915	622207	590	377793	035121	88	964879	16		4
31	0	45	9.587386	502	10.412614	9.622561	590	10.377439	10.035174	88	9.964826	15	29	0
	4	46	587688	501	412312	622915	590	377085	035227	88	964773	14		56
	8	47	587989	501	412011	623269	589	376731	035280	88	964720	13		52
	12	48	588289	501	411711	623623	589	376377	035334	89	964666	12		48
	16	49	588590	500	411410	623976	589	376024	035387	89	964613	11		44
	20	50	588890	500	411110	624330	588	375670	035440	89	964560	10		40
	24	51	589190	499	410810	624683	588	375317	035493	89	964507	9		36
	28	52	589489	499	410511	625036	588	374964	035546	89	964454	8		32
	32	53	589789	499	410211	625388	587	374612	035600	89	964400	7		28
	36	54	590088	498	409912	625741	587	374259	035653	89	964347	6		24
	40	55	590387	498	409613	626093	587	373907	035706	89	964294	5		20
	44	56	590686	497	409314	626445	586	373555	035760	89	964240	4		16
	48	57	590984	497	409016	626797	586	373203	035813	89	964187	3		12
	52	58	591282	497	408718	627149	586	372851	035867	89	964133	2		8
	56	59	591580	496	408420	627501	585	372499	035920	89	964080	1		4
32	0	60	591878	496	408122	627852	585	372148	035974	89	964026	0	28	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

4 Hours, or 67 Degrees.

P. P. to s or ″	1s	15″	76	1s	15″	89	1s	15″	13	P. P. to s or ″
	2	30	152	2	30	178	2	30	26	
	3	45	229	3	45	268	3	45	39	

1 Hour,							or				23 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
32	0	0	9.591878	496	10.408122	9.627852	585	10.372148	10.035974	89	9.964026	60	28	0
	4	1	592176	495	407824	628203	585	371797	036028	89	963972	59		56
	8	2	592473	495	407527	628554	585	371446	036081	89	963919	58		52
	12	3	592770	495	407230	628905	584	371095	036135	90	963865	57		48
	16	4	593067	494	406933	629255	584	370745	036189	90	963811	56		44
	20	5	593363	494	406637	629606	583	370394	036243	90	963757	55		40
	24	6	593659	493	406341	629956	583	370044	036296	90	963704	54		36
	28	7	593955	493	406045	630306	583	369694	036350	90	963650	53		32
	32	8	594251	493	405749	630656	583	369344	036404	90	963596	52		28
	36	9	594547	492	405453	631005	582	368995	036458	90	963542	51		24
	40	10	594842	492	405158	631355	582	368645	036512	90	963488	50		20
	44	11	595137	491	404863	631704	582	368296	036566	90	963434	49		16
	48	12	595432	491	404568	632053	581	367947	036621	90	963379	48		12
	52	13	595727	491	404273	632401	581	367599	036675	90	963325	47		8
	56	14	596021	490	403979	632750	581	367250	036729	90	963271	46		4
33	0	15	9.596315	490	10.403685	9.633098	580	10.366902	10.036783	90	9.963217	45	27	0
	4	16	596609	489	403391	633447	580	366553	036837	90	963163	44		56
	8	17	596903	489	403097	633795	580	366205	036892	91	963108	43		52
	12	18	597196	489	402804	634143	579	365857	036946	91	963054	42		48
	16	19	597490	488	402510	634490	579	365510	037001	91	962999	41		44
	20	20	597783	488	402217	634838	579	365162	037055	91	962945	40		40
	24	21	598075	487	401925	635185	578	364815	037110	91	962890	39		36
	28	22	598368	487	401632	635532	578	364468	037164	91	962836	38		32
	32	23	598660	487	401340	635879	578	364121	037219	91	962781	37		28
	36	24	598952	486	401048	636226	577	363774	037273	91	962727	36		24
	40	25	599244	486	400756	636572	577	363428	037328	91	962672	35		20
	44	26	599536	485	400464	636919	577	363081	037383	91	962617	34		16
	48	27	599827	485	400173	637265	577	362735	037438	91	962562	33		12
	52	28	600118	485	399882	637611	576	362389	037492	91	962508	32		8
	56	29	600409	484	399591	637956	576	362044	037547	91	962453	31		4
34	0	30	9.600700	484	10.399300	9.638302	576	10.361698	10.037602	92	9.962398	30	26	0
	4	31	600990	484	399010	638647	575	361353	037657	92	962343	29		56
	8	32	601280	483	398720	638992	575	361008	037712	92	962288	28		52
	12	33	601570	483	398430	639337	575	360663	037767	92	962233	27		48
	16	34	601860	482	398140	639682	574	360318	037822	92	962178	26		44
	20	35	602150	482	397850	640027	574	359973	037877	92	962123	25		40
	24	36	602439	482	397561	640371	574	359629	037933	92	962067	24		36
	28	37	602728	481	397272	640716	573	359284	037988	92	962012	23		32
	32	38	603017	481	396983	641060	573	358940	038043	92	961957	22		28
	36	39	603305	481	396695	641404	573	358596	038098	92	961902	21		24
	40	40	603594	480	396406	641747	572	358253	038154	92	961846	20		20
	44	41	603882	480	396118	642091	572	357909	038209	92	961791	19		16
	48	42	604170	479	395830	642434	572	357566	038265	92	961735	18		12
	52	43	604457	479	395543	642777	572	357223	038320	92	961680	17		8
	56	44	604745	479	395255	643120	571	356880	038376	93	961624	16		4
35	0	45	9.605032	478	10.394968	9.643463	571	10.356537	10.038431	93	9.961569	15	25	0
	4	46	605319	478	394681	643806	571	356194	038487	93	961513	14		56
	8	47	605606	478	394394	644148	570	355852	038542	93	961458	13		52
	12	48	605892	477	394108	644490	570	355510	038598	93	961402	12		48
	16	49	606179	477	393821	644832	570	355168	038654	93	961346	11		44
	20	50	606465	476	393535	645174	569	354826	038710	93	961290	10		40
	24	51	606751	476	393249	645516	569	354484	038765	93	961235	9		36
	28	52	607036	476	392964	645857	569	354143	038821	93	961179	8		32
	32	53	607322	475	392678	646199	569	353801	038877	93	961123	7		28
	36	54	607607	475	392393	646540	568	353460	038933	93	961067	6		24
	40	55	607892	474	392108	646881	568	353119	038989	93	961011	5		20
	44	56	608177	474	391823	647222	568	352778	039045	93	960955	4		16
	48	57	608461	474	391539	647562	567	352438	039101	93	960899	3		12
	52	58	608745	473	391255	647903	567	352097	039157	94	960843	2		8
	56	59	609029	473	390971	648243	567	351757	039214	94	960786	1		4
36	0	60	609313	473	390687	648583	566	351417	039270	94	960730	0	24	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
4 Hours,							or				66 Degrees.			

P. P. to s or ″	1s	15″	73	1s	15″	86	1s	15″	14	P. P. to s or ″
	2	30	145	2	30	173	2	30	28	
	3	45	218	3	45	259	3	45	41	

1 Hour,							or				24 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
36	0	0	9.609313	473	10.390687	9.648583	566	10.351417	10.039270	94	9.960730	60	24	0
	4	1	609597	472	390403	648923	566	351077	039326	94	960674	59		56
	8	2	609880	472	390120	649263	566	350737	039382	94	960618	58		52
	12	3	610164	472	389836	649602	566	350398	039439	94	960561	57		48
	16	4	610447	471	389553	649942	565	350058	039495	94	960505	56		44
	20	5	610729	471	389271	650281	565	349719	039552	94	960448	55		40
	24	6	611012	470	388988	650620	565	349380	039608	94	960392	54		36
	28	7	611294	470	388706	650959	564	349041	039665	94	960335	53		32
	32	8	611576	470	388424	651297	564	348703	039721	94	960279	52		28
	36	9	611858	469	388142	651636	564	348364	039778	94	960222	51		24
	40	10	612140	469	387860	651974	563	348026	039835	94	960165	50		20
	44	11	612421	469	387579	652312	563	347688	039891	95	960109	49		16
	48	12	612702	468	387298	652650	563	347350	039948	95	960052	48		12
	52	13	612983	468	387017	652988	563	347012	040005	95	959995	47		8
	56	14	613264	467	386736	653326	562	346674	040062	95	959938	46		4
37	0	15	9.613545	467	10.386455	9.653663	562	10.346337	10.040118	95	9.959882	45	23	0
	4	16	613825	467	386175	654000	562	346000	040175	95	959825	44		56
	8	17	614105	466	385895	654337	561	345663	040232	95	959768	43		52
	12	18	614385	466	385615	654674	561	345326	040289	95	959711	42		48
	16	19	614665	466	385335	655011	561	344989	040346	95	959654	41		44
	20	20	614944	465	385056	655348	561	344652	040404	95	959596	40		40
	24	21	615223	465	384777	655684	560	344316	040461	95	959539	39		36
	28	22	615502	465	384498	656020	560	343980	040518	95	959482	38		32
	32	23	615781	464	384219	656356	560	343644	040575	95	959425	37		28
	36	24	616060	464	383940	656692	559	343308	040632	95	959368	36		24
	40	25	616338	464	383662	657028	559	342972	040690	96	959310	35		20
	44	26	616616	463	383384	657364	559	342636	040747	96	959253	34		16
	48	27	616894	463	383106	657699	559	342301	040805	96	959195	33		12
	52	28	617172	462	382828	658034	558	341966	040862	96	959138	32		8
	56	29	617450	462	382550	658369	558	341631	040920	96	959080	31		4
38	0	30	9.617727	462	10.382273	9.658704	558	10.341296	10.040977	96	9.959023	30	22	0
	4	31	618004	461	381996	659039	558	340961	041035	96	958965	29		56
	8	32	618281	461	381719	659373	557	340627	041092	96	958908	28		52
	12	33	618558	461	381442	659708	557	340292	041150	96	958850	27		48
	16	34	618834	460	381166	660042	557	339958	041208	96	958792	26		44
	20	35	619110	460	380890	660376	557	339624	041266	96	958734	25		40
	24	36	619386	460	380614	660710	556	339290	041323	96	958677	24		36
	28	37	619662	459	380338	661043	556	338957	041381	96	958619	23		32
	32	38	619938	459	380062	661377	556	338623	041439	96	958561	22		28
	36	39	620213	459	379787	661710	555	338290	041497	97	958503	21		24
	40	40	620488	458	379512	662043	555	337957	041555	97	958445	20		20
	44	41	620763	458	379237	662376	555	337624	041613	97	958387	19		16
	48	42	621038	457	378962	662709	554	337291	041671	97	958329	18		12
	52	43	621313	457	378687	663042	554	336958	041729	97	958271	17		8
	56	44	621587	457	378413	663375	554	336625	041787	97	958213	16		4
39	0	45	9.621861	456	10.378139	9.663707	554	10.336293	10.041846	97	9.958154	15	21	0
	4	46	622135	456	377865	664039	553	335961	041904	97	958096	14		56
	8	47	622409	456	377591	664371	553	335629	041962	97	958038	13		52
	12	48	622682	455	377318	664703	553	335297	042021	97	957979	12		48
	16	49	622956	455	377044	665035	553	334965	042079	97	957921	11		44
	20	50	623229	455	376771	665366	552	334634	042137	97	957863	10		40
	24	51	623502	454	376498	665697	552	334303	042196	97	957804	9		36
	28	52	623774	454	376226	666029	552	333971	042254	98	957746	8		32
	32	53	624047	454	375953	666360	551	333640	042313	98	957687	7		28
	36	54	624319	453	375681	666691	551	333309	042372	98	957628	6		24
	40	55	624591	453	375409	667021	551	332979	042430	98	957570	5		20
	44	56	624863	453	375137	667352	551	332648	042489	98	957511	4		16
	48	57	625135	452	374865	667682	550	332318	042548	98	957452	3		12
	52	58	625406	452	374594	668013	550	331987	042607	98	957393	2		8
	56	59	625677	452	374323	668343	550	331657	042665	98	957335	1		4
40	0	60	625948	451	374052	668672	550	331328	042724	98	957276	0	20	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
4 Hours,							or				65 Degrees.			

P. P. to s or ″	1s	15″	69	1s	15″	84	1s	15″	14	P. P. to s or ″
	2	30	139	2	30	167	2	30	29	
	3	45	208	3	45	251	3	45	43	

1 Hour,						or				25 Degrees.				
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
40	0	0	9.625948	451	10.374052	9.668673	550	10.331327	10.042724	98	9.957276	60	20	0
	4	1	626219	451	373781	669002	549	330998	042783	98	957217	59		56
	8	2	626490	451	373510	669332	549	330668	042842	98	957158	58		52
	12	3	626760	450	373240	669661	549	330339	042901	98	957099	57		48
	16	4	627030	450	372970	669991	548	330009	042960	98	957040	56		44
	20	5	627300	450	372700	670320	548	329680	043019	98	956981	55		40
	24	6	627570	449	372430	670649	548	329351	043079	99	956921	54		36
	28	7	627840	449	372160	670977	548	329023	043138	99	956862	53		32
	32	8	628109	449	371891	671306	547	328694	043197	99	956803	52		28
	36	9	628378	448	371622	671634	547	328366	043256	99	956744	51		24
	40	10	628647	448	371353	671963	547	328037	043316	99	956684	50		20
	44	11	628916	447	371084	672291	547	327709	043375	99	956625	49		16
	48	12	629185	447	370815	672619	546	327381	043434	99	956566	48		12
	52	13	629453	447	370547	672947	546	327053	043494	99	956506	47		8
	56	14	629721	446	370279	673274	546	326726	043553	99	956447	46		4
41	0	15	9.629989	446	10.370011	9.673602	546	10.326398	10.043613	99	9.956387	45	19	0
	4	16	630257	446	369743	673929	545	326071	043673	99	956327	44		56
	8	17	630524	446	369476	674257	545	325743	043732	99	956268	43		52
	12	18	630792	445	369208	674584	545	325416	043792	100	956208	42		48
	16	19	631059	445	368941	674910	544	325090	043852	100	956148	41		44
	20	20	631326	445	368674	675237	544	324763	043911	100	956089	40		40
	24	21	631593	444	368407	675564	544	324436	043971	100	956029	39		36
	28	22	631859	444	368141	675890	544	324110	044031	100	955969	38		32
	32	23	632125	444	367875	676217	543	323783	044091	100	955909	37		28
	36	24	632392	443	367608	676543	543	323457	044151	100	955849	36		24
	40	25	632658	443	367342	676869	543	323131	044211	100	955789	35		20
	44	26	632923	443	367077	677194	543	322806	044271	100	955729	34		16
	48	27	633189	442	366811	677520	542	322480	044331	100	955669	33		12
	52	28	633454	442	366546	677846	542	322154	044391	100	955609	32		8
	56	29	633719	442	366281	678171	542	321829	044452	100	955548	31		4
42	0	30	9.633984	441	10.366016	9.678496	542	10.321504	10.044512	100	9.955488	30	18	0
	4	31	634249	441	365751	678821	541	321179	044572	101	955428	29		56
	8	32	634514	440	365486	679146	541	320854	044632	101	955368	28		52
	12	33	634778	440	365222	679471	541	320529	044693	101	955307	27		48
	16	34	635042	440	364958	679795	541	320205	044753	101	955247	26		44
	20	35	635306	439	364694	680120	540	319880	044814	101	955186	25		40
	24	36	635570	439	364430	680444	540	319556	044874	101	955126	24		36
	28	37	635834	439	364166	680768	540	319232	044935	101	955065	23		32
	32	38	636097	438	363903	681092	540	318908	044995	101	955005	22		28
	36	39	636360	438	363640	681416	539	318584	045056	101	954944	21		24
	40	40	636623	438	363377	681740	539	318260	045117	101	954883	20		20
	44	41	636886	437	363114	682063	539	317937	045177	101	954823	19		16
	48	42	637148	437	362852	682387	539	317613	045238	101	954762	18		12
	52	43	637411	437	362589	682710	538	317290	045299	101	954701	17		8
	56	44	637673	437	362327	683038	538	316967	045360	101	954640	16		4
43	0	45	9.637935	436	10.362065	9.683356	538	10.316644	10.045421	101	9.954579	15	17	0
	4	46	638197	436	361803	683679	538	316321	045482	102	954518	14		56
	8	47	638458	436	361542	684001	537	315999	045543	102	954457	13		52
	12	48	638720	435	361280	684324	537	315676	045604	102	954396	12		48
	16	49	638981	435	361019	684646	537	315354	045665	102	954335	11		44
	20	50	639242	435	360758	684968	537	315032	045726	102	954274	10		40
	24	51	639503	434	360497	685290	536	314710	045787	102	954213	9		36
	28	52	639764	434	360236	685612	536	314388	045848	102	954152	8		32
	32	53	640024	434	359976	685934	536	314066	045910	102	954090	7		28
	36	54	640284	433	359716	686255	536	313745	045971	102	954029	6		24
	40	55	640544	433	359456	686577	535	313423	046032	102	953968	5		20
	44	56	640804	433	359196	686898	535	313102	046094	102	953906	4		16
	48	57	641064	432	358936	687219	535	312781	046155	102	953845	3		12
	52	58	641324	432	358676	687540	535	312460	046217	102	953783	2		8
	56	59	641583	432	358417	687861	534	312139	046278	103	953722	1		4
44	0	60	641842	431	358158	688182	534	311818	046340	103	953660	0	16	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
4 Hours,						or				64 Degrees.				

P. P. to s or ″	1s	15″	66	1s	15″	81	1s	15″	15	P. P. to s or ″
	2	30	132	2	30	163	2	30	30	
	3	45	199	3	45	244	3	45	45	

1 Hour, or 26 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
44	0	0	9.641842	431	10.358158	9.688182	534	10.311818	10.046340	103	9.953660	60	16	0
	4	1	642101	431	357899	688502	534	311498	046401	103	953599	59		56
	8	2	642360	431	357640	688823	534	311177	046463	103	953537	58		52
	12	3	642618	430	357382	689143	533	310857	046525	103	953475	57		48
	16	4	642877	430	357123	689463	533	310537	046587	103	953413	56		44
	20	5	643135	430	356865	689783	533	310217	046648	103	953352	55		40
	24	6	643393	430	356607	690103	533	309897	046710	103	953290	54		36
	28	7	643650	429	356350	690423	533	309577	046772	103	953228	53		32
	32	8	643908	429	356092	690742	532	309258	046834	103	953166	52		28
	36	9	644165	429	355835	691062	532	308938	046896	103	953104	51		24
	40	10	644423	428	355577	691381	532	308619	046958	103	953042	50		20
	44	11	644680	428	355320	691700	531	308300	047020	104	952980	49		16
	48	12	644936	428	355064	692019	531	307981	047082	104	952918	48		12
	52	13	645193	427	354807	692338	531	307662	047145	104	952855	47		8
	56	14	645450	427	354550	692656	531	307344	047207	104	952793	46		4
45	0	15	9.645706	427	10.354294	9.692975	531	10.307025	10.047269	104	9.952731	45	15	0
	4	16	645962	426	354038	693293	530	306707	047331	104	952669	44		56
	8	17	646218	426	353782	693612	530	306388	047394	104	952606	43		52
	12	18	646474	426	353526	693930	530	306070	047456	104	952544	42		48
	16	19	646729	425	353271	694248	530	305752	047519	104	952481	41		44
	20	20	646984	425	353016	694566	529	305434	047581	104	952419	40		40
	24	21	647240	425	352760	694883	529	305117	047644	104	952356	39		36
	28	22	647494	424	352506	695201	529	304799	047706	104	952294	38		32
	32	23	647749	424	352251	695518	529	304482	047769	104	952231	37		28
	36	24	648004	424	351996	695836	529	304164	047832	105	952168	36		24
	40	25	648258	424	351742	696153	528	303847	047894	105	952106	35		20
	44	26	648512	423	351488	696470	528	303530	047957	105	952043	34		16
	48	27	648766	423	351234	696787	528	303213	048020	105	951980	33		12
	52	28	649020	423	350980	697103	528	302897	048083	105	951917	32		8
	56	29	649274	422	350726	697420	527	302580	048146	105	951854	31		4
46	0	30	9.649527	422	10.350473	9.697736	527	10.302264	10.048209	105	9.951791	30	14	0
	4	31	649781	422	350219	698053	527	301947	048272	105	951728	29		56
	8	32	650034	422	349966	698369	527	301631	048335	105	951665	28		52
	12	33	650287	421	349713	698685	526	301315	048398	105	951602	27		48
	16	34	650539	421	349461	699001	526	300999	048461	105	951539	26		44
	20	35	650792	421	349208	699316	526	300684	048524	105	951476	25		40
	24	36	651044	420	348956	699632	526	300368	048588	105	951412	24		36
	28	37	651297	420	348703	699947	526	300053	048651	106	951349	23		32
	32	38	651549	420	348451	700263	525	299737	048714	106	951286	22		28
	36	39	651800	419	348200	700578	525	299422	048778	106	951222	21		24
	40	40	652052	419	347948	700893	525	299107	048841	106	951159	20		20
	44	41	652304	419	347696	701208	524	298792	048904	106	951096	19		16
	48	42	652555	418	347445	701523	524	298477	048968	106	951032	18		12
	52	43	652806	418	347194	701837	524	298163	049032	106	950968	17		8
	56	44	653057	418	346943	702152	524	297848	049095	106	950905	16		4
47	0	45	9.653308	418	10.346692	9.702466	524	10.297534	10.049159	106	9.950841	15	13	0
	4	46	653558	417	346442	702780	523	297220	049222	106	950778	14		56
	8	47	653808	417	346194	703095	523	296905	049286	106	950714	13		52
	12	48	654059	417	345941	703409	523	296591	049350	106	950650	12		48
	16	49	654309	416	345691	703723	523	296277	049414	106	950586	11		44
	20	50	654558	416	345442	704036	522	295964	049478	107	950522	10		40
	24	51	654808	416	345192	704350	522	295650	049542	107	950458	9		36
	28	52	655058	416	344942	704663	522	295337	049606	107	950394	8		32
	32	53	655307	415	344693	704977	522	295023	049670	107	950330	7		28
	36	54	655556	415	344444	705290	522	294710	049734	107	950266	6		24
	40	55	655805	415	344195	705603	521	294397	049798	107	950202	5		20
	44	56	656054	414	343946	705916	521	294084	049862	107	950138	4		16
	48	57	656302	414	343698	706228	521	293772	049926	107	950074	3		12
	52	58	656551	414	343449	706541	521	293459	049990	107	950010	2		8
	56	59	656799	413	343201	706854	521	293146	050055	107	949945	1		4
48	0	60	657047	413	342953	707166	520	292834	050119	107	949881	0	12	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

4 Hours, or 63 Degrees.

P. P. to s or ″	1ˢ	15″	63	1ˢ	15″	79	1ˢ	15″	16	P. P. to s or ″
	2	30	127	2	30	158	2	30	31	
	3	45	190	3	45	238	3	45	47	

1 Hour, or 27 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
48	0	0	9.657047	413	10.342953	9.707166	520	10.292834	10.050119	107	9.949881	60	12	0
	4	1	657295	413	342705	707478	520	292522	050184	107	949816	59		56
	8	2	657542	412	342458	707790	520	292210	050248	107	949752	58		52
	12	3	657790	412	342210	708102	520	291898	050312	108	949688	57		48
	16	4	658037	412	341963	708414	519	291586	050377	108	949623	56		44
	20	5	658284	412	341716	708726	519	291274	050442	108	949558	55		40
	24	6	658531	411	341469	709037	519	290963	050506	108	949494	54		36
	28	7	658778	411	341222	709349	519	290651	050571	108	949429	53		32
	32	8	659025	411	340975	709660	519	290340	050636	108	949364	52		28
	36	9	659271	410	340729	709971	518	290029	050700	108	949300	51		24
	40	10	659517	410	340483	710282	518	289718	050765	108	949235	50		20
	44	11	659763	410	340237	710593	518	289407	050830	108	949170	49		16
	48	12	660009	409	339991	710904	518	289096	050895	108	949105	48		12
	52	13	660255	409	339745	711215	518	288785	050960	108	949040	47		8
	56	14	660501	409	339499	711525	517	288475	051025	108	948975	46		4
49	0	15	9.660746	409	10.339254	9.711836	517	10.288164	10.051090	108	9.948910	45	11	0
	4	16	660991	408	339009	712146	517	287854	051155	108	948845	44		56
	8	17	661236	408	338764	712456	517	287544	051220	109	948780	43		52
	12	18	661481	408	338519	712766	516	287234	051285	109	948715	42		48
	16	19	661726	407	338274	713076	516	286924	051350	109	948650	41		44
	20	20	661970	407	338030	713386	516	286614	051416	109	948584	40		40
	24	21	662214	407	337786	713696	516	286304	051481	109	948519	39		36
	28	22	662459	407	337541	714005	516	285995	051546	109	948454	38		32
	32	23	662703	406	337297	714314	515	285686	051612	109	948388	37		28
	36	24	662946	406	337054	714624	515	285376	051677	109	948323	36		24
	40	25	663190	406	336810	714933	515	285067	051743	109	948257	35		20
	44	26	663433	405	336567	715242	515	284758	051808	109	948192	34		16
	48	27	663677	405	336323	715551	514	284449	051874	109	948126	33		12
	52	28	663920	405	336080	715860	514	284140	051940	109	948060	32		8
	56	29	664163	405	335837	716168	514	283832	052005	110	947995	31		4
50	0	30	9.664406	404	10.335594	9.716477	514	10.283523	10.052071	110	9.947929	30	10	0
	4	31	664648	404	335352	716785	514	283215	052137	110	947863	29		56
	8	32	664891	404	335109	717093	513	282907	052203	110	947797	28		52
	12	33	665133	403	334867	717401	513	282599	052269	110	947731	27		48
	16	34	665375	403	334625	717709	513	282291	052335	110	947665	26		44
	20	35	665617	403	334383	718017	513	281983	052400	110	947600	25		40
	24	36	665859	402	334141	718325	513	281675	052467	110	947533	24		36
	28	37	666100	402	333900	718633	512	281367	052533	110	947467	23		32
	32	38	666342	402	333658	718940	512	281060	052599	110	947401	22		28
	36	39	666583	402	333417	719248	512	280752	052665	110	947335	21		24
	40	40	666824	401	333176	719555	512	280445	052731	110	947269	20		20
	44	41	667065	401	332935	719862	512	280138	052797	110	947203	19		16
	48	42	667305	401	332695	720169	511	279831	052864	111	947136	18		12
	52	43	667546	401	332454	720476	511	279524	052930	111	947070	17		8
	56	44	667786	400	332214	720783	511	279217	052996	111	947004	16		4
51	0	45	9.668027	400	10.331973	9.721089	511	10.278911	10.053063	111	9.946937	15	9	0
	4	46	668267	400	331733	721396	511	278604	053129	111	946871	14		56
	8	47	668506	399	331494	721702	510	278298	053196	111	946804	13		52
	12	48	668746	399	331254	722009	510	277991	053262	111	946738	12		48
	16	49	668986	399	331014	722315	510	277685	053329	111	946671	11		44
	20	50	669225	399	330775	722621	510	277379	053396	111	946604	10		40
	24	51	669464	398	330536	722927	510	277073	053462	111	946538	9		36
	28	52	669703	398	330297	723232	509	276768	053529	111	946471	8		32
	32	53	669942	398	330058	723538	509	276462	053596	111	946404	7		28
	36	54	670181	397	329819	723844	509	276156	053663	111	946337	6		24
	40	55	670419	397	329581	724149	509	275851	053730	112	946270	5		20
	44	56	670658	397	329342	724454	509	275546	053797	112	946203	4		16
	48	57	670896	397	329104	724759	508	275241	053864	112	946136	3		12
	52	58	671134	396	328866	725065	508	274935	053931	112	946069	2		8
	56	59	671372	396	328628	725369	508	274631	053998	112	946002	1		4
52	0	60	671609	396	328391	725674	508	274326	054065	112	945935	0	8	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

4 Hours, or 62 Degrees.

P. P. to s or ″	1s	15″	61	1s	15″	77	1s	15″	16	P. P. to s or ″
	2	30	121	2	30	154	2	30	33	
	3	45	182	3	45	231	3	45	49	

1 Hour,							or				28 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
52	0	0	9.671609	396	10.328391	9.725674	508	10.274326	10.054065	112	9.945935	60	8	0
	4	1	671847	395	328153	725979	508	274021	054132	112	945868	59		56
	8	2	672084	395	327916	726284	507	273716	054200	112	945800	58		52
	12	3	672321	395	327679	726588	507	273412	054267	112	945733	57		48
	16	4	672558	395	327442	726892	507	273108	054334	112	945666	56		44
	20	5	672795	394	327205	727197	507	272803	054402	112	945598	55		40
	24	6	673032	394	326968	727501	507	272499	054469	112	945531	54		36
	28	7	673268	394	326732	727805	506	272195	054536	113	945464	53		32
	32	8	673505	394	326495	728109	506	271891	054604	113	945396	52		28
	36	9	673741	393	326259	728412	506	271588	054672	113	945328	51		24
	40	10	673977	393	326023	728716	506	271284	054739	113	945261	50		20
	44	11	674213	393	325787	729020	506	270980	054807	113	945193	49		16
	48	12	674448	392	325552	729323	505	270677	054875	113	945125	48		12
	52	13	674684	392	325316	729626	505	270374	054942	113	945058	47		8
	56	14	674919	392	325081	729929	505	270071	055010	113	944990	46		4
53	0	15	9.675155	392	10.324845	9.730233	505	10.269767	10.055078	113	9.944922	45	7	0
	4	16	675390	391	324610	730535	505	269465	055146	113	944854	44		56
	8	17	675624	391	324376	730838	504	269162	055214	113	944786	43		52
	12	18	675859	391	324141	731141	504	268859	055282	113	944718	42		48
	16	19	676094	391	323906	731444	504	268556	055350	113	944650	41		44
	20	20	676328	390	323672	731746	504	268254	055418	114	944582	40		40
	24	21	676562	390	323438	732048	504	267952	055486	114	944514	39		36
	28	22	676796	390	323204	732351	503	267649	055554	114	944446	38		32
	32	23	677030	300	322970	732653	503	267347	055623	114	944377	37		28
	36	24	677264	389	322736	732955	503	267045	055691	114	944309	36		24
	40	25	677498	389	322502	733257	503	266743	055759	114	944241	35		20
	44	26	677731	389	322269	733558	503	266442	055828	114	944172	34		16
	48	27	677964	388	322036	733860	502	266140	055896	114	944104	33		12
	52	28	678197	388	321803	734162	502	265838	055964	114	944036	32		8
	56	29	678430	388	321570	734463	502	265537	056033	114	943967	31		4
54	0	30	9.678663	388	10.321337	9.734764	502	10.265236	10.056101	114	9.943899	30	6	0
	4	31	678895	387	321105	735066	502	264934	056170	114	943830	29		56
	8	32	679128	387	320872	735367	502	264633	056239	114	943761	28		52
	12	33	679360	387	320640	735668	501	264332	056307	115	943693	27		48
	16	34	679592	387	320408	735969	501	264031	056376	115	943624	26		44
	20	35	679824	386	320176	736269	501	263731	056445	115	943555	25		40
	24	36	680056	386	319944	736570	501	263430	056514	115	943486	24		36
	28	37	680288	386	319712	736871	501	263129	056583	115	943417	23		32
	32	38	680519	385	319481	737171	500	262829	056652	115	943348	22		28
	36	39	680750	385	319250	737471	500	262529	056721	115	943279	21		24
	40	40	680982	385	319018	737771	500	262229	056790	115	943210	20		20
	44	41	681213	385	318787	738071	500	261929	056859	115	943141	19		16
	48	42	681443	384	318557	738371	500	261629	056928	115	943072	18		12
	52	43	681674	384	318326	738671	499	261329	056997	115	943003	17		8
	56	44	681905	384	318095	738971	499	261029	057066	115	942934	16		4
55	0	45	9.682135	384	10.317865	9.739271	499	10.260729	10.057136	115	9.942864	15	5	0
	4	46	682365	383	317635	739570	499	260430	057205	116	942795	14		56
	8	47	682595	383	317405	739870	499	260130	057274	116	942726	13		52
	12	48	682825	383	317175	740169	499	259831	057344	116	942656	12		48
	16	49	683055	383	316945	740468	498	259532	057413	116	942587	11		44
	20	50	683284	382	316716	740767	498	259233	057483	116	942517	10		40
	24	51	683514	382	316486	741066	498	258934	057552	116	942448	9		36
	28	52	683743	382	316257	741365	498	258635	057622	116	942378	8		32
	32	53	683972	382	316028	741664	498	258336	057692	116	942308	7		28
	36	54	684201	381	315799	741962	497	258038	057761	116	942239	6		24
	40	55	684430	381	315570	742261	497	257739	057831	116	942169	5		20
	44	56	684658	381	315342	742559	497	257441	057901	116	942099	4		16
	48	57	684887	380	315113	742858	497	257142	057971	116	942029	3		12
	52	58	685115	380	314885	743156	497	256844	058041	116	941959	2		8
	56	59	685343	380	314657	743454	497	256546	058111	117	941889	1		4
56	0	60	685571	380	314429	743752	496	256248	058181	117	941819	0	4	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
4 Hours,							or				61 Degrees.			

P. P. to s or ″	1s	15″	58	1s	15″	75	1s	15″	17	P. P. to s or ″
	2	30	116	2	30	151	2	30	34	
	3	45	175	3	45	226	3	45	51	

			1 Hour,			or			29 Degrees.					
m.	s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D	Cosine.	'	m.	s.
56	0	0	9.685571	380	10.314429	9.743752	496	10.256248	10.058181	117	9.941819	60	4	0
	4	1	685799	379	314201	744050	496	255950	058251	117	941749	59		56
	8	2	686027	379	313973	744348	496	255652	058321	117	941679	58		52
	12	3	686254	379	313746	744645	496	255355	058391	117	941609	57		48
	16	4	686482	379	313518	744943	496	255057	058461	117	941539	56		44
	20	5	686709	378	313291	745240	495	254760	058531	117	941469	55		40
	24	6	686936	378	313064	745538	495	254462	058602	117	941398	54		36
	28	7	687163	378	312837	745835	495	254165	058672	117	941328	53		32
	32	8	687389	378	312611	746132	495	253868	058742	117	941258	52		28
	36	9	687616	377	312384	746429	495	253571	058813	117	941187	51		24
	40	10	687843	377	312157	746726	495	253274	058883	117	941117	50		20
	44	11	688069	377	311931	747023	494	252977	058954	118	941046	49		16
	48	12	688295	377	311705	747319	494	252681	059025	118	940975	48		12
	52	13	688521	376	311479	747616	494	252384	059095	118	940905	47		8
	56	14	688747	376	311253	747913	494	252087	059166	118	940834	46		4
57	0	15	9.688972	376	10.311028	9.748209	494	10.251791	10.059237	118	9.940763	45	3	0
	4	16	689198	376	310802	748505	493	251495	059307	118	940693	44		56
	8	17	689423	375	310577	748801	493	251199	059378	118	940622	43		52
	12	18	689648	375	310352	749097	493	250903	059449	118	940551	42		48
	16	19	689873	375	310127	749393	493	250607	059520	118	940480	41		44
	20	20	690098	375	309902	749689	493	250311	059591	118	940409	40		40
	24	21	690323	374	309677	749985	493	250015	059662	118	940338	39		36
	28	22	690548	374	309452	750281	492	249719	059733	118	940267	38		32
	32	23	690772	374	309228	750576	492	249424	059804	118	940196	37		28
	36	24	690996	374	309004	750872	492	249128	059875	119	940125	36		24
	40	25	691220	373	308780	751167	492	248833	059946	119	940054	35		20
	44	26	691444	373	308556	751462	492	248538	060018	119	939982	34		16
	48	27	691668	373	308332	751757	492	248243	060089	119	939911	33		12
	52	28	691892	373	308108	752052	491	247948	060160	119	939840	32		8
	56	29	692115	372	307885	752347	491	247653	060232	119	939768	31		4
58	0	30	9.692339	372	10.307661	9.752642	491	10.247358	10.060303	119	9.939697	30	2	0
	4	31	692562	372	307438	752937	491	247063	060375	119	939625	29		56
	8	32	692785	371	307215	753231	491	246769	060446	119	939554	28		52
	12	33	693008	371	306992	753526	491	246474	060518	119	939482	27		48
	16	34	693231	371	306769	753820	490	246180	060590	119	939410	26		44
	20	35	693453	371	306547	754115	490	245885	060661	119	939339	25		40
	24	36	693676	370	306324	754409	490	245591	060733	120	939267	24		36
	28	37	693898	370	306102	754703	490	245297	060805	120	939195	23		32
	32	38	694120	370	305880	754997	490	245003	060877	120	939123	22		28
	36	39	694342	370	305658	755291	490	244709	060948	120	939052	21		24
	40	40	694564	369	305436	755585	489	244415	061020	120	938980	20		20
	44	41	694786	369	305214	755878	489	244122	061092	120	938908	19		16
	48	42	695007	369	304993	756172	489	243828	061164	120	938836	18		12
	52	43	695229	369	304771	756465	489	243535	061237	120	938763	17		8
	56	44	695450	368	304550	756759	489	243241	061309	120	938691	16		4
59	0	45	9.695671	368	10.304329	9.757052	489	10.242948	10.061381	120	9.938619	15	1	0
	4	46	695892	368	304108	757345	488	242655	061453	120	938547	14		56
	8	47	696113	368	303887	757638	488	242362	061525	120	938475	13		52
	12	48	696334	367	303666	757931	488	242069	061598	121	938402	12		48
	16	49	696554	367	303446	758224	488	241776	061670	121	938330	11		44
	20	50	696775	367	303225	758517	488	241483	061742	121	938258	10		40
	24	51	696995	367	303005	758810	488	241190	061815	121	938185	9		36
	28	52	697215	366	302785	759102	487	240898	061887	121	938113	8		32
	32	53	697435	366	302565	759395	487	240605	061960	121	938040	7		28
	36	54	697654	366	302346	759687	487	240313	062033	121	937967	6		24
	40	55	697874	366	302126	759979	487	240021	062105	121	937895	5		20
	44	56	698094	365	301906	760272	487	239728	062178	121	937822	4		16
	48	57	698313	365	301687	760564	487	239436	062251	121	937749	3		12
	52	58	698532	365	301468	760856	486	239144	062324	121	937676	2		8
	56	59	698751	365	301249	761148	486	238852	062396	121	937604	1		4
60	0	60	698970	364	301030	761439	486	238561	062469	121	937531	0	0	0
m.	s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	'	m.	s.
			4 Hours,			or			60 Degrees.					

P. P. to s or "	1s	15"	56	1s	15"	74	1s	15"	18	P. P. to s or "
	2	30	111	2	30	147	2	30	36	
	3	45	177	3	45	221	3	45	54	

2 Hours,							or				30 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
0	0	0	9.698970	364	10.301030	9.761439	486	10.238561	10.062469	121	9.937531	60	60	0
	4	1	699189	364	300811	761731	486	238269	062542	122	937458	59		56
	8	2	699407	364	300593	762023	486	237977	062615	122	937385	58		52
	12	3	699626	364	300374	762314	486	237686	062688	122	937312	57		48
	16	4	699844	363	300156	762606	485	237394	062762	122	937238	56		44
	20	5	700062	363	299938	762897	485	237103	062835	122	937165	55		40
	24	6	700280	363	299720	763188	485	236812	062908	122	937092	54		36
	28	7	700498	363	299502	763479	485	236521	062981	122	937019	53		32
	32	8	700716	363	299284	763770	485	236230	063054	122	936946	52		28
	36	9	700933	362	299067	764061	485	235939	063128	122	936872	51		24
	40	10	701151	362	298849	764352	484	235648	063201	122	936799	50		20
	44	11	701368	362	298632	764643	484	235357	063275	122	936725	49		16
	48	12	701585	362	298415	764933	484	235067	063348	123	936652	48		12
	52	13	701802	361	298198	765224	484	234776	063422	123	936578	47		8
	56	14	702019	361	297981	765514	484	234486	063495	123	936505	46		4
1	0	15	9.702236	361	10.297764	9.765805	484	10.234195	10.063569	123	9.936431	45	59	0
	4	16	702452	361	297548	766095	484	233905	063643	123	936357	44		56
	8	17	702669	360	297331	766385	483	233615	063716	123	936284	43		52
	12	18	702885	360	297115	766675	483	233325	063790	123	936210	42		48
	16	19	703101	360	296899	766965	483	233035	063864	123	936136	41		44
	20	20	703317	360	296683	767255	483	232745	063938	123	936062	40		40
	24	21	703533	359	296467	767545	483	232455	064012	123	935988	39		36
	28	22	703749	359	296251	767834	483	232166	064086	123	935914	38		32
	32	23	703964	359	296036	768124	482	231876	064160	123	935840	37		28
	36	24	704179	359	295821	768414	482	231586	064234	124	935766	36		24
	40	25	704395	359	295605	768703	482	231297	064308	124	935692	35		20
	44	26	704610	358	295390	768992	482	231008	064382	124	935618	34		16
	48	27	704825	358	295175	769281	482	230719	064457	124	935543	33		12
	52	28	705040	358	294960	769570	482	230430	064531	124	935469	32		8
	56	29	705254	358	294746	769860	481	230140	064605	124	935395	31		4
2	0	30	9.705469	357	10.294531	9.770148	481	10.229852	10.064680	124	9.935320	30	58	0
	4	31	705683	357	294317	770437	481	229563	064754	124	935246	29		56
	8	32	705898	357	294102	770726	481	229274	064829	124	935171	28		52
	12	33	706112	357	293888	771015	481	228985	064903	124	935097	27		48
	16	34	706326	356	293674	771303	481	228697	064978	124	935022	26		44
	20	35	706539	356	293461	771592	481	228408	065052	124	934948	25		40
	24	36	706753	356	293247	771880	480	228120	065127	124	934873	24		36
	28	37	706967	356	293033	772168	480	227832	065202	125	934798	23		32
	32	38	707180	355	292820	772457	480	227543	065277	125	934723	22		28
	36	39	707393	355	292607	772745	480	227255	065351	125	934649	21		24
	40	40	707606	355	292394	773033	480	226967	065426	125	934574	20		20
	44	41	707819	355	292181	773321	480	226679	065501	125	934499	19		16
	48	42	708032	354	291968	773608	479	226392	065576	125	934424	18		12
	52	43	708245	354	291755	773896	479	226104	065651	125	934349	17		8
	56	44	708458	354	291542	774184	479	225816	065726	125	934274	16		4
3	0	45	9.708670	354	10.291330	9.774471	479	10.225529	10.065801	125	9.934199	15	57	0
	4	46	708882	353	291118	774759	479	225241	065877	125	934123	14		56
	8	47	709094	353	290906	775046	479	224954	065952	125	934048	13		52
	12	48	709306	353	290694	775333	479	224667	066027	125	933973	12		48
	16	49	709518	353	290482	775621	478	224379	066102	126	933898	11		44
	20	50	709730	353	290270	775908	478	224092	066178	126	933822	10		40
	24	51	709941	352	290059	776195	478	223805	066253	126	933747	9		36
	28	52	710153	352	289847	776482	478	223518	066329	126	933671	8		32
	32	53	710364	352	289636	776769	478	223231	066404	126	933596	7		28
	36	54	710575	352	289425	777055	478	222945	066480	126	933520	6		24
	40	55	710786	351	289214	777342	478	222658	066555	126	933445	5		20
	44	56	710997	351	289003	777628	477	222372	066631	126	933369	4		16
	48	57	711208	351	288792	777915	477	222085	066707	126	933293	3		12
	52	58	711419	351	288581	778201	477	221799	066783	126	933217	2		8
	56	59	711629	350	288371	778487	477	221513	066859	126	933141	1		4
4	0	60	711839	350	288161	778774	477	221226	066934	126	933066	0	56	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
3 Hours,							or				59 Degrees.			

P. P. to s or ″	1^s	15″	54	1^s	15″	72	1^s	15″	19	P. P. to s or ″
	2	30	107	2	30	144	2	30	37	
	3	45	161	3	45	217	3	45	56	

m.	s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	'	m.	s.
			2 Hours,				or				**31 Degrees.**			
4	0	0	9.711839	350	10.288161	9.778774	477	10.221226	10.066934	126	9.933066	60	56	0
	4	1	712050	350	287950	779060	477	220940	067010	127	932990	59		56
	8	2	712260	350	287740	779346	476	220654	067086	127	932914	58		52
	12	3	712469	349	287531	779632	476	220368	067162	127	932838	57		48
	16	4	712679	349	287321	779918	476	220082	067238	127	932762	56		44
	20	5	712889	349	287111	780203	476	219797	067315	127	932685	55		40
	24	6	713098	349	286902	780489	476	219511	067391	127	932609	54		36
	28	7	713308	349	286692	780775	476	219225	067467	127	932533	53		32
	32	8	713517	348	286483	781060	476	218940	067543	127	932457	52		28
	36	9	713726	348	286274	781346	475	218654	067620	127	932380	51		24
	40	10	713935	348	286065	781631	475	218369	067696	127	932304	50		20
	44	11	714144	348	285856	781916	475	218084	067772	127	932228	49		16
	48	12	714352	347	285648	782201	475	217799	067849	127	932151	48		12
	52	13	714561	347	285439	782486	475	217514	067925	128	932075	47		8
	56	14	714769	347	285231	782771	475	217229	068002	128	931998	46		4
5	0	15	9.714978	347	10.285022	9.783056	475	10.216944	10.068079	128	9.931921	45	55	0
	4	16	715186	347	284814	783341	475	216659	068155	128	931845	44		56
	8	17	715394	346	284606	783626	474	216374	068232	128	931768	43		52
	12	18	715602	346	284398	783910	474	216090	068309	128	931691	42		48
	16	19	715809	346	284191	784195	474	215805	068386	128	931614	41		44
	20	20	716017	346	283983	784479	474	215521	068463	128	931537	40		40
	24	21	716224	345	283776	784764	474	215236	068540	128	931460	39		36
	28	22	716432	345	283568	785048	474	214952	068617	128	931383	38		32
	32	23	716639	345	283361	785332	473	214668	068694	128	931306	37		28
	36	24	716846	345	283154	785616	473	214384	068771	129	931229	36		24
	40	25	717053	345	282947	785900	473	214100	068848	129	931152	35		20
	44	26	717259	344	282741	786184	473	213816	068925	129	931075	34		16
	48	27	717466	344	282534	786468	473	213532	069002	129	930998	33		12
	52	28	717673	344	282327	786752	473	213248	069079	129	930921	32		8
	56	29	717879	344	282121	787036	473	212964	069157	129	930843	31		4
6	0	30	9.718085	343	10.281915	9.787319	472	10.212681	10.069234	129	9.930766	30	54	0
	4	31	718291	343	281709	787603	472	212397	069312	129	930688	29		56
	8	32	718497	343	281503	787886	472	212114	069389	129	930611	28		52
	12	33	718703	343	281297	788170	472	211830	069467	129	930533	27		48
	16	34	718909	343	281091	788453	472	211547	069544	129	930456	26		44
	20	35	719114	342	280886	788736	472	211264	069622	129	930378	25		40
	24	36	719320	342	280680	789019	472	210981	069700	130	930300	24		36
	28	37	719525	342	280475	789302	471	210698	069777	130	930223	23		32
	32	38	719730	342	280270	789585	471	210415	069855	130	930145	22		28
	36	39	719935	341	280065	789868	471	210132	069933	130	930067	21		24
	40	40	720140	341	279860	790151	471	209849	070011	130	929989	20		20
	44	41	720345	341	279655	790433	471	209567	070089	130	929911	19		16
	48	42	720549	341	279451	790716	471	209284	070167	130	929833	18		12
	52	43	720754	340	279246	790999	471	209001	070245	130	929755	17		8
	56	44	720958	340	279042	791281	471	208719	070323	130	929677	16		4
7	0	45	9.721162	340	10.278838	9.791563	470	10.208437	10.070401	130	9.929599	15	53	0
	4	46	721366	340	278634	791846	470	208154	070479	130	929521	14		56
	8	47	721570	340	278430	792128	470	207872	070558	130	929442	13		52
	12	48	721774	339	278226	792410	470	207590	070636	131	929364	12		48
	16	49	721978	339	278022	792692	470	207308	070714	131	929286	11		44
	20	50	722181	339	277819	792974	470	207026	070793	131	929207	10		40
	24	51	722385	339	277615	793256	470	206744	070871	131	929129	9		36
	28	52	722588	339	277412	793538	469	206462	070950	131	929050	8		32
	32	53	722791	338	277209	793819	469	206181	071028	131	928972	7		28
	36	54	722994	338	277006	794101	469	205899	071107	131	928893	6		24
	40	55	723197	338	276803	794383	469	205617	071185	131	928815	5		20
	44	56	723400	338	276600	794664	469	205336	071264	131	928736	4		16
	48	57	723603	337	276397	794945	469	205055	071343	131	928657	3		12
	52	58	723805	337	276195	795227	469	204773	071422	131	928578	2		8
	56	59	724007	337	275993	795508	468	204492	071501	131	928499	1		4
8	0	60	724210	337	275790	795789	468	204211	071580	131	928420	0	52	0
m.	s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	'	m.	s.
			3 Hours,				or				**58 Degrees.**			

P. P. to s or "	1s	15"	51	1s	15"	71	1s	15"	19	P. P. to s or "
	2	30	103	2	30	142	2	30	39	
	3	45	154	3	45	212	3	45	58	

2 Hours, or 32 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
8	0	0	9.724210	337	10.275790	9.795789	468	10.204211	10.071580	132	9.928420	60	52	0
	4	1	724412	337	275588	796070	468	203930	071658	132	928342	59		56
	8	2	724614	336	275386	796351	468	203649	071737	132	928263	58		52
	12	3	724816	336	275184	796632	468	203368	071817	132	928183	57		48
	16	4	725017	336	274983	796913	468	203087	071896	132	928104	56		44
	20	5	725219	336	274781	797194	468	202806	071975	132	928025	55		40
	24	6	725420	335	274580	797475	468	202525	072054	132	927946	54		36
	28	7	725622	335	274378	797755	468	202245	072133	132	927867	53		32
	32	8	725823	335	274177	798036	467	201964	072213	132	927787	52		28
	36	9	726024	335	273976	798316	467	201684	072292	132	927708	51		24
	40	10	726225	335	273775	798596	467	201404	072371	132	927629	50		20
	44	11	726426	334	273574	798877	467	201123	072451	132	927549	49		16
	48	12	726626	334	273374	799157	467	200843	072530	133	927470	48		12
	52	13	726827	334	273173	799437	467	200563	072610	133	927390	47		8
	56	14	727027	334	272973	799717	467	200283	072690	133	927310	46		4
9	0	15	9.727228	334	10.272772	9.799997	466	10.200003	10.072769	133	9.927231	45	51	0
	4	16	727428	333	272572	800277	466	199723	072849	133	927151	44		56
	8	17	727628	333	272372	800557	466	199443	072929	133	927071	43		52
	12	18	727828	333	272172	800836	466	199164	073009	133	926991	42		48
	16	19	728027	333	271973	801116	466	198884	073089	133	926911	41		44
	20	20	728227	333	271773	801396	466	198604	073169	133	926831	40		40
	24	21	728427	332	271573	801675	466	198325	073249	133	926751	39		36
	28	22	728626	332	271374	801955	466	198045	073329	133	926671	38		32
	32	23	728825	332	271175	802234	465	197766	073409	133	926591	37		28
	36	24	729024	332	270976	802513	465	197487	073489	134	926511	36		24
	40	25	729223	331	270777	802792	465	197208	073569	134	926431	35		20
	44	26	729422	331	270578	803072	465	196928	073649	134	926351	34		16
	48	27	729621	331	270379	803351	465	196649	073730	134	926270	33		12
	52	28	729820	331	270180	803630	465	196370	073810	134	926190	32		8
	56	29	730018	330	269982	803908	465	196092	073890	134	926110	31		4
10	0	30	9.730217	330	10.269783	9.804187	465	10.195813	10.073971	134	9.926029	30	50	0
	4	31	730415	330	269585	804466	464	195534	074051	134	925949	29		56
	8	32	730613	330	269387	804745	464	195255	074132	134	925868	28		52
	12	33	730811	330	269189	805023	464	194977	074212	134	925788	27		48
	16	34	731009	329	268991	805302	464	194698	074293	134	925707	26		44
	20	35	731206	329	268794	805580	464	194420	074374	134	925626	25		40
	24	36	731404	329	268596	805859	464	194141	074455	135	925545	24		36
	28	37	731602	329	268398	806137	464	193863	074535	135	925465	23		32
	32	38	731799	329	268201	806415	463	193585	074616	135	925384	22		28
	36	39	731996	328	268004	806693	463	193307	074697	135	925303	21		24
	40	40	732193	328	267807	806971	463	193029	074778	135	925222	20		20
	44	41	732390	328	267610	807249	463	192751	074859	135	925141	19		16
	48	42	732587	328	267413	807527	463	192473	074940	135	925060	18		12
	52	43	732784	328	267216	807805	463	192195	075021	135	924979	17		8
	56	44	732980	327	267020	808083	463	191917	075103	135	924897	16		4
11	0	45	9.733177	327	10.266823	9.808361	463	10.191639	10.075184	135	9.924816	15	49	0
	4	46	733373	327	266627	808638	462	191362	075265	136	9.4735	14		56
	8	47	733569	327	266431	808916	462	191084	075346	136	924654	13		52
	12	48	733765	327	266235	809193	462	190807	075428	136	924572	12		48
	16	49	733961	326	266039	809471	462	190529	075509	136	924491	11		44
	20	50	734157	326	265843	809748	462	190252	075591	136	924409	10		40
	24	51	734353	326	265647	810025	462	189975	075672	136	924328	9		36
	28	52	734549	326	265451	810302	462	189698	075754	136	924246	8		32
	32	53	734744	325	265256	810580	462	189420	075836	136	924164	7		28
	36	54	734939	325	265061	810857	462	189143	075917	136	924083	6		24
	40	55	735135	325	264865	811134	461	188866	075999	136	924001	5		20
	44	56	735330	325	264670	811410	461	188590	076081	136	923919	4		16
	48	57	735525	325	264475	811687	461	188313	076163	136	923837	3		12
	52	58	735719	324	264281	811964	461	188036	076245	137	923755	2		8
	56	59	735914	324	264086	812241	461	187759	076327	137	923673	1		4
12	0	60	736109	324	263891	812517	461	187483	076409	137	923591	0	48	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

3 Hours, or 57 Degrees.

P. P. to s or ″	1s	15″	49	1s	15″	70	1s	15″	20	P. P. to s or ″
	2	30	99	2	30	140	2	30	40	
	3	45	148	3	45	209	3	45	60	

2 Hours, or 33 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
12	0	0	9.736109	324	10.263891	9.812517	461	10.187483	10.076409	137	9.923591	60	48	0
	4	1	736303	324	263697	812794	461	187206	076491	137	923509	59		56
	8	2	736498	324	263502	813070	461	186930	076573	137	923427	58		52
	12	3	736692	323	263308	813347	460	186653	076655	137	923345	57		48
	16	4	736886	323	263114	813623	460	186377	076737	137	923263	56		44
	20	5	737080	323	262920	813899	460	186101	076819	137	923181	55		40
	24	6	737274	323	262726	814175	460	185825	076902	137	923098	54		36
	28	7	737467	323	262533	814452	460	185548	076984	137	923016	53		32
	32	8	737661	322	262339	814728	460	185272	077067	137	922933	52		28
	36	9	737855	322	262145	815004	460	184996	077149	137	922851	51		24
	40	10	738048	322	261952	815279	460	184721	077232	138	922768	50		20
	44	11	738241	322	261759	815555	459	184445	077314	138	922686	49		16
	48	12	738434	322	261566	815831	459	184169	077397	138	922603	48		12
	52	13	738627	321	261373	816107	459	183893	077480	138	922520	47		8
	56	14	738820	321	261180	816382	459	183618	077562	138	922438	46		4
13	0	15	9.739013	321	10.260987	9.816658	459	10.183342	10.077645	138	9.922355	45	47	0
	4	16	739206	321	260794	816933	459	183067	077728	138	922272	44		56
	8	17	739398	321	260602	817209	459	182791	077811	138	922189	43		52
	12	18	739590	320	26041[illegible]	817484	459	182516	077894	138	922106	42		48
	16	19	739783	320	260217	817759	459	182241	077977	138	922023	41		44
	20	20	739975	320	260025	818035	458	181965	078060	138	921940	40		40
	24	21	740167	320	259833	818310	458	181690	078143	139	921857	39		36
	28	22	740359	320	259641	818585	458	181415	078226	139	921774	38		32
	32	23	740550	319	259450	818860	458	181140	078309	139	921691	37		28
	36	24	740742	319	259258	819135	458	180865	078393	139	921607	36		24
	40	25	740934	319	259066	819410	458	180590	078476	139	921524	35		20
	44	26	741125	319	258875	819684	458	180316	078559	139	921441	34		16
	48	27	741316	319	258684	819959	458	180041	078643	139	921357	33		12
	52	28	741508	318	258492	820234	458	179766	078726	139	921274	32		8
	56	29	741699	318	258301	820508	457	179492	078810	139	921190	31		4
14	0	30	9.741889	318	10.258111	9.820783	457	10.179217	10.078893	139	9.921107	30	46	0
	4	31	742080	318	257920	821057	457	178943	078977	139	921023	29		56
	8	32	742271	318	257729	821332	457	178668	079061	140	920939	28		52
	12	33	742462	317	257538	821606	457	178394	079144	140	920856	27		48
	16	34	742652	317	257348	821880	457	178120	079228	140	920772	26		44
	20	35	742842	317	257158	822154	457	177846	079312	140	920688	25		40
	24	36	743033	317	256967	822429	457	177571	079396	140	920604	24		36
	28	37	743223	317	256777	822703	457	177297	079480	140	920520	23		32
	32	38	743413	316	256587	822977	456	177023	079564	140	920436	22		28
	36	39	743602	316	256398	823250	456	176750	079648	140	920352	21		24
	40	40	743792	316	256208	823524	456	176476	079732	140	920268	20		20
	44	41	743982	316	256018	823798	456	176202	079816	140	920184	19		16
	48	42	744171	316	255829	824072	456	175928	079901	140	920099	18		12
	52	43	744361	315	255639	824345	456	175655	079985	140	920015	17		8
	56	44	744550	315	255450	824619	456	175381	080069	141	919931	16		4
15	0	45	9.744739	315	10.255261	9.824893	456	10.175107	10.080154	141	9.919846	15	45	0
	4	46	744928	315	255072	825166	456	174834	080238	141	919762	14		56
	8	47	745117	315	254883	825439	455	174561	080323	141	919677	13		52
	12	48	745306	314	254694	825713	455	174287	080407	141	919593	12		48
	16	49	745494	314	254506	825986	455	174014	080492	141	919508	11		44
	20	50	745683	314	254317	826259	455	173741	080576	141	919424	10		40
	24	51	745871	314	254129	826532	455	173468	080661	141	919339	9		36
	28	52	746060	314	253940	826805	455	173195	080746	141	919254	8		32
	32	53	746248	313	253752	827078	455	172922	080831	141	919169	7		28
	36	54	746436	313	253564	827351	455	172649	080915	141	919085	6		24
	40	55	746624	313	253376	827624	455	172376	081000	141	919000	5		20
	44	56	746812	313	253188	827897	454	172103	081085	142	918915	4		16
	48	57	746999	313	253001	828170	454	171830	081170	142	918830	3		12
	52	58	747187	312	252813	828442	454	171558	081255	142	918745	2		8
	56	59	747374	312	252626	828715	454	171285	081341	142	918659	1		4
16	0	60	747562	312	252438	828987	454	171013	081426	142	918574	0	44	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

3 Hours, or 56 Degrees.

P. P. to s or ″	1s	15″	48	1s	15″	69	1s	15″	21	P. P. to s or ″
	2	30	95	2	30	137	2	30	42	
	3	45	143	3	45	206	3	45	63	

2 Hours,							or				34 Degrees.		
m. s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	'	m. s.
16 0	0	9.747562	312	10.252438	9.828987	454	10.171013	10.081426	142	9.918574	60	44 0
4	1	747749	312	252251	829260	454	170740	081511	142	918489	59	56
8	2	747936	312	252064	829532	454	170468	081596	142	918404	58	52
12	3	748123	311	251877	829805	454	170195	081682	142	918318	57	48
16	4	748310	311	251690	830077	454	169923	081767	142	918233	56	44
20	5	748497	311	251503	830349	453	169651	081853	142	918147	55	40
24	6	748683	311	251317	830621	453	169379	081938	142	918062	54	36
28	7	748870	311	251130	830893	453	169107	082024	143	917976	53	32
32	8	749056	310	250944	831165	453	168835	082109	143	917891	52	28
36	9	749243	310	250757	831437	453	168563	082195	143	917805	51	24
40	10	749429	310	250571	831709	453	168291	082281	143	917719	50	20
44	11	749615	310	250385	831981	453	168019	082366	143	917634	49	16
48	12	749801	310	250199	832253	453	167747	082452	143	917548	48	12
52	13	749987	309	250013	832525	453	167475	082538	143	917462	47	8
56	14	750172	309	249828	832796	453	167204	082624	143	917376	46	4
17 0	15	9.750358	309	10.249642	9.833068	452	10.166932	10.082710	143	9.917290	45	43 0
4	16	750543	309	249457	833339	452	166661	082796	143	917204	44	56
8	17	750729	309	249271	833611	452	166389	082882	144	917118	43	52
12	18	750914	308	249086	833882	452	166118	082968	144	917032	42	48
16	19	751099	308	248901	834154	452	165846	083054	144	916946	41	44
20	20	751284	308	248716	834425	452	165575	083141	144	916859	40	40
24	21	751469	308	248531	834696	452	165304	083227	144	916773	39	36
28	22	751654	308	248346	834967	452	165033	083313	144	916687	38	32
32	23	751839	308	248161	835238	452	164762	083400	144	916600	37	28
36	24	752023	307	247977	835509	452	164491	083486	144	916514	36	24
40	25	752208	307	247792	835780	451	164220	083573	144	916427	35	20
44	26	752392	307	247608	836051	451	163949	083659	144	916341	34	16
48	27	752576	307	247424	836322	451	163678	083746	144	916254	33	12
52	28	752760	307	247240	836593	451	163407	083833	145	916167	32	8
56	29	752944	306	247056	836864	451	163136	083919	145	916081	31	4
18 0	30	9.753128	306	10.246872	9.837134	451	10.162866	10.084006	145	9.915994	30	42 0
4	31	753312	306	246688	837405	451	162595	084093	145	915907	29	56
8	32	753495	306	246505	837675	451	162325	084180	145	915820	28	52
12	33	753679	306	246321	837946	451	162054	084267	145	915733	27	48
16	34	753862	305	246138	838216	451	161784	084354	145	915646	26	44
20	35	754046	305	245954	838487	450	161513	084441	145	915559	25	40
24	36	754229	305	245771	838757	450	161243	084528	145	915472	24	36
28	37	754412	305	245588	839027	450	160973	084615	145	915385	23	32
32	38	754595	305	245405	839297	450	160703	084703	145	915297	22	28
36	39	754778	304	245222	839568	450	160432	084790	145	915210	21	24
40	40	754960	304	245040	839838	450	160162	084877	146	915123	20	20
44	41	755143	304	244857	840108	450	159892	084965	146	915035	19	16
48	42	755326	304	244674	840378	450	159622	085052	146	914948	18	12
52	43	755508	304	244492	840647	450	159353	085140	146	914860	17	8
56	44	755690	304	244310	840917	449	159083	085227	146	914773	16	4
19 0	45	9.755872	303	10.244128	9.841187	449	10.158813	10.085315	146	9.914685	15	41 0
4	46	756054	303	243946	841457	449	158543	085402	146	914598	14	56
8	47	756236	303	243764	841726	449	158274	085490	146	914510	13	52
12	48	756418	303	243582	841996	449	158004	085578	146	914422	12	48
16	49	756600	303	243400	842266	449	157734	085666	146	914334	11	44
20	50	756782	302	243218	842535	449	157465	085754	147	914246	10	40
24	51	756963	302	243037	842805	449	157195	085842	147	914158	9	36
28	52	757144	302	242856	843074	449	156926	085930	147	914070	8	32
32	53	757326	302	242674	843343	449	156657	086018	147	913982	7	28
36	54	757507	302	242493	843612	449	156388	086106	147	913894	6	24
40	55	757688	301	242312	843882	448	156118	086194	147	913806	5	20
44	56	757869	301	242131	844151	448	155849	086282	147	913718	4	16
48	57	758050	301	241950	844420	448	155580	086370	147	913630	3	12
52	58	758230	301	241770	844689	448	155311	086459	147	913541	2	8
56	59	758411	301	241589	844958	448	155042	086547	147	913453	1	4
20 0	60	758591	301	241409	845227	448	154773	086635	147	913365	0	40 0
m. s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	'	m. s.
3 Hours,							or				55 Degrees.		

P. P. to s or "	1s	15"	46	1s	15"	68	1s	15"	22	P. P. to s or "
	2	30	92	2	30	135	2	30	43	
	3	45	138	3	45	203	3	45	65	

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
2 Hours,							or			35 Degrees.				
20	0	0	9.758591	301	10.241409	9.845227	448	10.154773	10.086635	147	9.913365	60	40	0
	4	1	758772	300	241228	845496	448	154504	086724	147	913276	59		56
	8	2	758952	300	241048	845764	448	154236	086813	148	913187	58		52
	12	3	759132	300	240868	846033	448	153967	086901	148	913099	57		48
	16	4	759312	300	240688	846302	448	153698	086990	148	913010	56		44
	20	5	759492	300	240508	846570	447	153430	087078	148	912922	55		40
	24	6	759672	299	240328	846839	447	153161	087167	148	912833	54		36
	28	7	759852	299	240148	847107	447	152893	087256	148	912744	53		32
	32	8	760031	299	239969	847376	447	152624	087345	148	912655	52		28
	36	9	760211	299	239789	847644	447	152356	087434	148	912566	51		24
	40	10	760390	299	239610	847913	447	152087	087523	148	912477	50		20
	44	11	760569	298	239431	848181	447	151819	087612	148	912388	49		16
	48	12	760748	298	239252	848449	447	151551	087701	149	912299	48		12
	52	13	760927	298	239073	848717	447	151283	087790	149	912210	47		8
	56	14	761106	298	238894	848986	447	151014	087879	149	912121	46		4
21	0	15	9.761285	298	10.238715	9.849254	447	10.150746	10.087969	149	9.912031	45	39	0
	4	16	761464	298	238536	849522	447	150478	088058	149	911942	44		56
	8	17	761642	297	238358	849790	446	150210	088147	149	911853	43		52
	12	18	761821	297	238179	850058	446	149942	088237	149	911763	42		48
	16	19	761999	297	238001	850325	446	149675	088326	149	911674	41		44
	20	20	762177	297	237823	850593	446	149407	088416	149	911584	40		40
	24	21	762356	297	237644	850861	446	149139	088505	149	911495	39		36
	28	22	762534	296	237466	851129	446	148871	088595	149	911405	38		32
	32	23	762712	296	237288	851396	446	148604	088685	150	911315	37		28
	36	24	762889	296	237111	851664	446	148336	088774	150	911226	36		24
	40	25	763067	296	236933	851931	446	148069	088864	150	911136	35		20
	44	26	763245	296	236755	852199	446	147801	088954	150	911046	34		16
	48	27	763422	296	236578	852466	446	147534	089044	150	910956	33		12
	52	28	763600	295	236400	852733	445	147267	089134	150	910866	32		8
	56	29	763777	295	236223	853001	445	146999	089224	150	910776	31		4
22	0	30	9.763954	295	10.236046	9.853268	445	10.146732	10.089314	150	9.910686	30	38	0
	4	31	764131	295	235869	853535	445	146465	089404	150	910596	29		56
	8	32	764308	295	235692	853802	445	146198	089494	150	910506	28		52
	12	33	764485	294	235515	854069	445	145931	089585	150	910415	27		48
	16	34	764662	294	235338	854336	445	145664	089675	151	910325	26		44
	20	35	764838	294	235162	854603	445	145397	089765	151	910235	25		40
	24	36	765015	294	234985	854870	445	145130	089856	151	910144	24		36
	28	37	765191	294	234809	855137	445	144863	089946	151	910054	23		32
	32	38	765367	294	234633	855404	445	144596	090037	151	909963	22		28
	36	39	765544	293	234456	855671	444	144329	090127	151	909873	21		24
	40	40	765720	293	234280	855938	444	144062	090218	151	909782	20		20
	44	41	765896	293	234104	856204	444	143796	090309	151	909691	19		16
	48	42	766072	293	233928	856471	444	143529	090399	151	909601	18		12
	52	43	766247	293	233753	856737	444	143263	090490	151	909510	17		8
	56	44	766423	293	233577	857004	444	142996	090581	151	909419	16		4
23	0	45	9.766598	292	10.233402	9.857270	444	10.142730	10.090672	152	9.909328	15	37	0
	4	46	766774	292	233226	857537	444	142463	090763	152	909237	14		56
	8	47	766949	292	233051	857803	444	142197	090854	152	909146	13		52
	12	48	767124	292	232876	858069	444	141931	090945	152	909055	12		48
	16	49	767300	292	232700	858336	444	141664	091036	152	908964	11		44
	20	50	767475	291	232525	858602	443	141398	091127	152	908873	10		40
	24	51	767649	291	232351	858868	443	141132	091219	152	908781	9		36
	28	52	767824	291	232176	859134	443	140866	091310	152	908690	8		32
	32	53	767999	291	232001	859400	443	140600	091401	152	908599	7		28
	36	54	768173	291	231827	859666	443	140334	091493	152	908507	6		24
	40	55	768348	290	231652	859932	443	140068	091584	153	908416	5		20
	44	56	768522	290	231478	860198	443	139802	091676	153	908324	4		16
	48	57	768697	290	231303	860464	443	139536	091767	153	908233	3		12
	52	58	768871	290	231129	860730	443	139270	091859	153	908141	2		8
	56	59	769045	290	230955	860995	443	139005	091951	153	908049	1		4
24	0	60	769219	290	230781	861261	443	138739	092042	153	907958	0	36	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
3 Hours,							or			54 Degrees.				

P. P. to s or ″	1s	15″	44	1s	15″	67	1s	15″	22	P. P. to s or ″
	2	30	88	2	30	133	2	30	45	
	3	45	133	3	45	200	3	45	67	

Table V. Logarithmic Sines, Tangents,

2 Hours, or 36 Degrees.

m.	s	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
24	0	0	9.769219	290	10.230781	9.861261	443	10.138739	10.092042	153	9.907958	60	36	0
	4	1	769393	289	230607	861527	443	138473	092134	153	907866	59		56
	8	2	769566	289	230434	861792	442	138208	092226	153	907774	58		52
	12	3	769740	289	230260	862058	442	137942	092318	153	907682	57		48
	16	4	769913	289	230087	862323	442	137677	092410	153	907590	56		44
	20	5	770087	289	229913	862589	442	137411	092502	153	907498	55		40
	24	6	770260	288	229740	862854	442	137146	092594	153	907406	54		36
	28	7	770433	288	229567	863119	442	136881	092686	154	907314	53		32
	32	8	770606	288	229394	863385	442	136615	092778	154	907222	52		28
	36	9	770779	288	229221	863650	442	136350	092871	154	907129	51		24
	40	10	770952	288	229048	863915	442	136085	092963	154	907037	50		20
	44	11	771125	288	228875	864180	442	135820	093055	154	906945	49		16
	48	12	771298	287	228702	864445	442	135555	093148	154	906852	48		12
	52	13	771470	287	228530	864710	442	135290	093240	154	906760	47		8
	56	14	771643	287	228357	864975	441	135025	093333	154	906667	46		4
25	0	15	9.771815	287	10.228185	9.865240	441	10.134760	10.093425	154	9.906575	45	35	0
	4	16	771987	287	228013	865505	441	134495	093518	154	906482	44		56
	8	17	772159	287	227841	865770	441	134230	093611	155	906389	43		52
	12	18	772331	286	227669	866035	441	133965	093704	155	906296	42		48
	16	19	772503	286	227497	866300	441	133700	093796	155	906204	41		44
	20	20	772675	286	227325	866564	441	133436	093889	155	906111	40		40
	24	21	772847	286	227153	866829	441	133171	093982	155	906018	39		36
	28	22	773018	286	226982	867094	441	132906	094075	155	905925	38		32
	32	23	773190	286	226810	867358	441	132642	094168	155	905832	37		28
	36	24	773361	285	226639	867623	441	132377	094261	155	905739	36		24
	40	25	773533	285	226467	867887	441	132113	094355	155	905645	35		20
	44	26	773704	285	226296	868152	440	131848	094448	155	905552	34		16
	48	27	773875	285	226125	868416	440	131584	094541	155	905459	33		12
	52	28	774046	285	225954	868680	440	131320	094634	156	905366	32		8
	56	29	774217	285	225783	868945	440	131055	094728	156	905272	31		4
26	0	30	9.774388	284	10.225612	9.869209	440	10.130791	10.094821	156	9.905179	30	34	0
	4	31	774558	284	225442	869473	440	130527	094915	156	905085	29		56
	8	32	774729	284	225271	869737	440	130263	095008	156	904992	28		52
	12	33	774899	284	225101	870001	440	129999	095102	156	904898	27		48
	16	34	775070	284	224930	870265	440	129735	095196	156	904804	26		44
	20	35	775240	284	224760	870529	440	129471	095289	156	904711	25		40
	24	36	775410	283	224590	870793	440	129207	095383	156	904617	24		36
	28	37	775580	283	224420	871057	440	128943	095477	156	904523	23		32
	32	38	775750	283	224250	871321	440	128679	095571	157	904429	22		28
	36	39	775920	283	224080	871585	440	128415	095665	157	904335	21		24
	40	40	776090	283	223910	871849	439	128151	095759	157	904241	20		20
	44	41	776259	283	223741	872112	439	127888	095853	157	904147	19		16
	48	42	776429	282	223571	872376	439	127624	095947	157	904053	18		12
	52	43	776598	282	223402	872640	439	127360	096041	157	903959	17		8
	56	44	776768	282	223232	872903	439	127097	096136	157	903864	16		4
27	0	45	9.776937	282	10.223063	9.873167	439	10.126833	10.096230	157	9.903770	15	33	0
	4	46	777106	282	222894	873430	439	126570	096324	157	903676	14		56
	8	47	777275	281	222725	873694	439	126306	096419	157	903581	13		52
	12	48	777444	281	222556	873957	439	126043	096513	157	903487	12		48
	16	49	777613	281	222387	874220	439	125780	096608	158	903392	11		44
	20	50	777781	281	222219	874484	439	125516	096702	158	903298	10		40
	24	51	777950	281	222050	874747	439	125253	096797	158	903203	9		36
	28	52	778119	281	221881	875010	439	124990	096892	158	903108	8		32
	32	53	778287	280	221713	875273	438	124727	096986	158	903014	7		28
	36	54	778455	280	221545	875536	438	124464	097081	158	902919	6		24
	40	55	778624	280	221376	875800	438	124200	097176	158	902824	5		20
	44	56	778792	280	221208	876063	438	123937	097271	158	902729	4		16
	48	57	778960	280	221040	876326	438	123674	097366	158	902634	3		12
	52	58	779128	280	220872	876589	438	123411	097461	159	902539	2		8
	56	59	779295	279	220705	876851	438	123149	097556	159	902444	1		4
28	0	60	779463	279	220537	877114	438	122886	097651	159	902349	0	32	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

3 Hours, or 53 Degrees.

P. P. to s or ″										
	1ˢ	15″	43	1ˢ	15″	66	1ˢ	15″	23	P. P. to s or ″
	2	30	85	2	30	132	2	30	47	
	3	45	128	3	45	198	3	45	70	

2 Hours, or 37 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
28	0	0	9.779463	279	10.220537	9.877114	438	10.122886	10.097651	159	9.902349	60	32	0
	4	1	779631	279	220369	877377	438	122623	097747	159	902253	59		56
	8	2	779798	279	220202	877640	438	122360	097842	159	902158	58		52
	12	3	779966	279	220034	877903	438	122097	097937	159	902063	57		48
	16	4	780133	279	219867	878165	438	121835	098033	159	901967	56		44
	20	5	780300	278	219700	878428	438	121572	098128	159	901872	55		40
	24	6	780467	278	219533	878691	438	121309	098224	159	901776	54		36
	28	7	780634	278	219366	878953	437	121047	098319	159	901681	53		32
	32	8	780801	278	219199	879216	437	120784	098415	159	901585	52		28
	36	9	780968	278	219032	879478	437	120522	098510	159	901490	51		24
	40	10	781134	278	218866	879741	437	120259	098606	160	901394	50		20
	44	11	781301	277	218699	880003	437	119997	098702	160	901298	49		16
	48	12	781468	277	218532	880265	437	119735	098798	160	901202	48		12
	52	13	781634	277	218366	880528	437	119472	098894	160	901106	47		8
	56	14	781800	277	218200	880790	437	119210	098990	160	901010	46		4
29	0	15	9.781966	277	10.218034	9.881052	437	10.118948	10.099086	160	9.900914	45	31	0
	4	16	782132	277	217868	881314	437	118686	099182	160	900818	44		56
	8	17	782298	276	217702	881576	437	118424	099278	160	900722	43		52
	12	18	782464	276	217536	881839	437	118161	099374	160	900626	42		48
	16	19	782630	276	217370	882101	437	117899	099471	160	900529	41		44
	20	20	782796	276	217204	882363	436	117637	099567	161	900433	40		40
	24	21	782961	276	217039	882625	436	117375	099663	161	900337	39		36
	28	22	783127	276	216873	882887	436	117113	099760	161	900240	38		32
	32	23	783292	275	216708	883148	436	116852	099856	161	900144	37		28
	36	24	783458	275	216542	883410	436	116590	099953	161	900047	36		24
	40	25	783623	275	216377	883672	436	116328	100049	161	899951	35		20
	44	26	783788	275	216212	883934	436	116066	100146	161	899854	34		16
	48	27	783953	275	216047	884196	436	115804	100243	161	899757	33		12
	52	28	784118	275	215882	884457	436	115543	100340	161	899660	32		8
	56	29	784282	274	215718	884719	436	115281	100436	161	899564	31		4
30	0	30	9.784447	274	10.215553	9.884980	436	10.115020	10.100533	162	9.899467	30	30	0
	4	31	784612	274	215388	885242	436	114758	100630	162	899370	29		56
	8	32	784776	274	215224	885503	436	114497	100727	162	899273	28		52
	12	33	784941	274	215059	885765	436	114235	100824	162	899176	27		48
	16	34	785105	274	214895	886026	436	113974	100922	162	899078	26		44
	20	35	785269	273	214731	886288	436	113712	101019	162	898981	25		40
	24	36	785433	273	214567	886549	435	113451	101116	162	898884	24		36
	28	37	785597	273	214403	886810	435	113190	101213	162	898787	23		32
	32	38	785761	273	214239	887072	435	112928	101311	162	898689	22		28
	36	39	785925	273	214075	887333	435	112667	101408	162	898592	21		24
	40	40	786089	273	213911	887594	435	112406	101506	163	898494	20		20
	44	41	786252	272	213748	887855	435	112145	101603	163	898397	19		16
	48	42	786416	272	213584	888116	435	111884	101701	163	898299	18		12
	52	43	786579	272	213421	888377	435	111623	101798	163	898202	17		8
	56	44	786742	272	213258	888639	435	111361	101896	163	898104	16		4
31	0	45	9.786906	272	10.213094	9.888900	435	10.111100	10.101994	163	9.898006	15	29	0
	4	46	787069	272	212931	889160	435	110840	102092	163	897908	14		56
	8	47	787232	271	212768	889421	435	110579	102190	163	897810	13		52
	12	48	787395	271	212605	889682	435	110318	102288	163	897712	12		48
	16	49	787557	271	212443	889943	435	110057	102386	163	897614	11		44
	20	50	787720	271	212280	890204	434	109796	102484	163	897516	10		40
	24	51	787883	271	212117	890465	434	109535	102582	164	897418	9		36
	28	52	788045	271	211955	890725	434	109275	102680	164	897320	8		32
	32	53	788208	271	211792	890986	434	109014	102778	164	897222	7		28
	36	54	788370	270	211630	891247	434	108753	102877	164	897123	6		24
	40	55	788532	270	211468	891507	434	108493	102975	164	897025	5		20
	44	56	788694	270	211306	891768	434	108232	103074	164	896926	4		16
	48	57	788856	270	211144	892028	434	107972	103172	164	896828	3		12
	52	58	789018	270	210982	892289	434	107711	103271	164	896729	2		8
	56	59	789180	270	210820	892549	434	107451	103369	164	896631	1		4
32	0	60	789342	269	210658	892810	434	107190	103468	164	896532	0	28	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

3 Hours, or 52 Degrees.

P. P. to s or ″	1s	15″	41	1s	15″	65	1s	15″	24	P. P. to s or ″
	2	30	82	2	30	131	2	30	49	
	3	45	124	3	45	196	3	45	73	

2 Hours,							or				38 Degrees,			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
32	0	0	9.789342	269	10.210658	9.892810	434	10.107190	10.103468	164	9.896532	60	28	0
	4	1	789504	269	210496	893070	434	106930	103567	165	896433	59		56
	8	2	789665	269	210335	893331	434	106669	103665	165	896335	58		52
	12	3	789827	269	210173	893591	434	106409	103764	165	896236	57		48
	16	4	789988	269	200012	893851	434	106149	103863	165	896137	56		44
	20	5	790149	269	209851	894111	434	105889	103962	165	896038	55		40
	24	6	790310	268	209690	894371	434	105629	104061	165	895939	54		36
	28	7	790471	268	209529	894632	433	105368	104160	165	895840	53		32
	32	8	790632	268	209368	894892	433	105108	104259	165	895741	52		28
	36	9	790793	268	209207	895152	433	104848	104359	165	895641	51		24
	40	10	790954	268	209046	895412	433	104588	104458	165	895542	50		20
	44	11	791115	268	208885	895672	433	104328	104557	166	895443	49		16
	48	12	791275	267	208725	895932	433	104068	104657	166	895343	48		12
	52	13	791436	267	208564	896192	433	103808	104756	166	895244	47		8
	56	14	791596	267	208404	896452	433	103548	104855	166	895145	46		4
33	0	15	9.791757	267	10.208243	9.896712	433	10.103288	10.104955	166	9.895045	45	27	0
	4	16	791917	267	208083	896971	433	103029	105055	166	894945	44		56
	8	17	792077	267	207923	897231	433	102769	105154	166	894846	43		52
	12	18	792237	266	207763	897491	433	102509	105254	166	894746	42		48
	16	19	792397	266	207603	897751	433	102249	105354	166	894646	41		44
	20	20	792557	266	207443	898010	433	101990	105454	166	894546	40		40
	24	21	792716	266	207284	898270	433	101730	105554	167	894446	39		36
	28	22	792876	266	207124	898530	433	101470	105654	167	894346	38		32
	32	23	793035	266	206965	898789	433	101211	105754	167	894246	37		28
	36	24	793195	265	206805	899049	432	100951	105854	167	894146	36		24
	40	25	793354	265	206646	899308	432	100692	105954	167	894046	35		20
	44	26	793514	265	206486	899568	432	100432	106054	167	893946	34		16
	48	27	793673	265	206327	899827	432	100173	106154	167	893846	33		12
	52	28	793832	265	206168	900086	432	099914	106255	167	893745	32		8
	56	29	793991	265	206009	900346	432	099654	106355	167	893645	31		4
34	0	30	9.794150	264	10.205850	9.900605	432	10.099395	10.106456	167	9.893544	30	26	0
	4	31	794308	264	205692	900864	432	099136	106556	168	893444	29		56
	8	32	794467	264	205533	901124	432	098876	106657	168	893343	28		52
	12	33	794626	264	205374	901383	432	098617	106757	168	893243	27		48
	16	34	794784	264	205216	901642	432	098358	106858	168	893142	26		44
	20	35	794942	264	205058	901901	432	098099	106959	168	893041	25		40
	24	36	795101	264	204899	902160	432	097840	107060	168	892940	24		36
	28	37	795259	263	204741	902419	432	097581	107161	168	892839	23		32
	32	38	795417	263	204583	902679	432	097321	107261	168	892739	22		28
	36	39	795575	263	204425	902938	432	097062	107362	168	892638	21		24
	40	40	795733	263	204267	903197	431	096803	107464	168	892536	20		20
	44	41	795891	263	204109	903455	431	096545	107565	169	892435	19		16
	48	42	796049	263	203951	903714	431	096286	107666	169	892334	18		12
	52	43	796206	263	203794	903973	431	096027	107767	169	892233	17		8
	56	44	796364	262	203636	904232	431	095768	107868	169	892132	16		4
35	0	45	9.796521	262	10.203479	9.904491	431	10.095509	10.107970	169	9.892030	15	25	0
	4	46	796679	262	203321	904750	431	095250	108071	169	891929	14		56
	8	47	796836	262	203164	905008	431	094992	108173	169	891827	13		52
	12	48	796993	262	203007	905267	431	094733	108274	169	891726	12		48
	16	49	797150	261	202850	905526	431	094474	108376	169	891624	11		44
	20	50	797307	261	202693	905784	431	094216	108477	170	891523	10		40
	24	51	797464	261	202536	906043	431	093957	108579	170	891421	9		36
	28	52	797621	261	202379	906302	431	093698	108681	170	891319	8		32
	32	53	797777	261	202223	906560	431	093440	108783	170	891217	7		28
	36	54	797934	261	202066	906819	431	093181	108885	170	891115	6		24
	40	55	798091	261	201909	907077	431	092923	108987	170	891013	5		20
	44	56	798247	261	201753	907336	431	092664	109089	170	890911	4		16
	48	57	798403	260	201597	907594	431	092406	109191	170	890809	3		12
	52	58	798560	260	201440	997852	431	092148	109293	170	890707	2		8
	56	59	798716	260	201284	908111	430	091889	109395	170	890605	1		4
36	0	60	798872	260	201128	908369	430	091631	109497	170	890503	0	24	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
3 Hours,							or				51 Degrees.			

P. P. to s or ″										P. P. to s or ″
	1ˢ	15″	40	1ˢ	15″	65	1ˢ	15″	25	
	2	30	79	2	30	130	2	30	50	
	3	45	119	3	45	194	3	45	75	

m.	s	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
			2 Hours,				or			39 Degrees.				
36	0	0	9.798872	260	10.201128	9.908369	430	10.091631	10.109497	170	9.890503	60	24	0
	4	1	799028	260	200972	908628	430	091372	109600	171	890400	59		56
	8	2	799184	260	200816	908886	430	0911[illegible]4	109702	171	890298	58		52
	12	3	799339	259	200661	909144	430	090856	109805	171	890195	57		48
	16	4	799495	259	200505	909402	430	090598	109907	171	890093	56		44
	20	5	799651	259	200349	909660	430	090340	110010	171	889990	55		40
	24	6	799806	259	200194	909918	430	090082	110112	171	889888	54		36
	28	7	799962	259	200038	910177	430	089823	110215	171	889785	53		32
	32	8	800117	259	199883	910435	430	089565	110318	171	889682	52		28
	36	9	800272	258	199728	910693	430	089307	110421	171	889579	51		24
	40	10	800427	258	199573	910951	430	089049	110523	171	889477	50		20
	44	11	800582	258	199418	911209	430	088791	110626	172	889374	49		16
	48	12	800737	258	199263	911467	430	088533	110729	172	889271	48		12
	52	13	800892	258	199108	911724	430	088276	110832	172	889168	47		8
	56	14	801047	258	198953	911982	430	088018	110936	172	889064	46		4
37	0	15	9.801201	258	10.198799	9.912240	430	10.087760	10.111039	172	9.888961	45	23	0
	4	16	801356	257	198644	912498	430	087502	111142	172	888858	44		56
	8	17	801511	257	198489	912756	430	087244	111245	172	888755	43		52
	12	18	801665	257	198335	913014	429	086986	111349	172	888651	42		48
	16	19	801819	257	198181	913271	429	086729	111452	172	888548	41		44
	20	20	801973	257	198027	913529	429	086471	111556	173	888444	40		40
	24	21	802128	257	197872	913787	429	086213	111659	173	888341	39		36
	28	22	802282	256	197718	914044	429	085956	111763	173	888237	38		32
	32	23	802436	256	197564	914302	429	085698	111866	173	888134	37		28
	36	24	802589	256	197411	914560	429	085440	111970	173	888030	36		24
	40	25	802743	256	197257	914817	429	085183	112074	173	887926	35		20
	44	26	802897	256	197103	915075	429	084925	112178	173	887822	34		16
	48	27	803050	256	196950	915332	429	084668	112282	173	887718	33		12
	52	28	803204	256	196796	915590	429	084410	112386	173	887614	32		8
	56	29	803357	255	196643	915847	429	084153	112490	173	887510	31		4
38	0	30	9.803511	255	10.196489	9.916104	429	10.083896	10.112594	174	9.887406	30	22	0
	4	31	803664	255	196336	916362	429	083638	112698	174	887302	29		56
	8	32	803817	255	196183	916619	429	083381	112802	174	887198	28		52
	12	33	803970	255	196030	916877	429	083123	112907	174	887093	27		48
	16	34	804123	255	195877	917134	429	082866	113011	174	886989	26		44
	20	35	804276	254	195724	917391	429	082609	113115	174	886885	25		40
	24	36	804428	254	195572	917648	429	082352	113220	174	886780	24		36
	28	37	804581	254	195419	917905	429	082095	113324	174	886676	23		32
	32	38	804734	254	195266	918163	428	081837	113429	174	886571	22		28
	36	39	804886	254	195114	918420	428	081580	113534	174	886466	21		24
	40	40	805039	254	194961	918677	428	081323	113638	175	886362	20		20
	44	41	805191	254	194809	918934	428	081066	113743	175	886257	19		16
	48	42	805343	253	194657	919191	428	080809	113848	175	886152	18		12
	52	43	805495	253	194505	919448	428	080552	113953	175	886047	17		8
	56	44	805647	253	194353	919705	428	080295	114058	175	885942	16		4
39	0	45	9.805799	253	10.194201	9.919962	428	10.080038	10.114163	175	9.885837	15	21	0
	4	46	805951	253	194049	920219	428	079781	114268	175	885732	14		56
	8	47	806103	253	193897	920476	428	079524	114373	175	885627	13		52
	12	48	806254	253	193746	920733	428	079267	114478	175	885522	12		48
	16	49	806406	252	193594	920990	428	079010	114584	175	885416	11		44
	20	50	806557	252	193443	921247	428	078753	114689	176	885311	10		40
	24	51	806709	252	193291	921503	428	078497	114795	176	885205	9		36
	28	52	806860	252	193140	921760	428	078240	114900	176	885100	8		32
	32	53	807011	252	192989	922017	428	077983	115006	176	884994	7		28
	36	54	807163	252	192837	922274	428	077726	115111	176	884889	6		24
	40	55	807314	252	192686	922530	428	077470	115217	176	884783	5		20
	44	56	807465	251	192535	922787	428	077213	115323	176	884677	4		16
	48	57	807615	251	192385	923044	428	076956	115428	176	884572	3		12
	52	58	807766	251	192234	923300	428	076700	115534	176	884466	2		8
	56	59	807917	251	192083	923557	427	076443	115640	176	884360	1		4
40	0	60	808067	251	191933	923813	427	076187	115746	177	884254	0	20	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			3 Hours,				or			50 Degrees.				

P. P. to s or ″	1s	15″	38	1s	15″	64	1s	15″	26	P. P. to s or ″
	2	30	76	2	30	129	2	30	52	
	3	45	115	3	45	193	3	45	78	

			2 Hours,				or				40 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
40	0	0	9.808067	251	10.191933	9.923813	427	10.076187	10.115746	177	9.884254	60	20	0
	4	1	808218	251	191782	924070	427	075930	115852	177	884148	59		56
	8	2	808368	251	191632	924327	427	075673	115958	177	884042	58		52
	12	3	808519	250	191481	924583	427	075417	116064	177	883936	57		48
	16	4	808669	250	191331	924840	427	075160	116171	177	883829	56		44
	20	5	808819	250	191181	925096	427	074904	116277	177	883723	55		40
	24	6	808969	250	191031	925352	427	074648	116383	177	883617	54		36
	28	7	809119	250	190881	925609	427	074391	116490	177	883510	53		32
	32	8	809269	250	190731	925865	427	074135	116596	177	883404	52		28
	36	9	809419	249	190581	926122	427	073878	116703	178	883297	51		24
	40	10	809569	249	190431	926378	427	073622	116809	178	883191	50		20
	44	11	809718	249	190282	926634	427	073366	116916	178	883084	49		16
	48	12	809868	249	190132	926890	427	073110	117023	178	882977	48		12
	52	13	810017	249	189983	927147	427	072853	117129	178	882871	47		8
	56	14	810167	249	189833	927403	427	072597	117236	178	882764	46		4
41	0	15	9.810316	248	10.189684	9.927659	427	10.072341	10.117343	178	9.882657	45	19	0
	4	16	810465	248	189535	927915	427	072085	117450	178	882550	44		56
	8	17	810614	248	189386	928171	427	071829	117557	178	882443	43		52
	12	18	810763	248	189237	928427	427	071573	117664	179	882336	42		48
	16	19	810912	248	189088	928683	427	071317	117771	179	882229	41		44
	20	20	811061	248	188939	928940	427	071060	117879	179	882121	40		40
	24	21	811210	248	188790	929196	427	070804	117986	179	882014	39		36
	28	22	811358	247	188642	929452	427	070548	118093	179	881907	38		32
	32	23	811507	247	188493	929708	427	070292	118201	179	881799	37		28
	36	24	811655	247	188345	929964	426	070036	118308	179	881692	36		24
	40	25	811804	247	188196	930220	426	069780	118416	179	881584	35		20
	44	26	811952	247	188048	930475	426	069525	118523	179	881477	34		16
	48	27	812100	247	187900	930731	426	069269	118631	179	881369	33		12
	52	28	812248	247	187752	930987	426	069013	118739	180	881261	32		8
	56	29	812396	246	187604	931243	426	068757	118847	180	881153	31		4
42	0	30	9.812544	246	10.187456	9.931499	426	10.068501	10.118954	180	9.881046	30	18	0
	4	31	812692	246	187308	931755	426	068245	119062	180	880938	29		56
	8	32	812840	246	187160	932010	426	067990	119170	180	880830	28		52
	12	33	812988	246	187012	932266	426	067734	119278	180	880722	27		48
	16	34	813135	246	186865	932522	426	067478	119387	180	880613	26		44
	20	35	813283	246	186717	932778	426	067222	119495	180	880505	25		40
	24	36	813430	245	186570	933033	426	066967	119603	180	880397	24		36
	28	37	813578	245	186422	933289	426	066711	119711	181	880289	23		32
	32	38	813725	245	186275	933545	426	066455	119820	181	880180	22		28
	36	39	813872	245	186128	933800	426	066200	119928	181	880072	21		24
	40	40	814019	245	185981	934056	426	065944	120037	181	879963	20		20
	44	41	814166	245	185834	934311	426	065689	120145	181	879855	19		16
	48	42	814313	245	185687	934567	426	065433	120254	181	879746	18		12
	52	43	814460	244	185540	934823	426	065177	120363	181	879637	17		8
	56	44	814607	244	185393	935078	426	064922	120471	181	879529	16		4
43	0	45	9.814753	244	10.185247	9.935333	426	10.064667	10.120580	181	9.879420	15	17	0
	4	46	814900	244	185100	935589	426	064411	120689	181	879311	14		56
	8	47	815046	244	184954	935844	426	064156	120798	182	879202	13		52
	12	48	815193	244	184807	936100	426	063900	120907	182	879093	12		48
	16	49	815339	244	184661	936355	426	063645	121016	182	878984	11		44
	20	50	815485	243	184515	936610	426	063390	121125	182	878875	10		40
	24	51	815632	243	184368	936866	425	063134	121234	182	878766	9		36
	28	52	815778	243	184222	937121	425	062879	121344	182	878656	8		32
	32	53	815924	243	184076	937376	425	062624	121453	182	878547	7		28
	36	54	816069	243	183931	937632	425	062368	121562	182	878438	6		24
	40	55	816215	243	183785	937887	425	062113	121672	182	878328	5		20
	44	56	816361	243	183639	938142	425	061858	121781	183	878219	4		16
	48	57	816507	242	183493	938398	425	061602	121891	183	878109	3		12
	52	58	816652	242	183348	938653	425	061347	122001	183	877999	2		8
	56	59	816798	242	183202	938908	425	061092	122110	183	877890	1		4
44	0	60	816943	242	183057	939163	425	060837	122220	183	877780	0	16	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
			3 Hours,				or				49 Degrees.			

P. P. to s or ″	1s	15″	37	1s	15″	64	1s	15″	27	P. P. to s or ″
	2	30	74	2	30	128	2	30	54	
	3	45	111	3	45	192	3	45	81	

2 Hours, or 41 Degrees.

m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
44	0	0	9.816943	242	10.183057	9.939163	425	10.060837	10.122220	183	9.877780	60	16	0
	4	1	817088	242	182912	939418	425	060582	122330	183	877670	59		56
	8	2	817233	242	182767	939673	425	060327	122440	183	877560	58		52
	12	3	817379	242	182621	939928	425	060072	122550	183	877450	57		48
	16	4	817524	241	182476	940183	425	059817	122660	183	877340	56		44
	20	5	817668	241	182332	940438	425	059562	122770	184	877230	55		40
	24	6	817813	241	182187	940694	425	059306	122880	184	877120	54		36
	28	7	817958	241	182042	940949	425	059051	122990	184	877010	53		32
	32	8	818103	241	181897	941204	425	058796	123101	184	876899	52		28
	36	9	818247	241	181753	941458	425	058542	123211	184	876789	51		24
	40	10	818392	241	181608	941714	425	058286	123322	184	876678	50		20
	44	11	818536	240	181464	941968	425	058032	123432	184	876568	49		16
	48	12	818681	240	181319	942223	425	057777	123543	184	876457	48		12
	52	13	818825	240	181175	942478	425	057522	123653	184	876347	47		8
	56	14	818969	240	181031	942733	425	057267	123764	185	876236	46		4
45	0	15	9.819113	240	10.180887	9.942988	425	10.057012	10.123875	185	9.876125	45	15	0
	4	16	819257	240	180743	943243	425	056757	123986	185	876014	44		56
	8	17	819401	240	180599	943498	425	056502	124096	185	875904	43		52
	12	18	819545	239	180455	943752	425	056248	124207	185	875793	42		48
	16	19	819689	239	180311	944007	425	055993	124318	185	875682	41		44
	20	20	819832	239	180168	944262	425	055738	124429	185	875571	40		40
	24	21	819976	239	180024	944517	425	055483	124541	185	875459	39		36
	28	22	820120	239	179880	944771	424	055229	124652	185	875348	38		32
	32	23	820263	239	179737	945026	424	054974	124763	185	875237	37		28
	36	24	820406	239	179594	945281	424	054719	124874	186	875126	36		24
	40	25	820550	238	179450	945535	424	054465	124986	186	875014	35		20
	44	26	820693	238	179307	945790	424	054210	125097	186	874903	34		16
	48	27	820836	238	179164	946045	424	053955	125209	186	874791	33		12
	52	28	820979	238	179021	946299	424	053701	125320	186	874680	32		8
	56	29	821122	238	178878	946554	424	053446	125432	186	874568	31		4
46	0	30	9.821265	238	10.178735	9.946808	424	10.053192	10.125544	186	9.874456	30	14	0
	4	31	821407	238	178593	947063	424	052937	125656	186	874344	29		56
	8	32	821550	238	178450	947318	424	052682	125768	187	874232	28		52
	12	33	821693	237	178307	947572	424	052428	125879	187	874121	27		48
	16	34	821835	237	178165	947826	424	052174	125991	187	874009	26		44
	20	35	821977	237	178023	948081	424	051919	126104	187	873896	25		40
	24	36	822120	237	177880	948336	424	051664	126216	187	873784	24		36
	28	37	822262	237	177738	948590	424	051410	126328	187	873672	23		32
	32	38	822404	237	177596	948844	424	051156	126440	187	873560	22		28
	36	39	822546	237	177454	949099	424	050901	126552	187	873448	21		24
	40	40	822688	236	177312	949353	424	050647	126665	187	873335	20		20
	44	41	822830	236	177170	949607	424	050393	126777	187	873223	19		16
	48	42	822972	236	177028	949862	424	050138	126890	188	873110	18		12
	52	43	823114	236	176886	950116	424	049884	127002	188	872998	17		8
	56	44	823255	236	176745	950370	424	049630	127115	188	872885	16		4
47	0	45	9.823397	236	10.176603	9.950625	424	10.049375	10.127228	188	9.872772	15	13	0
	4	46	823539	236	176461	950879	424	049121	127341	188	872659	14		56
	8	47	823680	235	176320	951133	424	048867	127453	188	872547	13		52
	12	48	823821	235	176179	951388	424	048612	127566	188	872434	12		48
	16	49	823963	235	176037	951642	424	048358	127679	188	872321	11		44
	20	50	824104	235	175896	951896	424	048104	127792	188	872208	10		40
	24	51	824245	235	175755	952150	424	047850	127905	189	872095	9		36
	28	52	824386	235	175614	952405	424	047595	128019	189	871981	8		32
	32	53	824527	235	175473	952659	424	047341	128132	189	871868	7		28
	36	54	824668	234	175332	952913	424	047087	128245	189	871755	6		24
	40	55	824808	234	175192	953167	423	046833	128359	189	871641	5		20
	44	56	824949	234	175051	953421	423	046579	128472	189	871528	4		16
	48	57	825090	234	174910	953675	423	046325	128586	189	871414	3		12
	52	58	825230	234	174770	953929	423	046071	128699	189	871301	2		8
	56	59	825371	234	174629	954183	423	045817	128813	189	871187	1		4
48	0	60	825511	234	174489	954437	423	045563	128927	190	871073	0	12	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.

3 Hours, or 48 Degrees.

P. P. to s or ″	1s	15″	36	1s	15″	64	1s	15″	28	P. P. to s or ″
	2	30	71	2	30	127	2	30	56	
	3	45	107	3	45	191	3	45	84	

2 Hours,							or				42 Degrees.			
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
48	0	0	9.825511	234	10.174489	9.954437	423	10.045563	10.128927	190	9.871073	60	12	0
	4	1	825651	233	174349	954691	423	045309	129040	190	870960	59		56
	8	2	825791	233	174209	954945	423	045055	129154	190	870846	58		52
	12	3	825931	233	174069	955200	423	044800	129268	190	870732	57		48
	16	4	826071	233	173929	955454	423	044546	129382	190	870618	56		44
	20	5	826211	233	173789	955707	423	044293	129496	190	870504	55		40
	24	6	826351	233	173649	955961	423	044039	129610	190	870390	54		36
	28	7	826491	233	173509	956215	423	043785	129724	190	870276	53		32
	32	8	826631	233	173369	956469	423	043531	129839	190	870161	52		28
	36	9	826770	232	173230	956723	423	043277	129953	191	870047	51		24
	40	10	826910	232	173090	956977	423	043023	130067	191	869933	50		20
	44	11	827049	232	172951	957231	423	042769	130182	191	869818	49		16
	48	12	827189	232	172811	957485	423	042515	130296	191	869704	48		12
	52	13	827328	232	172672	957739	423	042261	130411	191	869589	47		8
	56	14	827467	232	172533	957993	423	042007	130526	191	869474	46		4
49	0	15	9.827606	232	10.172394	9.958246	423	10.041754	10.130640	191	9.869360	45	11	0
	4	16	827745	232	172255	958500	423	041500	130755	191	869245	44		56
	8	17	827884	231	172116	958754	423	041246	130870	191	869130	43		52
	12	18	828023	231	171977	959008	423	040992	130985	192	869015	42		48
	16	19	828162	231	171838	959262	423	040738	131100	192	868900	41		44
	20	20	828301	231	171699	959516	423	040484	131215	192	868785	40		40
	24	21	828439	231	171561	959769	423	040231	131330	192	868670	39		36
	28	22	828578	231	171422	960023	423	039977	131445	192	868555	38		32
	32	23	828716	231	171284	960277	423	039723	131560	192	868440	37		28
	36	24	828855	230	171145	960531	423	039469	131676	192	868324	36		24
	40	25	828993	230	171007	960784	423	039216	131791	192	868209	35		20
	44	26	829131	230	170869	961038	423	038962	131907	192	868093	34		16
	48	27	829269	230	170731	961291	423	038709	132022	193	867978	33		12
	52	28	829407	230	170593	961545	423	038455	132138	193	867862	32		8
	56	29	829545	230	170455	961799	423	038201	132253	193	867747	31		4
50	0	30	9.829683	230	10.170317	9.962052	423	10.037948	10.132369	193	9.867631	30	10	0
	4	31	829821	229	170179	962306	423	037694	132485	193	867515	29		56
	8	32	829959	229	170041	962560	423	037440	132601	193	867399	28		52
	12	33	830097	229	169903	962813	423	037187	132717	193	867283	27		48
	16	34	830234	229	169766	963067	423	036933	132833	193	867167	26		44
	20	35	830372	229	169628	963320	423	036680	132949	193	867051	25		40
	24	36	830509	229	169491	963574	423	036426	133065	194	866935	24		36
	28	37	830646	229	169354	963827	423	036173	133181	194	866819	23		32
	32	38	830784	229	169216	964081	423	035919	133297	194	866703	22		28
	36	39	830921	228	169079	964335	423	035665	133414	194	866586	21		24
	40	40	831058	228	168942	964588	422	035412	133530	194	866470	20		20
	44	41	831195	228	168805	964842	422	035158	133647	194	866353	19		16
	48	42	831332	228	168668	965095	422	034905	133763	194	866237	18		12
	52	43	831469	228	168531	965349	422	034651	133880	194	866120	17		8
	56	44	831606	228	168394	965602	422	034398	133996	195	866004	16		4
51	0	45	9.831742	228	10.168258	9.965855	422	10.034145	10.134113	195	9.865887	15	9	0
	4	46	831879	228	168121	966109	422	033891	134230	195	865770	14		56
	8	47	832015	227	167985	966362	422	033638	134347	195	865653	13		52
	12	48	832152	227	167848	966616	422	033384	134464	195	865536	12		48
	16	49	832288	227	167712	966869	422	033131	134581	195	865419	11		44
	20	50	832425	227	167575	967123	422	032877	134698	195	865302	10		40
	24	51	832561	227	167439	967376	422	032624	134815	195	865185	9		36
	28	52	832697	227	167303	967629	422	032371	134932	195	865068	8		32
	32	53	832833	227	167167	967883	422	032117	135050	195	864950	7		28
	36	54	832969	226	167031	968136	422	031864	135167	196	864833	6		24
	40	55	833105	226	166895	968389	422	031611	135284	196	864716	5		20
	44	56	833241	226	166759	968643	422	031357	135402	196	864598	4		16
	48	57	833377	226	166623	968896	422	031104	135519	196	864481	3		12
	52	58	833512	226	166488	969149	422	030851	135637	196	864363	2		8
	56	59	833648	226	166352	969403	422	030597	135755	196	864245	1		4
52	0	60	833783	226	166217	969656	422	030344	135873	196	864127	0	8	0
m.	s.	′	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
3 Hours,							or				47 Degrees.			

P. P. to s or ″	1s	15″	34	1s	15″	63	1s	15″	29	P. P. to s or ″
	2	30	69	2	30	127	2	30	58	
	3	45	103	3	45	190	3	45	87	

2 Hours,							or			43 Degrees.				
m.	s.	'	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	'	m.	s.
52	0	0	9.833783	226	10.166217	9.969656	422	10.030344	10.135873	196	9.864127	60	8	0
	4	1	833919	225	166081	969909	422	030091	135990	196	864010	59		56
	8	2	834054	225	165946	970162	422	029838	136108	197	863892	58		52
	12	3	834189	225	165811	970416	422	029584	136226	197	863774	57		48
	16	4	834325	225	165675	970669	422	029331	136344	197	863656	56		44
	20	5	834460	225	165540	970922	422	029078	136462	197	863538	55		40
	24	6	834595	225	165405	971175	422	028825	136581	197	863419	54		36
	28	7	834730	225	165270	971429	422	028571	136699	197	863301	53		32
	32	8	834865	225	165135	971682	422	028318	136817	197	863183	52		28
	36	9	834999	224	165001	971935	422	028065	136936	197	863064	51		24
	40	10	835134	224	164866	972188	422	027812	137054	198	862946	50		20
	44	11	835269	224	164731	972441	422	027559	137173	198	862827	49		16
	48	12	835403	224	164597	972694	422	027306	137291	198	862709	48		12
	52	13	835538	224	164462	972948	422	027052	137410	198	862590	47		8
	56	14	835672	224	164328	973201	422	026799	137529	198	862471	46		4
53	0	15	9.835807	224	10.164193	9.973454	422	10.026546	10.137647	198	9.862353	45	7	0
	4	16	835941	224	164059	973707	422	026293	137766	198	862234	44		56
	8	17	836075	223	163925	973960	422	026040	137885	198	862115	43		52
	12	18	836209	223	163791	974213	422	025787	138004	198	861996	42		48
	16	19	836343	223	163657	974466	422	025534	138123	198	861877	41		44
	20	20	836477	223	163523	974719	422	025281	138242	199	861758	40		40
	24	21	836611	223	163389	974973	422	025027	138362	199	861638	39		36
	28	22	836745	223	163255	975226	422	024774	138481	199	861519	38		32
	32	23	836878	223	163122	975479	422	024521	138600	199	861400	37		28
	36	24	837012	222	162988	975732	422	024268	138720	199	861280	36		24
	40	25	837146	222	162854	975985	422	024015	138839	199	861161	35		20
	44	26	837279	222	162721	976238	422	023762	138959	199	861041	34		16
	48	27	837412	222	162588	976491	422	023509	139078	199	860922	33		12
	52	28	837546	222	162454	976744	422	023256	139198	199	860802	32		8
	56	29	837679	222	162321	976997	422	023003	139318	200	860682	31		4
54	0	30	9.837812	222	10.162188	9.977250	422	10.022750	10.139438	200	9.860562	30	6	0
	4	31	837945	222	162055	977503	422	022497	139558	200	860442	29		56
	8	32	838078	221	161922	977756	422	022244	139678	200	860322	28		52
	12	33	838211	221	161789	978009	422	021991	139798	200	860202	27		48
	16	34	838344	221	161656	978262	422	021738	139918	200	860082	26		44
	20	35	838477	221	161523	978515	422	021485	140038	200	859962	25		40
	24	36	838610	221	161390	978768	422	021232	140158	200	859842	24		36
	28	37	838742	221	161258	979021	422	020979	140279	201	859721	23		32
	32	38	838875	221	161125	979274	422	020726	140399	201	859601	22		28
	36	39	839007	221	160993	979527	422	020473	140520	201	859480	21		24
	40	40	839140	220	160860	979780	422	020220	140640	201	859360	20		20
	44	41	839272	220	160728	980033	422	019967	140761	201	859239	19		16
	48	42	839404	220	160596	980286	422	019714	140881	201	859119	18		12
	52	43	839536	220	160464	980538	422	019462	141002	201	858998	17		8
	56	44	839668	220	160332	980791	421	019209	141123	201	858877	16		4
55	0	45	9.839800	220	10.160200	9.981044	421	10.018956	10.141244	202	9.858756	15	5	0
	4	46	839932	220	160068	981297	421	018703	141365	202	858635	14		56
	8	47	840064	219	159936	981550	421	018450	141486	202	858514	13		52
	12	48	840196	219	159804	981803	421	018197	141607	202	858393	12		48
	16	49	840328	219	159672	982056	421	017944	141728	202	858272	11		44
	20	50	840459	219	159541	982309	421	017691	141849	202	858151	10		40
	24	51	840591	219	159409	982562	421	017438	141971	202	858029	9		36
	28	52	840722	219	159278	982814	421	017186	142092	202	857908	8		32
	32	53	840854	219	159146	983067	421	016933	142214	202	857786	7		28
	36	54	840985	219	159015	983320	421	016680	142335	203	857665	6		24
	40	55	841116	218	158884	983573	421	016427	142457	203	857543	5		20
	44	56	841247	218	158753	983826	421	016174	142578	203	857422	4		16
	48	57	841378	218	158622	984079	421	015921	142700	203	857300	3		12
	52	58	841509	218	158491	984331	421	015669	142822	203	857178	2		8
	56	59	841640	218	158360	984584	421	015416	142944	203	857056	1		4
56	0	60	841771	218	158229	984837	421	015163	143066	203	856934	0	4	0
m.	s.	'	Cosine.		Secant.	Cotang.		Tang.	Cosec.		Sine	'	m.	s.
3 Hours,							or			46 Degrees.				

P. P. to s or "	1s	15"	33	1s	15"	63	1s	15"	30	P. P. to s or "
	2	30	67	2	30	127	2	30	60	
	3	45	100	3	45	190	3	45	90	

2 Hours,							or			44 Degrees.				
m.	s.	′	Sine.	D.	Cosec.	Tang.	D.	Cotang.	Secant.	D.	Cosine.	′	m.	s.
56	0	0	9.841771	218	10.158229	9.984837	421	10.015163	10.143066	203	9.856934	60	4	0
	4	1	841902	218	158098	985090	421	014910	143188	203	856812	59		56
	8	2	842033	218	157967	985343	421	014657	143310	204	856690	58		52
	12	3	842163	217	157837	985596	421	014404	143432	204	856568	57		48
	16	4	842294	217	157706	985848	421	014152	143554	204	856446	56		44
	20	5	842424	217	157576	986101	421	013899	143677	204	856323	55		40
	24	6	842555	217	157445	986354	421	013646	143799	204	856201	54		36
	28	7	842685	217	157315	986607	421	013393	143922	204	856078	53		32
	32	8	842815	217	157185	986860	421	013140	144044	204	855956	52		28
	36	9	842946	217	157054	987112	421	012888	144167	204	855833	51		24
	40	10	843076	217	156924	987365	421	012635	144289	205	855711	50		20
	44	11	843206	216	156794	987618	421	012382	144412	205	855588	49		16
	48	12	843336	216	156664	987871	421	012129	144535	205	855465	48		12
	52	13	843466	216	156534	988123	421	011877	144658	205	855342	47		8
	56	14	843595	216	156405	988376	421	011624	144781	205	855219	46		4
57	0	15	9.843725	216	10.156275	9.988629	421	10.011371	10.144904	205	9.855096	45	3	0
	4	16	843855	216	156145	988882	421	011118	145027	205	854973	44		56
	8	17	843984	216	156016	989134	421	010866	145150	205	854850	43		52
	12	18	844114	215	155886	989387	421	010613	145273	206	854727	42		48
	16	19	844243	215	155757	989640	421	010360	145397	206	854603	41		44
	20	20	844372	215	155628	989893	421	010107	145520	206	854480	40		40
	24	21	844502	215	155498	990145	421	009855	145644	206	854356	39		36
	28	22	844631	215	155369	990398	421	009602	145767	206	854233	38		32
	32	23	844760	215	155240	990651	421	009349	145891	206	854109	37		28
	36	24	844889	215	155111	990903	421	009097	146014	206	853986	36		24
	40	25	845018	215	154982	991156	421	008844	146138	206	853862	35		20
	44	26	845147	215	154853	991409	421	008591	146262	206	853738	34		16
	48	27	845276	214	154724	991662	421	008338	146386	207	853614	33		12
	52	28	845405	214	154595	991914	421	008086	146510	207	853490	32		8
	56	29	845533	214	154467	992167	421	007833	146634	207	853366	31		4
58	0	30	9.845662	214	10.154338	9.992420	421	10.007580	10.146758	207	9.853242	30	2	0
	4	31	845790	214	154210	992672	421	007328	146882	207	853118	29		56
	8	32	845919	214	154081	992925	421	007075	147006	207	852994	28		52
	12	33	846047	214	153953	993178	421	006822	147131	207	852869	27		48
	16	34	846175	214	153825	993430	421	006570	147255	207	852745	26		44
	20	35	846304	214	153696	993683	421	006317	147380	207	852620	25		40
	24	36	846432	213	153568	993936	421	006064	147504	208	852496	24		36
	28	37	846560	213	153440	994189	421	005811	147629	208	852371	23		32
	32	38	846688	213	153312	994441	421	005559	147753	208	852247	22		28
	36	39	846816	213	153184	994694	421	005306	147878	208	852122	21		24
	40	40	846944	213	153056	994947	421	005053	148003	208	851997	20		20
	44	41	847071	213	152929	995199	421	004801	148128	208	851872	19		16
	48	42	847199	213	152801	995452	421	004548	148253	208	851747	18		12
	52	43	847327	213	152673	995705	421	004295	148378	208	851622	17		8
	56	44	847454	212	152546	995957	421	004043	148503	209	851497	16		4
59	0	45	9.847582	212	10.152418	9.996210	421	10.003790	10.148628	209	9.851372	15	1	0
	4	46	847709	212	152291	996463	421	003537	148754	209	851246	14		56
	8	47	847836	212	152164	996715	421	003285	148879	209	851121	13		52
	12	48	847964	212	152036	996968	421	003032	149004	209	850996	12		48
	16	49	848091	212	151909	997221	421	002779	149130	209	850870	11		44
	20	50	848218	212	151782	997473	421	002527	149255	209	850745	10		40
	24	51	848345	212	151655	997726	421	002274	149381	209	850619	9		36
	28	52	848472	211	151528	997979	421	002021	149507	210	850493	8		32
	32	53	848599	211	151401	998231	421	001769	149632	210	850368	7		28
	36	54	848726	211	151274	998484	421	001516	149758	210	850242	6		24
	40	55	848852	211	151148	998737	421	001263	149884	210	850116	5		20
	44	56	848979	211	151021	998989	421	001011	150010	210	849990	4		16
	48	57	849106	211	150894	999242	421	000758	150136	210	849864	3		12
	52	58	849232	211	150768	999495	421	000505	150262	210	849738	2		8
	56	59	849359	211	150641	999747	421	000253	150389	210	849611	1		4
60	0	60	849485	211	150515	10.00000	421	000000	150515	210	849485	0	0	0
m.	s.	′	Cosine.	•	Secant.	Cotang.		Tang.	Cosec.		Sine.	′	m.	s.
3 Hours,							or			45 Degrees.				

P. P. to s or ″	1s	15″	32	1s	15″	63	1s	15″	31	P. P. to s or ″
	2	30	64	2	30	126	2	30	62	
	3	45	96	3	45	189	3	45	93	

TABLE VI.

NATURAL SINES, TANGENTS, SECANTS, AND VERSINES, TO EVERY DEGREE OF THE QUADRANT.

Arc.	Sine.	Cosine.	Tangent.	Cotan.	Secant.	Cosec.	Versine	Coversine	Arc.
0°	000000	1.000000	000000	Infinite.	1.000000	Infinite.	000000	1.000000	90°
1	017452	999848	017455	57.28996	1.000152	57.29869	000154	982548	89
2	034899	999391	034921	28.63625	1.000609	28.65371	000609	965100	88
3	052336	998630	052408	19.08114	1.001372	19.10732	001370	947664	87
4	069756	997564	069927	14 30067	1.002442	14.33559	002436	930243	86
5	087156	996195	087489	11.43005	1.003820	11.47371	003805	912844	85
6	104528	994522	105104	9.514365	1.005508	9.566772	005478	895471	84
7	121869	992546	122785	8.144346	1.007510	8.205509	007454	878131	83
8	139173	990268	140541	7.115370	1.009828	7.185297	009732	860827	82
9	156434	987688	158384	6.313752	1.012465	6.392453	012312	843565	81
10	173648	984808	176327	5.671282	1.015427	5.758771	015192	826352	80
11	190809	981627	194380	5.144554	1.018717	5.240843	018373	809191	79
12	207912	978148	212557	4.704630	1.022341	4.809734	021852	792088	78
13	224951	974370	230868	4.331476	1.026304	4.445411	025630	775049	77
14	241922	970296	249328	4.010781	1.030614	4.133566	029704	758078	76
15	258819	965926	267949	3.732051	1.035276	3.863703	034074	741181	75
16	275637	961262	286745	3.487414	1.040299	3.627955	038738	724363	74
17	292372	956305	305731	3.270853	1.045692	3.420304	043695	707628	73
18	309017	951056	324920	3.077684	1.051462	3.236068	048943	690983	72
19	325568	945519	344328	2.904211	1.057621	3.071554	054481	674432	71
20	342020	939693	363970	2.747477	1.064178	2.923804	060307	657980	70
21	358368	933580	383864	2.605089	1.071145	2.790428	066420	641632	69
22	374607	927184	404026	2.475087	1.078535	2.669467	072816	625393	68
23	390731	920505	424475	2.355852	1.086360	2.559305	079495	609269	67
24	406737	913546	445229	2.246037	1.094636	2.458593	086454	593263	66
25	422618	906308	466308	2.144507	1.103378	2.366202	093692	577382	65
26	438371	898794	487733	2.050304	1.112602	2.281172	101206	561629	64
27	453991	891007	509525	1.962611	1.122326	2.202689	108993	546009	63
28	469472	882948	531709	1.880727	1.132570	2.130055	117052	530528	62
29	484810	874620	554309	1.804048	1.143354	2.062665	125380	515190	61
30	500000	866025	577350	1.732051	1.154701	2.000000	133975	500000	60
31	515038	857167	600861	1.664280	1.166633	1.941604	142833	484962	59
32	529919	848048	624869	1.600335	1.179178	1.887080	151952	470081	58
33	544639	838671	649408	1.539865	1.192363	1.836079	161329	455361	57
34	559193	829038	674509	1.482561	1.206218	1.788292	170962	440807	56
35	573576	819152	700208	1.428148	1.220775	1.743447	180848	426424	55
36	587785	809017	726543	1.376382	1.236068	1.701302	190983	412215	54
37	601815	798636	753554	1.327045	1.252136	1.661640	201364	398185	53
38	615661	788011	781286	1.279942	1.269018	1.624269	211989	384338	52
39	629320	777146	809784	1.234897	1.286760	1.589016	222854	360680	51
40	642788	766044	839100	1.191754	1.305407	1.555724	233956	357212	50
41	656059	754710	869287	1.150368	1.325013	1.524253	245290	343941	49
42	669131	743145	900404	1.110613	1.345633	1.494477	256855	330869	48
43	681998	731354	932515	1.072369	1.367328	1.466279	268646	318002	47
44	694658	719340	965689	1.035530	1.390164	1.439557	280660	305342	46
45	707107	707107	1.000000	1.000000	1.414214	1.414214	292893	292893	45
Arc.	Cosine.	Sine.	Cotan.	Tangent.	Cosec.	Secant.	Covers.	Versine.	Arc.

TABLE VII.

MERIDIONAL PARTS TO EVERY DEGREE OF THE QUADRANT.

D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.	D.	M. P.
0	.0	10	603.1	20	1225.1	30	1888.4	40	2622.7	50	3474.5	60	4527.4	70	5965.9	80	8375.2
1	60.0	11	664.1	21	1289.2	31	1958.0	41	2701.6	51	3568.8	61	4649.2	71	6145.7	81	8739.1
2	120.0	12	725.3	22	1353.7	32	2028.4	42	2781.7	52	3665.2	62	4775.0	72	6334.8	82	9145.5
3	180.1	13	786.8	23	1418 6	33	2099.5	43	2863.1	53	3763.8	63	4904.9	73	6534.4	83	9605.8
4	240.2	14	848.5	24	1484.1	34	2171.5	44	2945.8	54	3864.6	64	5039.4	74	6745.7	84	10136.9
5	300.4	15	910.5	25	1550.0	35	2244.3	45	3029.9	55	3968.0	65	5178.8	75	6970.3	85	10764.6
6	360.7	16	972.7	26	1616.5	36	2318.0	46	3115.6	56	4073.9	66	5323.5	76	7210.1	86	11532.5
7	421.1	17	1035.3	27	1683.5	37	2392.6	47	3202.7	57	4182.6	67	5474.0	77	7467.2	87	12522.1
8	481.6	18	1098.2	28	1751.2	38	2468.3	48	3291.5	58	4294.3	68	5630.8	78	7744.6	88	13916.4
9	542.2	19	1161.5	29	1819.4	39	2544.9	49	3382.1	59	4409.1	69	5794.6	79	8045.7	89	16299.6

Course		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Course	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	0.9998	0.0175	1.9997	0.0349	2.9995	0.0524	3.9994	0.0698	4 9992	0.0873	89	
	2	0.9994	0 0349	1.9988	0.0698	2.9982	0.1047	3.9976	0.1396	4.9970	0.1745	88	
0 ¼		0.9988	0.0491	1.9976	0.0981	2.9964	0.1472	3.9952	0.1963	4.9940	0.2453		7 ¾
	3	0.9986	0.0523	1.9973	0.1047	2.9959	0.1570	3.9945	0.2093	4.9931	0.2617	87	
	4	0.9976	0.0698	1.9951	0.1395	2.9927	0.2093	3.9903	0.2790	4.9878	0.3488	86	
	5	0.9962	0.0872	1.9924	0.1743	2.9886	0.2615	3.9848	0.3486	4.9810	0.4358	85	
0 ½		0.9952	0.0980	1.9904	0.1960	2.9856	0.2941	3.9807	0.3921	4.9759	0.4901		7 ½
	6	0.9945	0.1045	1.9890	0.2091	2.9836	0.3136	3.9781	0.4181	4.9726	0.5226	84	
	7	0.9925	0.1219	1.9851	0.2437	2.9776	0.3656	3.9702	0.4875	4.9627	0.6093	83	
	8	0.9903	0.1392	1.9805	0.2783	2.9708	0.4175	3.9611	0.5567	4.9513	0.6959	82	
0 ¾		0.9892	0.1467	1.9784	0.2935	2.9675	0.4402	3.9567	0.5869	4.9459	0.7337		7 ¼
	9	0.9877	0.1564	1.9754	0.3129	2.9631	0.4693	3.9508	0.6257	4.9384	0.7822	81	
	10	0.9848	0.1736	1.9696	0.3473	2.9544	0.5209	3.9392	0.6946	4.9240	0.8682	80	
	11	0.9816	0.1908	1.9633	0.3816	2.9449	0.5724	3.9265	0.7632	4.9081	0.9540	79	
1		0.9808	0.1951	1.9616	0.3902	2.9424	0.5853	3.9231	0.7804	4.9039	0.9755		7
	12	0.9781	0.2079	1.9563	0.4158	2.9344	0.6237	3.9126	0.8316	4.8907	1.0396	78	
	13	0.9744	0 2250	1.9487	0.4499	2.9231	0.6749	3.8975	0.8998	4.8719	1.1248	77	
	14	0.9703	0.2419	1.9406	0.4838	2.9109	0.7258	3.8812	0.9677	4.8515	1.2096	76	
1 ¼		0.9700	0.2430	1.9401	0.4860	2.9101	0.7289	3.8801	0.9719	4.8502	1.2149		6 ¾
	15	0.9659	0.2588	1.9319	0.5176	2.8978	0.7765	3.8637	1 0353	4.8296	1.2941	75	
	16	0.9613	0.2756	1.9225	0.5513	2.8838	0.8269	3.8450	1.1025	4.8063	1.3782	74	
1 ½		0.9569	0.2903	1.9139	0.5806	2.8708	0.8709	3.8278	1.1611	4.7847	1.4514		6 ½
	17	0.9563	0.2924	1.9126	0.5847	2.8689	0.8771	3.8252	1.1695	4.7815	1.4619	73	
	18	0.9511	0.3090	1.9021	0.6180	2.8532	0.9271	3.8042	1.2361	4.7553	1.5451	72	
	19	0.9455	0.3256	1.8910	0.6511	2.8366	0.9767	3.7821	1.3023	4.7276	1.6278	71	
1 ¾		0.9415	0.3369	1.8831	0.6738	2.8246	1.0107	3.7662	1.3476	4.7077	1.6844		6 ¼
	20	0.9397	0.3420	1.8794	0.6840	2.8191	1.0261	3.7588	1.3681	4.6985	1.7101	70	
	21	0.9336	0.3584	1.8672	0.7167	2.8007	1.0751	3.7343	1.4335	4.6679	1.7918	69	
	22	0.9272	0.3746	1.8544	0.7492	2.7816	1.1238	3.7087	1.4984	4.6359	1.8730	68	
2		0.9239	0.3827	1.8478	0.7654	2.7716	1.1481	3.6955	1.5307	4.6194	1.9134		6
	23	0.9205	0.3907	1.8410	0.7815	2.7615	1.1722	3.6820	1.5629	4.6025	1.9537	67	
	24	0.9135	0.4067	1.8271	0.8135	2.7406	1.2202	3.6542	1.6269	4.5677	2.0337	66	
	25	0.9063	0.4226	1.8126	0.8452	2.7189	1.2679	3.6252	1.6905	4.5315	2.1131	65	
2 ¼		0.9040	0.4276	1.8080	0.8551	2.7120	1.2827	3.6160	1.7102	4.5199	2.1378		5 ¾
	26	0.8988	0.4384	1.7976	0.8767	2.6964	1.3151	3.5952	1.7535	4.4940	2.1919	64	
	27	0.8910	0.4540	1.7820	0.9080	2.6730	1.3620	3.5640	1.8160	4.4550	2.2700	63	
	28	0.8829	0.4695	1.7659	0.9389	2.6488	1.4084	3.5318	1.8779	4.4147	2.3474	62	
2 ½		0.8819	0 4714	1.7638	0.9428	2.6458	1.4142	3.5277	1.8856	4.4096	2.3570		5 ½
	29	0.8746	0.4848	1.7492	0.9696	2.6239	1.4544	3.4985	1.9392	4.3731	2.4240	61	
	30	0.8660	0.5000	1.7321	1.0000	2.5981	1.5000	3.4641	2.0000	4.3301	2.5000	60	
2 ¾		0.8577	0.5141	1.7155	1.0282	2.5732	1.5423	3.4309	2.0564	4.2886	2.5705		5 ¼
	31	0.8572	0.5150	1.7143	1.0301	2.5715	1.5451	3.4287	2.0602	4.2858	2.5752	59	
	32	0.8480	0.5299	1.6961	1.0598	2.5441	1.5898	3.3922	2.1197	4.2402	2.6496	58	
	33	0.8387	0.5446	1.6773	1.0893	2.5160	1.6339	3.3547	2.1786	4.1934	2.7232	57	
3		0.8315	0.5556	1.6629	1.1111	2.4944	1.6667	3.3259	2.2223	4.1573	2.7779		5
	34	0.8290	0.5592	1.6581	1.1184	2.4871	1.6776	3.3162	2.2368	4.1452	2.7960	56	
	35	0.8192	0.5736	1.6383	1.1472	2.4575	1.7207	3.2766	2.2943	4.0958	2.8679	55	
	36	0.8090	0.5878	1.6180	1.1756	2.4271	1.7634	3.2361	2.3511	4.0451	2.9389	54	
3 ¼		0.8032	0.5957	1.6064	1.1914	2.4096	1.7871	3.2128	2.3828	4.0160	2.9785		4 ¾
	37	0.7986	0.6018	1.5973	1.2036	2.3959	1.8054	3.1945	2.4073	3.9932	3.0091	53	
	38	0.7880	0.6157	1.5760	1.2313	2.3640	1.8470	3.1520	2.4626	3.9401	3.0783	52	
	39	0.7771	0.6293	1.5543	1.2586	2.3314	1.8880	3.1086	2.5173	3.8857	3.1466	51	
3 ½		0.7730	0.6344	1.5460	1.2688	2.3190	1.9032	3.0920	2.5376	3.8650	3.1720		4 ½
	40	0.7660	0.6428	1.5321	1.2856	2.2981	1.9284	3.0642	2.5712	3.8302	3.2139	50	
	41	0.7547	0.6561	1.5094	1.3121	2.2641	1.9682	3.0188	2.6242	3.7735	3.3803	49	
	42	0.7431	0.6691	1.4803	1.3383	2.2294	2.0074	2.9726	2.6765	3.7157	3.3457	48	
3 ¾		0.7410	0.6716	1.4819	1.3431	2.2229	2.0147	2.9638	2.6862	3.7048	3.3578		4 ¼
	43	0.7314	0.6820	1.4627	1.3640	2.1941	2.0460	2.9254	2.7280	3.6568	3.4100	47	
	44	0.7193	0.6947	1.4387	1.3893	2.1580	2.0840	2.8774	2.7786	3.5967	3.4733	46	
4	45	0.7071	0.7071	1.4142	1.4142	2.1213	2.1213	2.8284	2.8284	3.5355	3.5355	45	4
Pts.	Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.	Pts.
		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.			

Course		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		Course	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	5.9991	0.1047	6.9989	0.1222	7.9988	0.1396	8.9986	0.1571	9.9985	0.1745	89	
	2	5.9963	0.2094	6.9957	0.2443	7.9951	0.2792	8.9945	0.3141	9.9939	0.3490	88	
0 ¼		5.9928	0.2944	6.9916	0.3435	7.9904	0.3925	8.9892	0.4416	9.9880	0.4907		7 ¾
	3	5.9918	0.3140	6.9904	0.3664	7.9890	0.4187	8.9877	0.4710	9.9863	0.5234	87	
	4	5.9854	0.4185	6.9829	0.4883	7.9805	0.5581	8.9781	0.6278	9.9756	0.6976	86	
	5	5.9772	0.5229	6.9734	0.6101	7.9696	0.6972	8.9658	0.7844	9.9619	0.8716	85	
0 ½		5.9711	0.5881	6.9663	0.6861	7.9615	0.7841	8.9567	0.8822	9.9518	0.9802		7 ½
	6	5.9671	0.6272	6.9617	0.7317	7.9562	0 8362	8.9507	0.9408	9.9452	1.0453	84	
	7	5.9553	0.7312	6.9478	0.8531	7.9404	0.9750	8.9329	1.0968	9.9255	1.2187	83	
	8	5.9416	0.8350	6.9319	0.9742	7.9221	1.1134	8.9124	1.2526	9.9027	1.3917	82	
0 ¾		5.9351	0.8804	6.9242	1.0271	7.9134	1.1738	8.9026	1.3206	9.8918	1.4673		7 ¼
	9	5.9261	0.9386	6.9138	1.0950	7.9015	1.2515	8.8892	1.4079	9.8769	1.5643	81	
	10	5.9088	1.0419	6.8937	1.2155	7.8785	1.3892	8.8633	1.5628	9.8481	1.7365	80	
	11	5.8898	1.1449	6.8714	1.3357	7.8530	1.5265	8.8346	1.7173	9.8163	1.9081	79	
1		5.8847	1.1705	6.8655	1.3656	7.8463	1.5607	8.8271	1.7558	9.8079	1.9509		7
	12	5.8689	1.2475	6.8470	1.4554	7.8252	1.6633	8.8033	1.8712	9.7815	2.0791	78	
	13	5.8462	1.3497	6.8206	1.5747	7.7950	1.7996	8.7693	2.0246	9.7437	2.2495	77	
	14	5.8218	1.4515	6.7921	1.6935	7.7624	1.9354	8.7327	2.1773	9.7030	2.4192	76	
1 ¼		5.8202	1.4579	6.7902	1.7009	7.7602	1.9438	8.7303	2.1868	9.7003	2.4298		6 ¾
	15	5.7956	1.5529	6.7615	1.8117	7.7274	2.0706	8.6933	2.3294	9.6593	2.5882	75	
	16	5.7676	1.6538	6.7288	1.9295	7.6901	2.2051	8.6513	2.4807	9.6120	2.7562	74	
1 ½		5.7416	1.7417	6.6986	2.0320	7.6555	2.3223	8.6125	2.6126	9.5694	2.9028		6 ½
	17	5.7378	1.7542	6.6941	2.0466	7.6504	2.3390	8.6067	2.6313	9.5630	2.9237	73	
	18	5.7063	1.8541	6.6574	2.1631	7.6085	2.4721	8.5595	2.7812	9.5106	3.0902	72	
	19	5.6731	1.9534	6.6186	2.2790	7.5642	2.6045	8.5097	2.9301	9.4552	3.2557	71	
1 ¾		5.6493	2.0213	6.5908	2.3582	7.5324	2.6951	8.4739	3.0320	9.4154	3.3689		6 ¼
	20	5.6382	2.0521	6.5778	2.3941	7.5175	2.7362	8.4572	3.0782	9.3969	3.4202	70	
	21	5.6015	2.1502	6.5351	2.5086	7.4686	2.8669	8.4022	3.2253	9.3358	3.5837	69	
	22	5.5631	2.2476	6.4903	2.6222	7.4175	2.9969	8.3447	3.3715	9.2718	3.7461	68	
2		5.5433	2.2961	6.4672	2.6788	7.3910	3.0615	8.3149	3.4442	9.2388	3.8268		6
	23	5.5230	2.3444	6.4435	2.7351	7.3640	3.1258	8.2845	3.5166	9.2050	3.9073	67	
	24	5.4813	2.4404	6.3948	2.8472	7.3084	3.2539	8.2219	3.6606	9.1355	4.0674	66	
	25	5.4378	2.5357	6.3442	2.9583	7.2505	3.3809	8.1568	3.8036	9.0631	4.2262	65	
2 ¼		5.4239	2.5653	6.3279	2.9929	7.2319	3.4204	8.1359	3.8480	9.0399	4.2756		5 ¾
	26	5.3928	2.6302	6.2916	3.0686	7.1904	3.5070	8.0891	3.9453	8.9879	4.3837	64	
	27	5.3460	2.7239	6.2370	3.1779	7.1280	3.6319	8.0191	4.0859	8.9101	4.5399	63	
	28	5.2977	2.8168	6.1806	3.2863	7.0636	3.7558	7.9465	4.2252	8.8295	4.6947	62	
2 ½		5.2915	2.8284	6.1734	3.2998	7.0554	3.7712	7.9373	4.2426	8.8192	4.7140		5 ½
	29	5.2477	2.9089	6.1223	3.3937	6.9970	3.8785	7.8716	4.3633	8.7462	4.8481	61	
	30	5.1962	3.0000	6.0622	3.5000	6.9282	4.0000	7.7942	4.5000	8.6603	5.0000	60	
2 ¾		5.1464	3.0846	6.0041	3.5987	6.8618	4.1128	7.7196	4.6269	8.5773	5.1410		5 ¼
	31	5.1430	3.0902	6.0002	3.6053	6.8573	4.1203	7.7145	4.6353	8.5717	5.1504	59	
	32	5.0883	3.1795	5.9363	3.7094	6.7844	4.2394	7.6324	4.7093	8.4805	5.2992	58	
	33	5.0320	3.2678	5.8707	3.8125	6.7094	4.3571	7.5480	4.9018	8.3867	5.4464	57	
3		4.9888	3.3334	5.8203	3.8890	6.6518	4.4446	7.4832	5.0001	8.3147	5.5557		5
	34	4.9742	3.3552	5.8033	3.9144	6.6323	4.4735	7.4613	5.0327	8.2904	5.5919	56	
	35	4.9149	3.4415	5.7341	4.0150	6.5532	4.5886	7.3724	5.1622	8.1915	5.7358	55	
	36	4.8541	3.5267	5.6631	4.1145	6.5721	4.7023	7.2812	5.2901	8.0902	5.8779	54	
3 ¼		4.8192	3.5742	5.6225	4.1699	6.4257	4.7656	7.2289	5.3613	8.0321	5.9570		4 ¾
	37	4.7918	3.6109	5.5904	4.2127	6.3891	4.8145	7.1877	5.4163	7.9864	6.0182	53	
	38	4.7281	3.6940	5.5161	4.3096	6.3041	4.9253	7.0921	5.5409	7.8801	6.1566	52	
	39	4.6629	3.7759	5.4400	4.4052	6.2172	5.0346	6.9943	5.6639	7.7715	6.2932	51	
3 ½		4.6381	3.8064	5.4111	4.4408	6.1841	5.0751	6.9571	5.7095	7.7301	6.3439		4 ½
	40	4.5963	3.8567	5.3623	4.4995	6.1284	5.1423	6.8944	5.7851	7.6604	6.4279	50	
	41	4.5283	3.9363	5.2830	4.5924	6.0377	5.2485	6.7924	5.9045	7.5471	6.5606	49	
	42	4.4589	4.0148	5.2020	4.6839	5.9452	5.3530	6.6883	6.0222	7.4314	6.6913	48	
3 ¾		4.4457	4.0294	5.1867	4.7009	5.9276	5.3725	6.6686	6.0440	7.4095	6.7156		4 ¼
	43	4.3881	4.0920	5.1195	4.7740	5.8508	5.4560	6.5822	6.1380	7.3135	6.8200	47	
	44	4.3160	4.1680	5.0354	4.8626	5.7547	5.5573	6.4741	6.2519	7.1934	6.9466	46	
4	45	4.2426	4.2426	4.9497	4.9497	5.6569	5.6569	6.3640	6.3640	7.0711	7.0711	45	4
Pts.	Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.	Pts.
		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.			

Moon		Sun	h. m. 0 0	h. m. 0 30	h. m. 1 0	h. m. 1 30	h. m. 2 0	h. m. 2 30	h. m. 3 0	h. m. 3 30	h. m. 4 0	h. m. 4 30	h. m. 5 0	h. m. 5 30
m.	s.	m.	h. 0	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11
0	0	0		1.38021	1.07918	90309	77815	68124	60206	53511	47712	42597	38021	33882
0	30	1	3.15836	1.37303	1.07558	90069	77635	67980	60086	53408	47622	42517	37949	33816
1	0	2	2.85733	1.36597	1.07200	89829	77455	67836	59966	53305	47532	42436	37877	33751
1	30	3	2.68124	1.35902	1.06846	89591	77276	67692	59846	53202	47442	42356	37805	33685
2	0	4	2.55630	1.35218	1.06494	89355	77097	67549	59726	53090	47352	42276	37733	33620
2	30	5	2.45939	1.34545	1.06145	89119	76920	67406	59607	52997	47262	42197	37661	33554
3	0	6	2.38021	1.33882	1.05799	88885	76743	67264	59488	52895	47173	42117	37589	33489
3	30	7	2.31327	1.33229	1.05456	88652	76567	67123	59370	52794	47083	42038	37518	33424
4	0	8	2.25527	1.32585	1.05115	88421	76391	66981	59252	52692	46994	41958	37446	33359
4	30	9	2.20412	1.31951	1.04777	88190	76216	66841	59134	52591	46905	41879	37375	33294
5	0	10	2.15836	1.31327	1.04442	87961	76042	66700	59016	52490	46817	41800	37303	33229
5	30	11	2.11697	1.30711	1.04109	87733	75869	66560	58899	52389	46728	41721	37232	33164
6	0	12	2.07918	1.30103	1.03779	87506	75696	66421	58782	52288	46640	41642	37161	33099
6	30	13	2.04442	1.29504	1.03451	87281	75524	66282	58665	52188	46552	41564	37090	33035
7	0	14	2.01224	1.28913	1.03126	87056	75353	66143	58549	52087	46464	41485	37020	32970
7	30	15	1.98227	1.28330	1.02803	86833	75182	66005	58433	51987	46376	41407	36949	32906
8	0	16	1.95424	1.27755	1.02482	86611	75012	65868	58318	51888	46288	41329	36878	32842
8	30	17	1.92791	1.27187	1.02164	86390	74843	65730	58202	51788	46201	41251	36808	32778
9	0	18	1.90309	1.26627	1.01848	86170	74674	65594	58087	51689	46113	41173	36738	32713
9	30	19	1.87961	1.26074	1.01535	85951	74506	65457	57972	51590	46026	41095	36667	32649
10	0	20	1.85733	1.25527	1.01224	85733	74339	65321	57858	51491	45939	41018	36597	32585
10	30	21	1.83614	1.24988	1.00914	85517	74172	65186	57744	51393	45853	40940	36527	32522
11	0	22	1.81594	1.24455	1.00608	85301	74006	65051	57630	51294	45766	40863	36457	32458
11	30	23	1.79664	1.23929	1.00303	85087	73841	64916	57516	51196	45680	40786	36388	32394
12	0	24	1.77815	1.23408	1.00000	84873	73676	64782	57403	51098	45593	40709	36318	32331
12	30	25	1.76042	1.22894	0.99700	84661	73512	64648	57290	51000	45507	40632	36248	32267
13	0	26	1.74339	1.22387	99401	84450	73348	64515	57178	50903	45421	40555	36179	32204
13	30	27	1.72700	1.21884	99105	84239	73185	64382	57065	50806	45336	40478	36110	32141
14	0	28	1.71121	1.21388	98810	84030	73023	64249	56953	50709	45250	40402	36040	32078
14	30	29	1.69597	1.20897	98518	83822	72861	64117	56841	50612	45165	40325	35971	32014
15	0	30	1.68124	1.20412	0.98227	83614	72700	63985	56730	50515	45079	40249	35902	31951
15	30	31	1.66700	1.19932	97939	83408	72539	63854	56619	50419	44994	40173	35833	31889
16	0	32	1.65321	1.19458	97652	83203	72379	63723	56508	50323	44909	40097	35765	31826
16	30	33	1.63985	1.18988	97367	82998	72220	63592	56397	50227	44825	40021	35696	31763
17	0	34	1.62688	1.18524	97084	82795	72061	63462	56287	50131	44740	39945	35627	31700
17	30	35	1.61430	1.18064	96803	82593	71903	63332	56177	50035	44656	39870	35559	31638
18	0	36	1.60206	1.17609	96524	82391	71745	63202	56067	49940	44571	39794	35491	31575
18	30	37	1.59016	1.17159	96246	82190	71588	63073	55957	49845	44487	39719	35422	31513
19	0	38	1.57858	1.16714	95971	81991	71432	62945	55848	49750	44403	39644	35354	31451
19	30	39	1.56730	1.16273	95697	81792	71276	62816	55739	49655	44320	39569	35286	31389
20	0	40	1.55630	1.15836	0.95424	81594	71121	62688	55630	49561	44236	39494	35218	31327
20	30	41	1.54558	1.15404	95154	81397	70966	62561	55522	49466	44153	39419	35151	31265
21	0	42	1.53511	1.14976	94885	81201	70811	62434	55414	49372	44069	39344	35083	31203
21	30	43	1.52490	1.14554	94618	81006	70658	62307	55306	49278	43986	39270	35015	31141
22	0	44	1.51491	1.14133	94352	80812	70505	62181	55198	49185	43903	39195	34948	31079
22	30	45	1.50515	1.13717	94088	80618	70352	62054	55091	49091	43820	39121	34880	31017
23	0	46	1.49561	1.13306	93826	80426	70200	61929	54984	48998	43738	39047	34813	30956
23	30	47	1.48627	1.12898	93565	80234	70048	61803	54877	48905	43655	38973	34746	30894
24	0	48	1.47712	1.12494	93305	80043	69897	61678	54770	48812	43573	38899	34679	30833
24	30	49	1.46817	1.12094	93048	79853	69747	61554	54664	48719	43491	38825	34612	30772
25	0	50	1.45939	1.11697	0.92791	79664	69597	61430	54558	48627	43409	38751	34545	30711
25	30	51	1.45079	1.11304	92537	79475	69447	61306	54452	48534	43327	38678	34478	30649
26	0	52	1.44236	1.10915	92284	79288	69298	61182	54347	48442	43245	38604	34412	30588
26	30	53	1.43409	1.10529	92032	79101	69150	61059	54241	48350	43164	38531	34345	30527
27	0	54	1.42597	1.10146	91781	78915	69002	60936	54136	48259	43082	38458	34279	30467
27	30	55	1.41800	1.09767	91533	78730	68854	60814	54032	48167	43001	38385	34212	30406
28	0	56	1.41018	1.09391	91285	78545	68707	60691	53927	48076	42920	38312	34146	30345
28	30	57	1.40249	1.09018	91039	78362	68561	60570	53823	47985	42839	38239	34080	30284
29	0	58	1.39494	1.08648	90794	78179	68415	60448	53719	47894	42758	38166	34014	30224
29	30	59	1.38751	1.08282	90551	77997	68269	60327	53615	47803	42677	38094	33948	30163

Moon m.	Moon s.	Sun. m.	h. m. 6 0 / h. 12	h. m. 6 30 / h. 13	h. m. 7 0 / h. 14	h. m. 7 30 / h. 15	h. m. 8 0 / h. 16	h. m. 8 30 / h. 17	h. m. 9 0 / h. 18	h. m. 9 30 / h. 19	h. m. 10 0 / h. 20	h. m. 10 30 / h. 21	h. m. 11 0 / h. 22	h. m. 11 30 / h. 23
0	0	0	30103	26627	23408	20412	17609	14976	12494	10146	07918	05799	03779	01848
0	30	1	30043	26571	23357	20364	17564	14934	12454	10108	07882	05765	03746	01817
1	0	2	29983	26516	23305	20316	17519	14891	12414	10070	07846	05730	03713	01786
1	30	3	29923	26460	23254	20268	17474	14849	12374	10032	07810	05696	03680	01754
2	0	4	29863	26405	23202	20220	17429	14806	12333	09994	07774	05662	03648	01723
2	30	5	29803	26349	23151	20172	17384	14764	12293	09956	07738	05627	03615	01691
3	0	6	29743	26294	23099	20124	17339	14722	12253	09918	07702	05593	03582	01660
3	30	7	29683	26239	23048	20076	17294	14679	12213	09880	07666	05559	03549	01629
4	0	8	29623	26184	22997	20028	17249	14637	12173	09842	07630	05524	03517	01597
4	30	9	29564	26129	22946	19980	17204	14595	12134	09804	07594	05490	03484	01566
5	0	10	29504	26074	22894	19932	17159	14553	12094	09767	07558	05456	03451	01535
5	30	11	29445	26019	22843	19885	17114	14511	12054	09729	07522	05422	03419	01504
6	0	12	29385	25964	22792	19837	17070	14468	12014	09691	07486	05388	03386	01472
6	30	13	29326	25909	22741	19789	17025	14426	11974	09653	07450	05354	03353	01441
7	0	14	29267	25854	22691	19742	16980	14384	11935	09616	07415	05319	03321	01410
7	30	15	29208	25800	22640	19694	16936	14342	11895	09578	07379	05285	03288	01379
8	0	16	29149	25745	22589	19647	16891	14300	11855	09541	07343	05251	03256	01348
8	30	17	29090	25691	22538	19599	16847	14258	11816	09503	07307	05217	03223	01317
9	0	18	29031	25636	22488	19552	16802	14217	11776	09466	07272	05183	03191	01286
9	30	19	28972	25582	22437	19505	16758	14175	11737	09428	07236	05149	03158	01255
10	0	20	28913	25527	22387	19458	16714	14133	11697	09391	07200	05115	03126	01224
10	30	21	28855	25473	22336	19410	16669	14091	11658	09353	07165	05081	03093	01193
11	0	22	28796	25419	22286	19363	16625	14050	11618	09316	07129	05048	03061	01162
11	30	23	28737	25365	22235	19316	16581	14008	11579	09278	07094	05014	03029	01131
12	0	24	28679	25311	22185	19269	16537	13966	11539	09241	07058	04980	02996	01100
12	30	25	28621	25257	22135	19222	16493	13925	11500	09204	07023	04946	02964	01069
13	0	26	28562	25203	22085	19175	16449	13883	11461	09167	06987	04912	02932	01038
13	30	27	28504	25149	22034	19128	16405	13842	11422	09129	06952	04879	02900	01007
14	0	28	28446	25095	21984	19082	16361	13800	11382	09092	06917	04845	02867	00976
14	30	29	28388	25042	21934	19035	16317	13759	11343	09055	06881	04811	02835	00945
15	0	30	28330	24988	21884	18988	16273	13717	11304	09018	06846	04777	02803	00914
15	30	31	28272	24934	21835	18941	16229	13676	11265	08981	06811	04744	02771	00884
16	0	32	28215	24881	21785	18895	16185	13635	11226	08944	06775	04710	02739	00853
16	30	33	28157	24827	21735	18848	16141	13594	11187	08907	06740	04677	02707	00822
17	0	34	28099	24774	21685	18802	16098	13552	11148	08870	06705	04643	02675	00791
17	30	35	28042	24721	21636	18755	16054	13511	11109	08833	06670	04609	02642	00761
18	0	36	27984	24667	21586	18709	16010	13470	11070	08796	06635	04576	02610	00730
18	30	37	27927	24614	21536	18662	15967	13429	11031	08759	06599	04542	02578	00699
19	0	38	27869	24561	21487	18616	15923	13388	10992	08722	06564	04509	02546	00669
19	30	39	27812	24508	21437	18570	15880	13347	10953	08685	06529	04475	02514	00638
20	0	40	27755	24455	21388	18524	15836	13306	10915	08648	06494	04442	02482	00608
20	30	41	27698	24402	21339	18477	15793	13265	10876	08611	06459	04409	02451	00577
21	0	42	27641	24349	21290	18431	15750	13224	10837	08575	06424	04375	02419	00546
21	30	43	27584	24296	21240	18385	15706	13183	10798	08538	06389	04342	02387	00516
22	0	44	27527	24244	21191	18339	15663	13142	10760	08501	06354	04309	02355	00485
22	30	45	27470	24191	21142	18293	15620	13101	10721	08465	06319	04275	02323	00455
23	0	46	27413	24138	21093	18247	15577	13061	10683	08428	06285	04242	02291	00424
23	30	47	27357	24086	21044	18201	15533	13020	10644	08391	06250	04209	02260	00394
24	0	48	27300	24033	20995	18156	15490	12979	10605	08355	06215	04176	02228	00364
24	30	49	27244	23981	20946	18110	15447	12939	10567	08318	06180	04142	02196	00333
25	0	50	27187	23929	20897	18064	15404	12898	10529	08282	06145	04109	02164	00303
25	30	51	27132	23876	20849	18018	15361	12857	10490	08245	06111	04076	02133	00272
26	0	52	27075	23824	20800	17973	15318	12817	10452	08209	06076	04043	02101	00242
26	30	53	27018	23772	20751	17927	15275	12776	10413	08172	06041	04010	02069	00212
27	0	54	26962	23720	20703	17882	15233	12736	10375	08136	06007	03977	02038	00181
27	30	55	26906	23668	20654	17836	15190	12696	10337	08100	05972	03944	02006	00151
28	0	56	26850	23616	20606	17791	15147	12655	10299	08063	05937	03911	01975	00121
28	30	57	26794	23564	20557	17745	15104	12615	10260	08027	05903	03878	01943	00091
29	0	58	26738	23512	20509	17700	15062	12574	10222	07991	05868	03845	01911	00060
29	30	59	26683	23460	20460	17655	15019	12534	10184	07954	05834	03812	01880	00030

0 Degree, or 0 Hour.

″ s	0′ 0m	1′ 1m	2′ 2m	3′ 3m	4′ 4m	5′ 5m	6′ 6m	7′ 7m	8′ 8m	9′ 9m
0		2.25527	1.95424	1.77815	1.65321	1.55630	1.47712	1.41017	1.35218	1.30103
1	4.03342	24809	95064	77575	65141	55486	47592	40914	35128	30023
2	3.73239	24103	94706	77335	64961	55342	47472	40811	35038	29942
3	55630	23408	94352	77097	64782	55198	47352	40708	34948	29862
4	43136	22724	94000	76861	64603	55055	47232	40606	34858	29782
5	33445	22051	93651	76625	64426	54912	47113	40503	34768	29703
6	25527	21388	93305	76391	64249	54770	46994	40401	34679	29623
7	18833	20735	92962	76158	64073	54629	46876	40300	34589	29544
8	13033	20091	92621	75927	63897	54487	46758	40198	34500	29464
9	07918	19457	92283	75696	63722	54347	46640	40097	34411	29385
10	3.03342	2.18833	1.91948	1.75467	1.63548	1.54206	1.46522	1.39996	1.34323	1.29306
11	2.99203	18217	91615	75239	63375	54066	46405	39895	34234	29227
12	95424	17609	91285	75012	63202	53927	46288	39794	34146	29148
13	91948	17010	90957	74787	63030	53788	46171	39694	34058	29070
14	88730	16419	90632	74562	62859	53649	46055	39593	33970	28991
15	85733	15836	90309	74339	62688	53511	45939	39493	33882	28913
16	82930	15261	89988	74117	62518	53374	45824	39394	33794	28835
17	80297	14693	89670	73896	62349	53236	45708	39294	33707	28757
18	77815	14133	89354	73676	62180	53100	45593	39195	33619	28679
19	75467	13580	89041	73457	62012	52963	45478	39096	33532	28601
20	2.73239	2.13033	1.88730	1.73239	1.61845	1.52827	1.45364	1.38997	1.33445	1.28524
21	71120	12494	88420	73023	61678	52692	45250	38899	33359	28446
22	69100	11961	88114	72807	61512	52557	45136	38800	33272	28369
23	67170	11435	87809	72593	61347	52422	45022	38702	33186	28292
24	65321	10914	87506	72379	61182	52288	44909	38604	33099	28215
25	63548	10400	87206	72167	61018	52154	44796	38506	33013	28138
26	61845	09893	86907	71956	60854	52021	44684	38409	32927	28061
27	60206	09390	86611	71745	60691	51888	44571	38312	32842	27984
28	58627	08894	86316	71536	60529	51755	44459	38215	32756	27908
29	57103	08403	86024	71328	60367	51623	44347	38118	32671	27831
30	2.55630	2.07918	1.85733	1.71120	1.60206	1.51491	1.44236	1.38021	1.32585	1.27755
31	54206	07438	85445	70914	60045	51360	44125	37925	32500	27679
32	52827	06964	85158	70709	59885	51229	44014	37829	32415	27603
33	51491	06494	84873	70504	59726	51098	43903	37733	32331	27527
34	50194	06030	84590	70301	59567	50968	43793	37637	32246	27451
35	48936	05570	84309	70099	59409	50838	43683	37541	32162	27376
36	47712	05115	84030	69897	59251	50708	43573	37446	32077	27300
37	46522	04665	83752	69696	59094	50579	43463	37351	31993	27225
38	45364	04220	83477	69497	58938	50451	43354	37256	31909	27150
39	44236	03779	83203	69298	58782	50322	43245	37161	31826	27075
40	2.43136	2.03342	1.82930	1.69100	1.58627	1.50194	1.43136	1.37067	1.31742	1.27000
41	42064	02910	82660	68903	58472	50067	43028	36972	31659	26925
42	41017	02482	82391	68707	58317	49940	42920	36878	31575	26850
43	39996	02060	82124	68512	58164	49813	42812	36784	31492	26776
44	38997	01639	81858	68318	58011	49687	42704	36691	31409	26701
45	38021	01223	81594	68124	57858	49560	42597	36597	31326	26627
46	37067	00812	81332	67932	57706	49435	42490	36504	31244	26553
47	36133	00404	81071	67740	57554	49309	42383	36411	31161	26479
48	35218	00000	80811	67549	57403	49184	42276	36318	31079	26405
49	34323	1.99600	80554	67359	57253	49060	42170	36225	30997	26331
50	2.33445	1.99203	1.80297	1.67170	1.57103	1.48936	1.42064	1.36133	1.30915	1.26257
51	32585	98810	80043	66981	56953	48812	41958	36040	30833	26184
52	31742	98421	79790	66794	56804	48688	41853	35948	30751	26110
53	30915	98035	79538	66607	56656	48565	41747	35856	30670	26037
54	30103	97652	79287	66421	56508	48442	41642	35765	30588	25964
55	29306	97273	79039	66236	56360	48320	41538	35673	30507	25891
56	28524	96897	78791	66051	56213	48197	41433	35582	30426	25818
57	27755	96524	78545	65868	56067	48076	41329	35491	30345	25745
58	27000	96154	78300	65685	55921	47954	41225	35400	30264	25672
59	26257	95788	78057	65503	55775	47833	41121	35309	30183	25600

0 Degree, or 0 Hour.

″ s	$10'$ 10^m	$11'$ 11^m	$12'$ 12^m	$13'$ 13^m	$14'$ 14^m	$15'$ 15^m	$16'$ 16^m	$17'$ 17^m	$18'$ 18^m	$19'$ 19^m
0	1.25527	1.21388	1.17609	1.14133	1.10914	1.07918	1.05115	1.02482	1.00000	0.97652
1	25455	21322	17549	14077	10863	07870	05070	02440	0.99960	97614
2	25383	21257	17489	14022	10811	07822	05025	02397	99920	97576
3	25311	21191	17429	13966	10760	07774	04980	02355	99880	97538
4	25239	21126	17369	13911	10708	07726	04935	02312	99839	97500
5	25167	21060	17309	13855	10657	07678	04890	02270	99799	97462
6	25095	20995	17249	13800	10605	07630	04845	02228	99759	97424
7	25024	20930	17189	13745	10554	07582	04800	02185	99719	97386
8	24952	20865	17129	13690	10503	07534	04755	02143	99679	97348
9	24881	20800	17070	13635	10452	07486	04710	02101	99640	97310
10	1.24809	1.20735	1.17010	1.13580	1.10400	1.07438	1.04665	1.02059	0.99600	0.97273
11	24738	20670	16951	13525	10349	07391	04620	02017	99560	97235
12	24667	20605	16891	13470	10298	07343	04576	01974	99520	97197
13	24596	20541	16832	13415	10247	07295	04531	01932	99480	97159
14	24526	20476	16773	13360	10197	07248	04486	01890	99441	97122
15	24455	20412	16714	13306	10146	07200	04442	01848	99401	97084
16	24384	20348	16655	13251	10095	07153	04397	01806	99361	97047
17	24314	20284	16596	13197	10044	07105	04353	01764	99322	97009
18	24244	20219	16537	13142	09994	07058	04308	01723	99282	96972
19	24173	20155	16478	13088	09943	07011	04264	01681	99243	96934
20	1.24103	1.20091	1.16419	1.13033	1.09893	1.06964	1.04220	1.01639	0.99203	0.96897
21	24033	20028	16361	12979	09842	06916	04175	01597	99164	96859
22	23963	19964	16302	12925	09792	06869	04131	01556	99124	96822
23	23894	19900	16243	12871	09741	06822	04087	01514	99085	96784
24	23824	19837	16185	12817	09691	06775	04043	01472	99045	96747
25	23754	19773	16127	12763	09641	06728	03999	01431	99006	96710
26	23685	19710	16068	12709	09591	06681	03955	01389	98967	96673
27	23616	19647	16010	12655	09540	06634	03911	01348	98928	96635
28	23546	19584	15952	12601	09490	06588	03867	01306	98888	96598
29	23477	19520	15894	12548	09440	06541	03823	01265	98849	96561
30	1.23408	1.19457	1.15836	1.12494	1.09390	1.06494	1.03779	1.01223	0.98810	0.96524
31	23339	19395	15778	12440	09341	06447	03735	01182	98771	96487
32	23271	19332	15721	12387	09291	06401	03691	01141	98732	96450
33	23202	19269	15663	12333	09241	06354	03647	01100	98693	96413
34	23133	19206	15605	12280	09191	06308	03604	01058	98654	96376
35	23065	19144	15548	12227	09142	06261	03560	01017	98615	96339
36	22997	19081	15490	12173	09092	06215	03516	00976	98576	96302
37	22928	19019	15433	12120	09042	06168	03473	00935	98537	96265
38	22860	18957	15375	12067	08993	06122	03429	00894	98498	96228
39	22792	18895	15318	12014	08943	06076	03386	00853	98459	96191
40	1.22724	1.18833	1.15261	1.11961	1.08894	1.06030	1.03342	1.00812	0.98421	0.96154
41	22657	18771	15204	11908	08845	05983	03299	00771	98382	96117
42	22589	18709	15147	11855	08796	05937	03256	00730	98343	96081
43	22521	18647	15090	11802	08746	05891	03212	00689	98304	96044
44	22454	18585	15033	11750	08697	05845	03169	00648	98266	96007
45	22386	18523	14976	11697	08648	05799	03126	00607	98227	95971
46	22319	18462	14919	11644	08599	05753	03083	00567	98189	95934
47	22252	18400	14863	11592	08550	05707	03039	00526	98150	95897
48	22185	18339	14806	11539	08501	05662	02996	00485	98111	95861
49	22118	18278	14750	11487	08452	05616	02953	00445	98073	95824
50	1.22051	1.18217	1.14693	1.11435	1.08403	1.05570	1.02910	1.00404	0.98035	0.95788
51	21984	18155	14637	11382	08355	05524	02867	00363	97996	95751
52	21918	18094	14581	11330	08306	05479	02824	00323	97958	95715
53	21851	18033	14524	11278	08257	05433	02781	00282	97919	95678
54	21785	17973	14468	11226	08209	05388	02739	00242	97881	95642
55	21718	17912	14412	11174	08160	05342	02696	00202	97843	95606
56	21652	17851	14356	11122	08112	05297	02653	00161	97805	95569
57	21586	17790	14300	11070	08063	05251	02610	00121	97766	95533
58	21520	17730	14244	11018	08015	05206	02568	00080	97728	95497
59	21454	17669	14189	10966	07966	05161	02525	00040	97690	95460

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	5	11	16	21	26	32	37	42	48

0 Degree, or 0 Hour.

″ s	20′ 20m	21′ 21m	22′ 22m	23′ 23m	24′ 24m	25′ 25m	26′ 26m	27′ 27m	28′ 28m	29′ 29m
0	95424	93305	91285	89354	87506	85733	84030	82391	80811	79287
1	95388	93271	91252	89323	87476	85704	84002	82364	80786	79263
2	95352	93236	91219	89292	87446	85675	83974	82337	80760	79238
3	95316	93202	91186	89260	87416	85646	83946	82311	80734	79213
4	95280	93168	91154	89229	87386	85618	83919	82284	80708	79188
5	95244	93133	91121	89197	87356	85589	83891	82257	80682	79163
6	95208	93099	91088	89166	87326	85560	83863	82230	80657	79138
7	95172	93065	91055	89135	87296	85531	83835	82204	80631	79113
8	95136	93030	91023	89103	87266	85502	83808	82177	80605	79088
9	95100	92996	90990	89072	87236	85473	83780	82150	80579	79063
10	95064	92962	90957	89041	87206	85445	83752	82124	80554	79039
11	95028	92928	90925	89010	87176	85416	83725	82097	80528	79014
12	94992	92894	90892	88978	87146	85387	83697	82070	80502	78989
13	94956	92860	90859	88947	87116	85358	83670	82044	80477	78964
14	94921	92825	90827	88916	87086	85330	83642	82017	80451	78939
15	94885	92791	90794	88885	87056	85301	83614	81991	80425	78915
16	94849	92757	90762	88854	87026	85272	83587	81964	80400	78890
17	94813	92723	90729	88823	86996	85244	83559	81938	80374	78865
18	94778	92689	90697	88792	86967	85215	83532	81911	80349	78840
19	94742	92655	90664	88761	86937	85187	83504	81884	80323	78816
20	94706	92621	90632	88730	86907	85158	83477	81858	80297	78791
21	94671	92587	90599	88699	86877	85129	83449	81832	80272	78766
22	94635	92554	90567	88668	86848	85101	83422	81805	80246	78742
23	94600	92520	90535	88637	86818	85072	83394	81779	80221	78717
24	94564	92486	90502	88606	86788	85044	83367	81752	80195	78693
25	94529	92452	90470	88575	86759	85015	83339	81726	80170	78668
26	94493	92418	90438	88544	86729	84987	83312	81699	80144	78643
27	94458	92385	90406	88513	86699	84958	83285	81673	80119	78619
28	94423	92351	90373	88482	86670	84930	83257	81647	80094	78594
29	94387	92317	90341	88451	86640	84902	83230	81620	80068	78570
30	94352	92283	90309	88420	86611	84873	83203	81594	80043	78545
31	94317	92250	90277	88390	86581	84845	83175	81568	80017	78521
32	94281	92216	90245	88359	86552	84816	83148	81541	79992	78496
33	94246	92183	90213	88328	86522	84788	83121	81515	79967	78472
34	94211	92149	90181	88297	86493	84760	83094	81489	79941	78447
35	94176	92115	90148	88267	86463	84732	83066	81463	79916	78423
36	94141	92082	90116	88236	86434	84703	83039	81436	79891	78398
37	94105	92048	90084	88205	86404	84675	83012	81410	79865	78374
38	94070	92015	90052	88175	86375	84647	82985	81384	79840	78349
39	94035	91981	90020	88144	86346	84619	82958	81358	79815	78325
40	94000	91948	89988	88114	86316	84590	82930	81332	79790	78300
41	93965	91915	89957	88083	86287	84562	82903	81305	79764	78276
42	93930	91881	89925	88052	86258	84534	82876	81279	79739	78252
43	93895	91848	89893	88022	86228	84506	82849	81253	79714	78227
44	93860	91815	89861	87991	86199	84478	82822	81227	79689	78203
45	93825	91781	89829	87961	86170	84450	82795	81201	79663	78179
46	93791	91748	89797	87930	86141	84421	82768	81175	79638	78154
47	93756	91715	89766	87900	86111	84393	82741	81149	79613	78130
48	93721	91682	89734	87870	86082	84365	82714	81123	79588	78106
49	93686	91648	89702	87839	86053	84337	82687	81097	79563	78081
50	93651	91615	89670	87809	86024	84309	82660	81071	79538	78057
51	93617	91582	89639	87778	85995	84281	82633	81045	79513	78033
52	93582	91549	89607	87748	85965	84253	82606	81019	79488	78009
53	93547	91516	89575	87718	85936	84225	82579	80993	79463	77984
54	93513	91483	89544	87687	85907	84197	82552	80967	79437	77960
55	93478	91450	89512	87657	85878	84169	82525	80941	79412	77936
56	93443	91417	89481	87627	85849	84141	82498	80915	79387	77912
57	93409	91384	89449	87597	85820	84114	82471	80889	79362	77888
58	93374	91351	89417	87566	85791	84086	82445	80863	79337	77863
59	93340	91318	89386	87536	85762	84058	82418	80837	79312	77839

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	3	6	9	12	15	18	21	24	27

0 Degree, or 0 Hour.

" s	30′ m	31′ m	32′ m	33′ m	34′ m	35′ m	36′ m	37′ m	38′ m	39′ m
0	77815	76391	75012	73676	72379	71120	69897	68707	67549	66421
1	77791	76368	74990	73654	72358	71100	69877	68688	67530	66402
2	77767	76344	74967	73632	72337	71079	69857	68668	67511	66384
3	77743	76321	74944	73610	72316	71058	69837	68648	67492	66365
4	77719	76298	74922	73588	72294	71038	69817	68629	67473	66347
5	77695	76275	74899	73566	72273	71017	69797	68609	67454	66328
6	77671	76251	74877	73544	72252	70997	69777	68590	67435	66310
7	77647	76228	74854	73523	72231	70976	69756	68570	67416	66291
8	77623	76205	74832	73501	72209	70955	69736	68551	67397	66273
9	77599	76181	74809	73479	72188	70935	69716	68531	67378	66254
10	77575	76158	74787	73457	72167	70914	69696	68512	67359	66236
11	77551	76135	74764	73435	72146	70894	69676	68492	67340	66217
12	77527	76112	74742	73413	72125	70873	69656	68473	67321	66199
13	77503	76089	74719	73392	72103	70852	69636	68454	67302	66180
14	77479	76065	74697	73370	72082	70832	69616	68434	67283	66162
15	77455	76042	74674	73348	72061	70811	69596	68415	67264	66143
16	77431	76019	74652	73326	72040	70791	69576	68395	67245	66125
17	77407	75996	74629	73305	72019	70770	69557	68376	67226	66106
18	77383	75973	74607	73283	71998	70750	69537	68356	67207	66088
19	77359	75950	74585	73261	71977	70729	69517	68337	67188	66070
20	77335	75927	74562	73239	71956	70709	69497	68318	67170	66051
21	77311	75903	74540	73218	71935	70688	69477	68298	67151	66033
22	77288	75880	74517	73196	71914	70668	69457	68279	67132	66014
23	77264	75857	74495	73174	71892	70647	69437	68259	67113	65996
24	77240	75834	74473	73153	71871	70627	69417	68240	67094	65978
25	77216	75811	74450	73131	71850	70606	69397	68221	67075	65959
26	77192	75788	74428	73109	71829	70586	69377	68201	67056	65941
27	77169	75765	74406	73088	71808	70566	69358	68182	67038	65923
28	77145	75742	74383	73066	71787	70545	69338	68163	67019	65904
29	77121	75719	74361	73044	71766	70525	69318	68143	67000	65886
30	77097	75696	74339	73023	71745	70504	69298	68124	66981	65868
31	77074	75673	74317	73001	71724	70484	69278	68105	66962	65849
32	77050	75650	74294	72980	71703	70464	69258	68086	66944	65831
33	77026	75627	74272	72958	71682	70443	69239	68066	66925	65813
34	77002	75604	74250	72936	71662	70423	69219	68047	66906	65794
35	76979	75581	74228	72915	71641	70403	69199	68028	66887	65776
36	76955	75559	74205	72893	71620	70382	69179	68008	66869	65758
37	76931	75536	74183	72872	71599	70362	69159	67989	66850	65739
38	76908	75513	74161	72850	71578	70342	69140	67970	66831	65721
39	76884	75490	74139	72829	71557	70321	69120	67951	66812	65703
40	76861	75467	74117	72807	71536	70301	69100	67932	66794	65685
41	76837	75444	74095	72786	71515	70281	69080	67912	66775	65666
42	76813	75421	74072	72764	71494	70260	69061	67893	66756	65648
43	76790	75398	74050	72743	71473	70240	69041	67874	66737	65630
44	76766	75376	74028	72721	71453	70220	69021	67855	66719	65612
45	76743	75353	74006	72700	71432	70200	69002	67836	66700	65594
46	76719	75330	73984	72678	71411	70179	68982	67816	66681	65575
47	76696	75307	73962	72657	71390	70159	68962	67797	66663	65557
48	76672	75285	73940	72636	71369	70139	68942	67778	66644	65539
49	76649	75262	73918	72614	71349	70119	68923	67759	66625	65521
50	76625	75239	73896	72593	71328	70099	68903	67740	66607	65503
51	76602	75216	73874	72571	71307	70078	68884	67721	66588	65484
52	76578	75194	73852	72550	71286	70058	68864	67702	66570	65466
53	76555	75171	73830	72529	71265	70038	68844	67682	66551	65448
54	76531	75148	73808	72507	71245	70018	68825	67663	66532	65430
55	76508	75125	73786	72486	71224	69998	68805	67644	66514	65412
56	76485	75103	73764	72465	71203	69977	68785	67625	66495	65394
57	76461	75080	73742	72443	71183	69957	68766	67606	66477	65376
58	76438	75058	73720	72422	71162	69937	68746	67587	66458	65357
59	76414	75035	73698	72401	71141	69917	68727	67568	66439	65339

Proportional Part to tenths of " or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	2	4	6	8	10	13	15	17	19

0 Degree, or 0 Hour.

″ s	40′m	41′m	42′m	43′m	44′m	45′m	46′m	47′m	48′m	49′m
0	65321	64249	63202	62180	61182	60206	59251	58317	57403	56508
1	65303	64231	63185	62164	61166	60190	59236	58302	57388	56493
2	65285	64214	63168	62147	61149	60174	59220	58287	57373	56478
3	65267	64196	63151	62130	61133	60158	59204	58271	57358	56463
4	65249	64178	63133	62113	61116	60142	59189	58256	57343	56449
5	65231	64161	63116	62096	61100	60126	59173	58241	57328	56434
6	65213	64143	63099	62080	61083	60110	59157	58225	57313	56419
7	65195	64125	63082	62063	61067	60094	59141	58210	57298	56404
8	65177	64108	63065	62046	61051	60078	59126	58194	57283	56390
9	65159	64090	63048	62029	61034	60061	59110	58179	57268	56375
10	65141	64073	63030	62012	61018	60045	59094	58164	57253	56360
11	65123	64055	63013	61996	61001	60029	59079	58148	57238	56345
12	65105	64038	62996	61979	60985	60013	59063	58133	57223	56331
13	65087	64020	62979	61962	60969	59997	59047	58118	57208	56316
14	65069	64002	62962	61945	60952	59981	59032	58102	57193	56301
15	65051	63985	62945	61929	60936	59965	59016	58087	57178	56287
16	65033	63967	62927	61912	60920	59949	59000	58072	57163	56272
17	65015	63950	62910	61895	60903	59933	58985	58056	57148	56257
18	64997	63932	62893	61878	60887	59917	58969	58041	57133	56243
19	64979	63915	62876	61862	60871	59901	58954	58026	57118	56228
20	64961	63897	62859	61845	60854	59885	58938	58011	57103	56213
21	64943	63880	62842	61828	60838	59870	58922	57995	57088	56199
22	64925	63862	62825	61812	60822	59854	58907	57980	57073	56184
23	64907	63845	62808	61795	60805	59838	58891	57965	57058	56169
24	64889	63827	62791	61778	60789	59822	58875	57949	57043	56155
25	64871	63810	62774	61762	60773	59806	58860	57934	57028	56140
26	64853	63792	62757	61745	60756	59790	58844	57919	57013	56125
27	64835	63775	62739	61728	60740	59774	58829	57904	56998	56111
28	64818	63757	62722	61712	60724	59758	58813	57888	56983	56096
29	64800	63740	62705	61695	60708	59742	58798	57873	56968	56081
30	64782	63722	62688	61678	60691	59726	58782	57858	56953	56067
31	64764	63705	62671	61662	60675	59710	58766	57843	56938	56052
32	64746	63688	62654	61645	60659	59694	58751	57827	56923	56037
33	64728	63610	62637	61628	60642	59678	58735	57812	56908	56023
34	64710	63653	62620	61612	60626	59663	58720	57797	56893	56008
35	64692	63635	62603	61595	60610	59647	58704	57782	56879	55994
36	64675	63618	62586	61579	60594	59631	58689	57767	56864	55979
37	64657	63601	62569	61562	60578	59615	58673	57751	56849	55965
38	64639	63583	62552	61545	60561	59599	58658	57736	56834	55950
39	64621	63566	62535	61529	60545	59583	58642	57721	56819	55935
40	64603	63548	62518	61512	60529	59567	58627	57706	56804	55921
41	64586	63531	62501	61496	60513	59551	58611	57691	56789	55906
42	64568	63514	62484	61479	60496	59536	58596	57675	56774	55892
43	64550	63496	62468	61463	60480	59520	58580	57660	56759	55877
44	64532	63479	62451	61446	60464	59504	58565	57645	56745	55862
45	64514	63462	62434	61429	60448	59488	58549	57630	56730	55848
46	64497	63444	62417	61413	60432	59472	58534	57615	56715	55833
47	64479	63427	62400	61396	60416	59457	58518	57600	56700	55819
48	64461	63410	62383	61380	60399	59441	58503	57584	56685	55804
49	64443	63392	62366	61363	60383	59425	58487	57569	56670	55790
50	64426	63375	62349	61347	60367	59409	58472	57554	56656	55775
51	64408	63358	62332	61330	60351	59393	58456	57539	56641	55761
52	64390	63340	62315	61314	60335	59378	58441	57524	56626	55746
53	64373	63323	62298	61297	60319	59362	58425	57509	56611	55732
54	64355	63306	62282	61281	60303	59346	58410	57494	56596	55717
55	64337	63289	62265	61264	60286	59330	58395	57479	56582	55703
56	64320	63271	62248	61248	60270	59314	58379	57463	56567	55688
57	64302	63254	62231	61231	60254	59299	58364	57448	56552	55674
58	64284	63237	62214	61215	60238	59283	58348	57433	56537	55659
59	64267	63220	62197	61198	60222	59267	58333	57418	56522	55645

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	2	3	5	6	8	10	11	13	14

0 Degree, or 0 Hour.

″ s	50′ 50m	51′ 51m	52′ 52m	53′ 53m	54′ 54m	55′ 55m	56′ 56m	57′ 57m	58′ 58m	59′ 59m
0	55630	54770	53927	53100	52288	51491	50708	49940	49184	48442
1	55616	54756	53913	53086	52274	51478	50696	49927	49172	48430
2	55601	54742	53899	53072	52261	51465	50683	49914	49159	48418
3	55587	54728	53885	53059	52248	51452	50670	49902	49147	48405
4	55572	54714	53871	53045	52234	51438	50657	49889	49135	48393
5	55558	54699	53857	53031	52221	51425	50644	49876	49122	48381
6	55543	54685	53843	53018	52208	51412	50631	49864	49110	48369
7	55529	54671	53830	53004	52194	51399	50618	49851	49097	48356
8	55515	54657	53816	52991	52181	51386	50605	49838	49085	48344
9	55500	54643	53802	52977	52167	51373	50592	49826	49072	48332
10	55486	54629	53788	52963	52154	51360	50579	49813	49060	48320
11	55471	54614	53774	52950	52141	51346	50566	49800	49047	48307
12	55457	54600	53760	52936	52127	51333	50554	49788	49035	48295
13	55442	54586	53746	52922	52114	51320	50541	49775	49023	48283
14	55428	54572	53732	52909	52101	51307	50528	49762	49010	48271
15	55414	54558	53719	52895	52087	51294	50515	49750	48998	48258
16	55399	54544	53705	52882	52074	51281	50502	49737	48985	48246
17	55385	54530	53691	52868	52061	51268	50489	49724	48973	48234
18	55370	54516	53677	52855	52047	51255	50476	49712	48960	48222
19	55356	54501	53663	52841	52034	51242	50464	49699	48948	48210
20	55342	54487	53649	52827	52021	51229	50451	49687	48936	48197
21	55327	54473	53636	52814	52007	51215	50438	49674	48923	48185
22	55313	54459	53622	52800	51994	51202	50425	49661	48911	48173
23	55299	54445	53608	52787	51981	51189	50412	49649	48898	48161
24	55284	54431	53594	52773	51967	51176	50399	49636	48886	48149
25	55270	54417	53580	52760	51954	51163	50387	49623	48874	48136
26	55255	54403	53567	52746	51941	51150	50374	49611	48861	48124
27	55241	54389	53553	52732	51927	51137	50361	49598	48849	48112
28	55227	54375	53539	52719	51914	51124	50348	49586	48836	48100
29	55212	54361	53525	52705	51901	51111	50335	49573	48824	48088
30	55198	54347	53511	52692	51888	51098	50322	49560	48812	48076
31	55184	54332	53498	52678	51874	51085	50310	49548	48799	48063
32	55169	54318	53484	52665	51861	51072	50297	49535	48787	48051
33	55155	54304	53470	52651	51848	51059	50284	49523	48775	48039
34	55141	54290	53456	52638	51835	51046	50271	49510	48762	48027
35	55127	54276	53442	52624	51821	51033	50258	49498	48750	48015
36	55112	54262	53429	52611	51808	51020	50246	49485	48737	48003
37	55098	54248	53415	52597	51795	51007	50233	49472	48725	47990
38	55084	54234	53401	52584	51781	50994	50220	49460	48713	47978
39	55069	54220	53387	52570	51768	50981	50207	49447	48700	47966
40	55055	54206	53374	52557	51755	50968	50194	49435	48688	47954
41	55041	54192	53360	52543	51742	50955	50182	49422	48676	47942
42	55026	54178	53346	52530	51729	50942	50169	49410	48663	47930
43	55012	54164	53332	52516	51715	50929	50156	49397	48651	47918
44	54998	54150	53319	52503	51702	50916	50143	49385	48639	47906
45	54984	54136	53305	52489	51689	50903	50131	49372	48626	47893
46	54969	54122	53291	52476	51676	50890	50118	49360	48614	47881
47	54955	54108	53278	52462	51662	50877	50105	49347	48602	47869
48	54941	54094	53264	52449	51649	50864	50092	49334	48590	47857
49	54927	54080	53250	52436	51636	50851	50080	49322	48577	47845
50	54912	54066	53236	52422	51623	50838	50067	49309	48565	47833
51	54898	54052	53223	52409	51610	50825	50054	49297	48553	47821
52	54884	54038	53209	52395	51596	50812	50041	49284	48540	47809
53	54870	54024	53195	52382	51583	50799	50029	49272	48528	47797
54	54855	54011	53182	52368	51570	50786	50016	49259	48516	47785
55	54841	53997	53168	52355	51557	50773	50003	49247	48503	47772
56	54827	53983	53154	52342	51544	50760	49991	49234	48491	47760
57	54813	53969	53141	52328	51530	50747	49978	49222	48479	47748
58	54799	53955	53127	52315	51517	50734	49965	49209	48467	47736
59	54784	53941	53113	52301	51504	50721	49952	49197	48454	47724

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	3	4	5	6	8	9	10	12

1 Degree, or 1 Hour.

″ s	0′ m	1′ m	2′ m	3′ m	4′ m	5′ m	6′ m	7′ m	8′ m	9′ m	10′ m	11′ m
0	47712	46994	46288	45593	44909	44236	43573	42920	42276	41642	41017	40401
1	47700	46982	46276	45582	44898	44225	43562	42909	42266	41632	41007	40391
2	47688	46971	46265	45570	44887	44214	43551	42898	42255	41621	40997	40381
3	47676	46959	46253	45559	44875	44203	43540	42887	42244	41611	40986	40371
4	47664	46947	46241	45547	44864	44191	43529	42877	42234	41600	40976	40361
5	47652	46935	46230	45536	44853	44180	43518	42866	42223	41590	40966	40350
6	47640	46923	46218	45524	44841	44169	43507	42855	42213	41579	40955	40340
7	47628	46911	46206	45513	44830	44158	43496	42844	42202	41569	40945	40330
8	47616	46899	46195	45501	44819	44147	43485	42833	42191	41559	40935	40320
9	47604	46888	46183	45490	44808	44136	43474	42823	42181	41548	40924	40310
10	47592	46876	46171	45478	44796	44125	43463	42812	42170	41538	40914	40300
11	47580	46864	46160	45467	44785	44114	43452	42801	42159	41527	40904	40289
12	47568	46852	46148	45456	44774	44102	43441	42790	42149	41517	40894	40279
13	47556	46840	46137	45444	44762	44091	43431	42780	42138	41506	40883	40269
14	47544	46828	46125	45433	44751	44080	43420	42769	42128	41496	40873	40259
15	47532	46817	46113	45421	44740	44069	43409	42758	42117	41485	40863	40249
16	47520	46805	46102	45410	44729	44058	43398	42747	42106	41475	40852	40239
17	47508	46793	46090	45398	44717	44047	43387	42737	42096	41464	40842	40228
18	47496	46781	46078	45387	44706	44036	43876	42726	42085	41454	40832	40218
19	47484	46769	46067	45375	44695	44025	43365	42715	42075	41443	40821	40208
20	47472	46758	46055	45364	44684	44014	43354	42704	42064	41433	40811	40198
21	47460	46746	46044	45353	44672	44003	43343	42693	42053	41423	40801	40188
22	47448	46734	46032	45341	44661	43992	43332	42683	42043	41412	40791	40178
23	47436	46722	46020	45330	44650	43981	43321	42672	42032	41402	40780	40168
24	47424	46710	46009	45318	44639	43969	43310	42661	42022	41391	40770	40157
25	47412	46699	45997	45307	44627	43958	43300	42651	42011	41381	40760	40147
26	47400	46687	45986	45295	44616	43947	43289	42640	42000	41370	40749	40137
27	47388	46675	45974	45284	44605	43936	43278	42629	41990	41360	40739	40127
28	47376	46663	45962	45273	44594	43925	43267	42618	41979	41350	40729	40117
29	47364	46652	45951	45261	44583	43914	43256	42608	41969	41339	40719	40107
30	47352	46640	45939	45250	44571	43903	43245	42597	41958	41329	40708	40097
31	47340	46628	45928	45238	44560	43892	43234	42586	41948	41318	40698	40087
32	47328	46616	45916	45227	44549	43881	43223	42575	41937	41308	40688	40076
33	47316	46604	45905	45216	44538	43870	43212	42565	41927	41298	40678	40066
34	47301	46593	45893	45204	44526	43859	43202	42554	41916	41287	40667	40056
35	47292	46581	45881	45193	44515	43848	43191	42543	41905	41277	40657	40046
36	47280	46569	45870	45182	44504	43837	43180	42533	41895	41266	40647	40036
37	47268	46557	45858	45170	44493	43826	43169	42522	41884	41256	40637	40026
38	47256	46546	45847	45159	44482	43815	43158	42511	41874	41246	40626	40016
39	47244	46534	45835	45147	44470	43804	43147	42500	41863	41235	40616	40006
40	47232	46522	45824	45136	44459	43793	43136	42490	41853	41225	40606	39996
41	47220	46510	45812	45125	44448	43782	43126	42479	41842	41214	40596	39985
42	47208	46499	45800	45113	44437	43771	43115	42468	41832	41204	40585	39975
43	47196	46487	45789	45102	44426	43760	43104	42458	41821	41194	40575	39965
44	47185	46475	45777	45091	44414	43749	43093	42447	41811	41183	40565	39955
45	47173	46464	45766	45079	44403	43738	43082	42436	41800	41173	40555	39945
46	47161	46452	45754	45068	44392	43727	43071	42426	41789	41162	40544	39935
47	47149	46440	45743	45057	44381	43716	43060	42415	41779	41152	40534	39925
48	47137	46428	45731	45045	44370	43705	43050	42404	41768	41142	40524	39915
49	47125	46417	45720	45034	44359	43694	43039	42394	41758	41131	40514	39905
50	47113	46405	45708	45022	44347	43683	43028	42383	41747	41121	40503	39895
51	47101	46393	45697	45011	44336	43672	43017	42372	41737	41111	40493	39885
52	47089	46382	45685	45000	44325	43661	43006	42362	41726	41100	40483	39674
53	47077	46370	45674	44988	44314	43650	42995	42351	41716	41090	40473	39864
54	47066	46358	45662	44977	44303	43639	42985	42340	41705	41080	40463	39854
55	47054	46346	45651	44966	44292	43628	42974	42330	41695	41069	40452	39844
56	47042	46335	45639	44955	44280	43617	42963	42319	41684	41059	40442	39834
57	47030	46323	45628	44943	44269	43606	42952	42308	41674	41048	40432	39824
58	47018	46311	45616	44932	44258	43595	42941	42298	41663	41038	40422	39814
59	47006	46300	45605	44921	44247	43584	42931	42287	41653	41028	40412	39804

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	9
	1	2	3	4	5	7	8	9	.10

1 Degree, or 1 Hour.

″ s.	12′ m	13′ m	14′ m	15′ m	16′ m	17′ m	18′ m	19′ m	20′ m	21′ m	22′ m	23′ m
0	39794	39195	38604	38021	37446	36878	36318	35765	35218	34679	34146	33619
1	39784	39185	38594	38011	37436	36869	36309	35755	35209	34670	34137	33611
2	39774	39175	38585	38002	37427	36859	36299	35746	35200	34661	34128	33602
3	39764	39165	38575	37992	37417	36850	36290	35737	35191	34652	34119	33593
4	39754	39155	38565	37983	37408	36841	36281	35728	35182	34643	34111	33585
5	39744	39145	38555	37973	37398	36831	36271	35719	35173	34634	34102	33576
6	39734	39136	38545	37963	37389	36822	36262	35710	35164	34625	34093	33567
7	39724	39126	38536	37954	37379	36812	36253	35700	35155	34616	34084	33558
8	39714	39116	38526	37944	37370	36803	36244	35691	35146	34607	34075	33550
9	39704	39106	38516	37934	37360	36794	36234	35682	35137	34598	34066	33541
10	39694	39096	38506	37925	37351	36784	36225	35673	35128	34589	34058	33532
11	39684	39086	38497	37915	37341	36775	36216	35664	35119	34581	34049	33524
12	39674	39076	38487	37905	37332	36766	36207	35655	35110	34572	34040	33515
13	39664	39066	38477	37896	37322	36756	36197	35646	35101	34563	34031	33506
14	39653	39056	38467	37886	37313	36747	36188	35636	35092	34554	34022	33498
15	39643	39046	38458	37877	37303	36737	36179	35627	35083	34545	34014	33489
16	39633	39037	38448	37867	37294	36728	36170	35618	35074	34536	34005	33480
17	39623	39027	38438	37857	37284	36719	36160	35609	35065	34527	33996	33471
18	39613	39017	38428	37848	37275	36709	36151	35600	35056	34518	33987	33463
19	39603	39007	38419	37838	37265	36700	36142	35591	35047	34509	33978	33454
20	39593	38997	38409	37829	37256	36691	36133	35582	35038	34500	33970	33445
21	39583	38987	38399	37819	37246	36681	36123	35573	35029	34491	33961	33437
22	39573	38977	38389	37809	37237	36672	36114	35563	35020	34483	33952	33428
23	39563	38968	38380	37800	37227	36663	36105	35554	35011	34474	33943	33419
24	39553	38958	38370	37790	37218	36653	36096	35545	35002	34465	33935	33411
25	39543	38948	38360	37781	37208	36644	36086	35536	34993	34456	33926	33402
26	39533	38938	38351	37771	37199	36634	36077	35527	34984	34447	33917	33393
27	39523	38928	38341	37761	37189	36625	36068	35518	34975	34438	33908	33385
28	39513	38918	38331	37752	37180	36616	36059	35509	34966	34429	33899	33376
29	39503	38908	38321	37742	37171	36606	36050	35500	34957	34420	33891	33367
30	39493	38899	38312	37733	37161	36597	36040	35491	34948	34411	33882	33359
31	39483	38889	38302	37723	37152	36588	36031	35481	34939	34403	33873	33350
32	39473	38879	38292	37713	37142	36578	36022	35472	34930	34394	33864	33341
33	39464	38869	38282	37704	37133	36569	36013	35463	34921	34385	33856	33333
34	39454	38859	38273	37694	37123	36560	36003	35454	34912	34376	33847	33324
35	39444	38849	38263	37685	37114	36550	35994	35445	34903	34367	33838	33315
36	39434	38839	38253	37675	37104	36541	35985	35436	34894	34358	33829	33307
37	39424	38830	38244	37665	37095	36532	35976	35427	34885	34349	33820	33298
38	39414	38820	38234	37656	37085	36522	35967	35418	34876	34340	33812	33289
39	39404	38810	38224	37646	37076	36513	35957	35409	34867	34332	33803	33281
40	39394	38800	38215	37637	37067	36504	35948	35400	34858	34323	33794	33272
41	39384	38790	38205	37627	37057	36494	35939	35391	34849	34314	33785	33263
42	39374	38781	38195	37618	37048	36485	35930	35381	34840	34305	33777	33255
43	39364	38771	38186	37608	37038	36476	35921	35372	34831	34296	33768	33246
44	39354	38761	38176	37599	37029	36467	35911	35363	34822	34287	33759	33237
45	39344	38751	38166	37589	37019	36457	35902	35354	34813	34278	33750	33229
46	39334	38741	38156	37579	37010	36448	35893	35345	34804	34270	33742	33220
47	39324	38731	38147	37570	37001	36439	35884	35336	34795	34261	33733	33211
48	39314	38722	38137	37560	36991	36429	35875	35327	34786	34252	33724	33203
49	39304	38712	38127	37551	36982	36420	35865	35318	34777	34243	33715	33194
50	39294	38702	38118	37541	36972	36411	35856	35309	34768	34234	33707	33186
51	39284	38692	38108	37532	36963	36401	35847	35300	34759	34225	33698	33177
52	39274	38682	38098	37522	36953	36392	35838	35291	34750	34217	33689	33168
53	39264	38673	38089	37513	36944	36383	35829	35282	34741	34208	33681	33160
54	39254	38663	38079	37503	36935	36374	35820	35273	34732	34199	33672	33151
55	39245	38653	38069	37494	36925	36364	35810	35264	34723	34190	33663	33142
56	39235	38643	38060	37484	36916	36355	35801	35254	34715	34181	33654	33134
57	39225	38633	38050	37474	36906	36346	35792	35245	34706	34172	33646	33125
58	39215	38624	38040	37465	36897	36336	35783	35236	34697	34164	33637	33117
59	39205	38614	38031	37455	36888	36327	35774	35227	34688	34155	33628	33108

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	2	3	4	4	5	6	7	8

1 Degree, or 1 Hour.

″ s	24′ m	25′ m	26′ m	27′ m	28′ m	29′ m	30′ m	31′ m	32′ m	33′ m	34′ m	35′ m
0	33099	32585	32077	31575	31079	30588	30103	29623	29148	28679	28214	27755
1	33091	32577	32069	31567	31071	30580	30095	29615	29141	28671	28207	27747
2	33082	32568	32061	31559	31063	30572	30087	29607	29133	28663	28199	27740
3	33073	32560	32052	31550	31054	30564	30079	29599	29125	28656	28191	27732
4	33065	32551	32044	31542	31046	30556	30071	29591	29117	28648	28184	27724
5	33056	32543	32035	31534	31038	30548	30063	29583	29109	28640	28176	27717
6	33048	32534	32027	31525	31030	30539	30055	29575	29101	28632	28168	27709
7	33039	32526	32019	31517	31021	30531	30047	29567	29093	28625	28161	27702
8	33030	32517	32010	31509	31013	30523	30039	29560	29086	28617	28153	27694
9	33022	32509	32002	31501	31005	30515	30031	29552	29078	28609	28145	27686
10	33013	32500	31993	31492	30997	30507	30023	29544	29070	28601	28138	27679
11	33005	32492	31985	31484	30989	30499	30015	29536	29062	28593	28130	27671
12	32996	32483	31977	31476	30980	30491	30007	29528	29054	28586	28122	27664
13	32987	32475	31968	31467	30972	30483	29999	29520	29046	28578	28114	27656
14	32979	32466	31960	31459	30964	30475	29991	29512	29038	28570	28107	27648
15	32970	32458	31951	31451	30956	30466	29983	29504	29031	28562	28099	27641
16	32962	32449	31943	31442	30948	30458	29975	29496	29023	28555	28091	27633
17	32953	32441	31935	31434	30939	30450	29967	29488	29015	28547	28084	27626
18	32944	32432	31926	31426	30931	30442	29958	29480	29007	28539	28076	27618
19	32936	32424	31918	31418	30923	30434	29950	29472	28999	28531	28068	27610
20	32927	32415	31909	31409	30915	30426	29942	29464	28991	28524	28061	27603
21	32919	32407	31901	31401	30907	30418	29934	29456	28984	28516	28053	27595
22	32910	32398	31893	31393	30898	30410	29926	29448	28976	28508	28045	27588
23	32902	32390	31884	31384	30890	30402	29918	29441	28968	28500	28038	27580
24	32893	32381	31876	31376	30882	30393	29910	29433	28960	28493	28030	27572
25	32884	32373	31867	31368	30874	30385	29902	29425	28952	28485	28022	27565
26	32876	32365	31859	31360	30866	30377	29894	29417	28944	28477	28015	27557
27	32867	32356	31851	31351	30857	30369	29886	29409	28937	28469	28007	27550
28	32859	32348	31842	31343	30849	30361	29878	29401	28929	28462	27999	27542
29	32850	32339	31834	31335	30841	30353	29870	29393	28921	28454	27992	27534
30	32842	32331	31826	31326	30833	30345	29862	29385	28913	28446	27984	27527
31	32833	32322	31817	31318	30825	30337	29854	29377	28905	28438	27976	27519
32	32824	32314	31809	31310	30817	30329	29846	29369	28897	28431	27969	27512
33	32816	32305	31801	31302	30808	30321	29838	29361	28890	28423	27961	27504
34	32807	32297	31792	31293	30800	30313	29830	29354	28882	28415	27953	27497
35	32799	32288	31784	31285	30792	30305	29822	29346	28874	28407	27946	27489
36	32790	32280	31775	31277	30784	30296	29814	29338	28866	28400	27938	27481
37	32782	32271	31767	31269	30776	30288	29806	29330	28858	28392	27930	27474
38	32773	32263	31759	31260	30768	30280	29798	29322	28851	28384	27923	27466
39	32765	32255	31750	31252	30759	30272	29790	29314	28843	28376	27915	27459
40	32756	32246	31742	31244	30751	30264	29782	29306	28835	28369	27908	27451
41	32747	32238	31734	31236	30743	30256	29775	29298	28827	28361	27900	27444
42	32739	32229	31725	31227	30735	30248	29767	29290	28819	28353	27892	27436
43	32730	32221	31717	31219	30727	30240	29759	29282	28811	28346	27885	27429
44	32722	32212	31709	31211	30719	30232	29751	29275	28804	28338	27877	27421
45	32713	32204	31700	31203	30710	30224	29743	29267	28796	28330	27869	27413
46	32705	32195	31692	31194	30702	30216	29735	29259	28788	28322	27862	27406
47	32696	32187	31684	31186	30694	30208	29727	29251	28780	28315	27854	27398
48	32688	32179	31675	31178	30686	30200	29719	29243	28772	28307	27846	27391
49	32679	32170	31667	31170	30678	30192	29711	29235	28765	28299	27839	27383
50	32671	32162	31659	31161	30670	30184	29703	29227	28757	28292	27831	27376
51	32662	32153	31650	31153	30662	30175	29695	29219	28749	28284	27824	27368
52	32654	32145	31642	31145	30653	30167	29687	29211	28741	28276	27816	27360
53	32645	32136	31634	31137	30645	30159	29679	29204	28733	28268	27808	27353
54	32636	32128	31625	31128	30637	30151	29671	29196	28726	28261	27801	27345
55	32628	32120	31617	31120	30629	30143	29663	29188	28718	28253	27793	27338
56	32619	32111	31609	31112	30621	30135	29655	29180	28710	28245	27785	27330
57	32611	32103	31600	31104	30613	30127	29647	29172	28702	28238	27778	27323
58	32602	32094	31592	31095	30605	30119	29639	29164	28695	28230	27770	27315
59	32504	32086	31584	31087	30596	30111	29631	29156	28687	28222	27763	27308

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	2	2	3	4	5	6	6	7

1 Degree, or 1 Hour.

″ s	36′ m	37′ m	38′ m	39′ m	40′ m	41′ m	42′ m	43′ m	44′ m	45′ m	46′ m	47′ m
0	27300	26850	26405	25964	25527	25095	24667	24244	23824	23408	22997	22589
1	27293	26843	26397	25956	25520	25088	24660	24237	23817	23401	22990	22582
2	27285	26835	26390	25949	25513	25081	24653	24229	23810	23395	22983	22575
3	27278	26828	26382	25942	25506	25074	24646	24222	23803	23388	22976	22569
4	27270	26820	26375	25934	25498	25066	24639	24215	23796	23381	22969	22562
5	27262	26813	26368	25927	25491	25059	24632	24208	23789	23374	22963	22555
6	27255	26805	26360	25920	25484	25052	24625	24201	23782	23367	22956	22548
7	27247	26798	26353	25913	25477	25045	24618	24194	23775	23360	22949	22542
8	27240	26790	26346	25905	25469	25038	24610	24187	23768	23353	22942	22535
9	27232	26783	26338	25898	25462	25031	24603	24180	23761	23346	22935	22528
10	27225	26776	26331	25891	25455	25024	24596	24173	23754	23339	22928	22521
11	27217	26768	26323	25883	25448	25016	24589	24166	23747	23333	22922	22515
12	27210	26761	26316	25876	25440	25009	24582	24159	23740	23326	22915	22508
13	27202	26753	26309	25869	25433	25002	24575	24152	23734	23319	22908	22501
14	27195	26746	26301	25861	25426	24995	24568	24145	23727	23312	22901	22494
15	27187	26738	26294	25854	25419	24988	24561	24138	23720	23305	22894	22488
16	27180	26731	26287	25847	25412	24981	24554	24131	23713	23298	22888	22481
17	27172	26723	26279	25840	25404	24973	24547	24124	23706	23291	22881	22474
18	27165	26716	26271	25832	25397	24966	24540	24117	23699	23284	22874	22467
19	27157	26709	26265	25825	25390	24959	24533	24110	23692	23278	22867	22461
20	27150	26701	26257	25818	25383	24952	24526	24103	23685	23271	22860	22454
21	27142	26694	26250	25810	25376	24945	24518	24096	23678	23264	22854	22447
22	27135	26686	26242	25803	25368	24938	24511	24089	23671	23257	22847	22440
23	27127	26679	26235	25796	25361	24931	24504	24082	23664	23250	22840	22434
24	27120	26671	26228	25789	25354	24923	24497	24075	23657	23243	22833	22427
25	27112	26664	26220	25781	25347	24916	24490	24068	23650	23236	22826	22420
26	27105	26656	26213	25774	25339	24909	24483	24061	23643	23229	22819	22413
27	27097	26649	26206	25767	25332	24902	24476	24054	23636	23223	22813	22407
28	27090	26642	26198	25759	25325	24895	24469	24047	23629	23216	22806	22400
29	27082	26634	26191	25752	25318	24888	24462	24040	23623	23209	22799	22393
30	27075	26627	26184	25745	25311	24881	24455	24033	23616	23202	22792	22386
31	27067	26619	26176	25738	25303	24874	24448	24026	23609	23195	22785	22380
32	27060	26612	26169	25730	25296	24866	24441	24019	23602	23188	22779	22373
33	27052	26605	26162	25723	25289	24859	24434	24012	23595	23181	22772	22366
34	27045	26597	26154	25716	25282	24852	24427	24005	23588	23175	22765	22359
35	27037	26590	26147	25709	25275	24845	24420	23998	23581	23168	22758	22353
36	27030	26582	26140	25701	25267	24838	24413	23991	23574	23161	22752	22346
37	27022	26575	26132	25694	25260	24831	24405	23984	23567	23154	22745	22339
38	27015	26567	26125	25687	25253	24824	24398	23977	23560	23147	22738	22333
39	27007	26560	26118	25680	25246	24817	24391	23970	23553	23140	22731	22326
40	27000	26553	26110	25672	25239	24809	24384	23963	23546	23133	22724	22319
41	26992	26545	26103	25665	25231	24802	24377	23956	23539	23127	22718	22312
42	26985	26538	26096	25658	25224	24795	24370	23949	23533	23120	22711	22306
43	26977	26530	26088	25650	25217	24788	24363	23942	23526	23113	22704	22299
44	26970	26523	26081	25643	25210	24781	24356	23935	23519	23106	22697	22292
45	26962	26516	26074	25636	25203	24774	24349	23928	23512	23099	22690	22286
46	26955	26508	26066	25629	25196	24767	24342	23921	23505	23092	22684	22279
47	26947	26501	26059	25621	25188	24760	24335	23914	23498	23086	22677	22272
48	26940	26493	26052	25614	25181	24752	24328	23908	23491	23079	22670	22265
49	26932	26486	26044	25607	25174	24745	24321	23901	23484	23072	22663	22259
50	26925	26479	26037	25600	25167	24738	24314	23894	23477	23065	22657	22252
51	26917	26471	26030	25592	25160	24731	24307	23887	23470	23058	22650	22245
52	26910	26464	26022	25585	25152	24724	24300	23880	23464	23051	22643	22239
53	26902	26456	26015	25578	25145	24717	24293	23873	23457	23044	22636	22232
54	26895	26449	26008	25571	25138	24710	24286	23866	23450	23038	22629	22225
55	26887	26442	26000	25563	25131	24703	24279	23859	23443	23031	22623	22218
56	26880	26434	25993	25556	25124	24696	24272	23852	23436	23024	22616	22212
57	26872	26427	25986	25549	25117	24689	24265	23845	23429	23017	22609	22205
58	26865	26419	25978	25542	25109	24681	24258	23838	23422	23010	22602	22198
59	26858	26412	25971	25534	25102	24674	24251	23831	23415	23004	22596	22192

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	1	2	3	3	4	5	6	6

1 Degree, or 1 Hour.

″ s	48′ 48m	49′ 49m	50′ 50m	51′ 51m	52′ 52m	53′ 53m	54′ 54m	55′ 55m	56′ 56m	57′ 57m	58′ 58m	59′ 59m
0	22185	21785	21388	20995	20605	20219	19837	19457	19081	18709	18339	17973
1	22178	21778	21381	20988	20599	20213	19830	19451	19075	18702	18333	17966
2	22171	21771	21375	20982	20593	20207	19824	19445	19069	18696	18327	17960
3	22165	21765	21368	20975	20586	20200	19818	19439	19063	18690	18321	17954
4	22158	21758	21362	20969	20580	20194	19811	19432	19056	18684	18315	17948
5	22151	21751	21355	20962	20573	20187	19805	19426	19050	18678	18308	17942
6	22145	21745	21349	20956	20567	20181	19799	19420	19044	18672	18302	17936
7	22138	21738	21342	20949	20560	20175	19792	19413	19038	18665	18296	17930
8	22131	21732	21335	20943	20554	20168	19786	19407	19032	18659	18290	17924
9	22125	21725	21329	20936	20547	20162	19780	19401	19025	18653	18284	17918
10	22118	21718	21322	20930	20541	20155	19773	19395	19019	18647	18278	17912
11	22111	21712	21316	20923	20534	20149	19767	19388	19013	18641	18272	17906
12	22105	21705	21309	20917	20528	20143	19761	19382	19007	18634	18266	17900
13	22098	21698	21303	20910	20522	20136	19754	19376	19000	18628	18259	17894
14	22091	21692	21296	20904	20515	20130	19748	19369	18994	18622	18253	17887
15	22084	21685	21289	20897	20509	20123	19742	19363	18988	18616	18247	17881
16	22078	21678	21283	20891	20502	20117	19735	19357	18982	18610	18241	17875
17	22071	21672	21276	20884	20496	20111	19729	19351	18976	18604	18235	17869
18	22064	21665	21270	20878	20489	20104	19723	19344	18969	18597	18229	17863
19	22058	21659	21263	20871	20483	20098	19716	19338	18963	18591	18223	17857
20	22051	21652	21257	20865	20476	20091	19710	19332	18957	18585	18217	17851
21	22044	21645	21250	20858	20470	20085	19704	19325	18951	18579	18210	17845
22	22038	21639	21243	20852	20464	20079	19697	19319	18944	18573	18204	17839
23	22031	21632	21237	20845	20457	20072	19691	19313	18938	18567	18198	17833
24	22024	21626	21230	20839	20451	20066	19685	19307	18932	18560	18192	17827
25	22018	21619	21224	20832	20444	20060	19678	19300	18926	18554	18186	17821
26	22011	21612	21217	20826	20438	20053	19672	19294	18920	18548	18180	17815
27	22004	21606	21211	20819	20431	20047	19666	19288	18913	18542	18174	17809
28	21998	21599	21204	20813	20425	20040	19659	19282	18907	18536	18168	17803
29	21991	21592	21198	20806	20418	20034	19653	19275	18901	18530	18162	17797
30	21984	21586	21191	20800	20412	20028	19647	19269	18895	18523	18155	17790
31	21978	21579	21184	20793	20406	20021	19640	19263	18888	18517	18149	17784
32	21971	21573	21178	20787	20399	20015	19634	19257	18882	18511	18143	17778
33	21964	21566	21171	20780	20393	20009	19628	19250	18876	18505	18137	17772
34	21958	21559	21165	20774	20386	20002	19621	19244	18870	18499	18131	17766
35	21951	21553	21158	20767	20380	19996	19615	19238	18864	18493	18125	17760
36	21944	21546	21152	20761	20373	19989	19609	19231	18857	18487	18119	17754
37	21938	21540	21145	20754	20367	19983	19602	19225	18851	18480	18113	17748
38	21931	21533	21139	20748	20361	19977	19596	19219	18845	18474	18107	17742
39	21924	21526	21132	20741	20354	19970	19590	19213	18839	18468	18100	17736
40	21918	21520	21126	20735	20348	19964	19584	19206	18833	18462	18094	17730
41	21911	21513	21119	20728	20341	19958	19577	19200	18826	18456	18088	17724
42	21904	21507	21112	20722	20335	19951	19571	19194	18820	18450	18082	17718
43	21898	21500	21106	20715	20328	19945	19565	19188	18814	18443	18076	17712
44	21891	21493	21099	20709	20322	19938	19558	19181	18808	18437	18070	17706
45	21884	21487	21093	20702	20316	19932	19552	19175	18802	18431	18064	17700
46	21878	21480	21086	20696	20309	19926	19546	19169	18795	18425	18058	17694
47	21871	21474	21080	20690	20303	19919	19539	19163	18789	18419	18052	17688
48	21864	21467	21073	20683	20296	19913	19533	19156	18783	18413	18046	17682
49	21858	21460	21067	20677	20290	19907	19527	19150	18777	18407	18040	17676
50	21851	21454	21060	20670	20284	19900	19520	19144	18771	18400	18033	17669
51	21844	21447	21054	20664	20277	19894	19514	19138	18764	18394	18027	17663
52	21838	21441	21047	20657	20271	19888	19508	19131	18758	18388	18021	17657
53	21831	21434	21041	20651	20264	19881	19502	19125	18752	18382	18015	17651
54	21824	21427	21034	20644	20258	19875	19495	19119	18746	18376	18009	17645
55	21818	21421	21028	20638	20251	19869	19489	19113	18740	18370	18003	17639
56	21811	21414	21021	20631	20245	19862	19483	19106	18733	18364	17997	17633
57	21805	21408	21015	20625	20239	19856	19476	19100	18727	18357	17991	17627
58	21798	21401	21008	20618	20232	19849	19470	19094	18721	18351	17985	17621
59	21791	21395	21001	20612	20226	19843	19464	19088	18715	18345	17979	17615

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	1	2	3	3	4	5	6	6

2 Degrees, or 2 Hours.

″ s	0′ m	1′ m	2′ m	3′ m	4′ m	5′ m	6′ m	7′ m	8′ m	9′ m	10′ m	11′ m
0	17609	17249	16891	16537	16185	15836	15490	15147	14806	14468	14133	13800
1	17603	17243	16885	16531	16179	15830	15484	15141	14801	14463	14127	13795
2	17597	17237	16879	16525	16173	15825	15479	15135	14795	14457	14122	13789
3	17591	17231	16873	16519	16168	15819	15473	15130	14789	14451	14116	13784
4	17585	17225	16868	16513	16162	15813	15467	15124	14784	14446	14111	13778
5	17579	17219	16862	16507	16156	15807	15461	15118	14778	14440	14105	13773
6	17573	17213	16856	16501	16150	15802	15456	15113	14772	14435	14100	13767
7	17567	17207	16850	16496	16144	15796	15450	15107	14767	14429	14094	13761
8	17561	17201	16844	16490	16138	15790	15444	15101	14761	14423	14088	13756
9	17555	17195	16838	16484	16133	15784	15439	15096	14755	14418	14083	13750
10	17549	17189	16832	16478	16127	15778	15433	15090	14750	14412	14077	13745
11	17543	17183	16826	16472	16121	15773	15427	15084	14744	14407	14072	13739
12	17537	17177	16820	16466	16115	15767	15421	15079	14738	14401	14066	13734
13	17531	17171	16814	16460	16109	15761	15416	15073	14733	14395	14061	13728
14	17525	17165	16808	16454	16103	15755	15410	15067	14727	14390	14055	13723
15	17519	17159	16802	16449	16098	15749	15404	15061	14722	14384	14049	13717
16	17513	17153	16796	16443	16092	15744	15398	15056	14716	14379	14044	13712
17	17507	17147	16791	16437	16086	15738	15393	15050	14710	14373	14038	13706
18	17501	17141	16785	16431	16080	15732	15387	15044	14705	14367	14033	13701
19	17495	17135	16779	16425	16074	15726	15381	15039	14699	14362	14027	13695
20	17489	17129	16773	16419	16068	15721	15375	15033	14693	14356	14022	13690
21	17483	17123	16767	16413	16063	15715	15370	15027	14688	14351	14016	13684
22	17477	17117	16761	16407	16057	15709	15364	15022	14682	14345	14011	13679
23	17471	17111	16755	16402	16051	15703	15358	15016	14676	14339	14005	13673
24	17465	17105	16749	16396	16045	15697	15353	15010	14671	14334	14000	13668
25	17459	17099	16743	16390	16039	15692	15347	15005	14665	14328	13994	13662
26	17453	17093	16737	16384	16034	15686	15341	14999	14659	14323	13988	13657
27	17447	17087	16731	16378	16028	15680	15335	14993	14654	14317	13983	13651
28	17441	17082	16725	16372	16022	15674	15330	14988	14648	14311	13977	13646
29	17435	17076	16720	16366	16016	15669	15324	14982	14643	14306	13972	13640
30	17429	17070	71614	16361	16010	15663	15318	14976	14637	14300	13966	13635
31	17423	17064	16708	16355	16005	15657	15312	14971	14631	14295	13961	13629
32	17417	17058	16702	16349	15999	15651	15307	14965	14626	14289	13955	13624
33	17411	17052	16696	16343	15993	15646	15301	14959	14620	14284	13950	13618
34	17405	17046	16690	16337	15987	15640	15295	14954	14614	14278	13944	13613
35	17399	17040	16684	16331	15981	15634	15290	14948	14609	14272	13938	13607
36	17393	17034	16678	16325	15975	15628	15284	14942	14603	14267	13933	13602
37	17387	17028	16672	16320	15970	15623	15278	14937	14598	14261	13927	13596
38	17381	17022	16666	16314	15964	15617	15272	14931	14592	14256	13922	13591
39	17375	17016	16660	16308	15958	15611	15267	14925	14586	14250	13916	13585
40	17369	17010	16655	16302	15952	15605	15261	14919	14581	14244	13911	13580
41	17363	17004	16649	16296	15946	15599	15255	14914	14575	14239	13905	13574
42	17357	16998	16643	16290	15941	15594	15250	14908	14569	14233	13900	13569
43	17351	16992	16637	16284	15935	15588	15244	14902	14564	14228	13894	13563
44	17345	16986	16631	16279	15929	15582	15238	14897	14558	14222	13889	13558
45	17339	16980	16625	16273	15923	15576	15232	14891	14553	14217	13883	13552
46	17333	16974	16619	16267	15917	15571	15227	14885	14547	14211	13878	13547
47	17327	16968	16613	16261	15912	15565	15221	14880	14541	14205	13872	13541
48	17321	16963	16607	16255	15906	15559	15215	14874	14536	14200	13866	13536
49	17315	16957	16602	16249	15900	15553	15210	14869	14530	14194	13861	13530
50	17309	16951	16596	16243	15894	15548	15204	14863	14524	14189	13855	13525
51	17303	16945	16590	16238	15888	15542	15198	14857	14519	14183	13850	13519
52	17297	16939	16584	16232	15883	15536	15192	14852	14513	14177	13844	13514
53	17291	16933	16578	16226	15877	15530	15187	14846	14508	14172	13839	13508
54	17285	16927	16572	16220	15871	15525	15181	14840	14502	14166	13833	13503
55	17279	16921	16566	16214	15865	15519	15175	14835	14496	14161	13828	13497
56	17273	16915	16560	16208	15859	15513	15170	14829	14491	14155	13822	13492
57	17267	16909	16551	16203	15854	15507	15164	14823	14485	14150	13817	13486
58	17261	16903	16549	16197	15848	15502	15158	14818	14480	14144	13811	13481
59	17255	16897	16543	16191	15842	15496	15152	14812	14474	14138	13806	13475

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	1	1	2	2	3	4	4	5	5

2 Degrees, or 2 Hours.

″ s	12′ m	13′ m	14′ m	15′ m	16′ m	17′ m	18′ m	19′ m	20′ m	21′ m	22′ m	23′ m
0	13470	13142	12817	12494	12173	11855	11539	11226	10914	10605	10298	09994
1	13464	13137	12811	12489	12168	11850	11534	11221	10909	10600	10293	09989
2	13459	13131	12806	12483	12163	11845	11529	11215	10904	10595	10288	09984
3	13453	13126	12801	12478	12157	11839	11524	11210	10899	10590	10283	09978
4	13448	13120	12795	12472	12152	11834	11518	11205	10894	10585	10278	09973
5	13442	13115	12790	12467	12147	11829	11513	11200	10889	10580	10273	09968
6	13437	13109	12784	12462	12141	11824	11508	11195	10883	10575	10268	09963
7	13431	13104	12779	12456	12136	11818	11503	11189	10878	10569	10263	09958
8	13426	13099	12774	12451	12131	11813	11497	11184	10873	10564	10258	09953
9	13421	13093	12768	12446	12125	11808	11492	11179	10868	10559	10253	09948
10	13415	13088	12763	12440	12120	11802	11487	11174	10863	10554	10247	09943
11	13410	13082	12757	12435	12115	11797	11482	11169	10858	10549	10242	09938
12	13404	13077	12752	12430	12110	11792	11476	11163	10852	10544	10237	09933
13	13399	13071	12747	12424	12104	11787	11471	11158	10847	10539	10232	09928
14	13393	13066	12741	12419	12099	11781	11466	11153	10842	10534	10227	09923
15	13388	13061	12736	12414	12094	11776	11461	11148	10837	10528	10222	09918
16	13382	13055	12730	12408	12088	11771	11456	11143	10832	10523	10217	09913
17	13377	13050	12725	12403	12083	11765	11450	11137	10827	10518	10212	09908
18	13371	13044	12720	12397	12078	11760	11445	11132	10821	10513	10207	09903
19	13366	13039	12714	12392	12072	11755	11440	11127	10816	10508	10202	09898
20	13360	13033	12709	12387	12067	11750	11435	11122	10811	10503	10197	09893
21	13355	13028	12703	12381	12062	11744	11429	11117	10806	10498	10192	09887
22	13349	13023	12698	12376	12056	11739	11424	11111	10801	10493	10186	09882
23	13344	13017	12693	12371	12051	11734	11419	11106	10796	10487	10181	09877
24	13338	13012	12687	12365	12046	11729	11414	11101	10791	10482	10176	09872
25	13333	13006	12682	12360	12041	11723	11408	11096	10785	10477	10171	09867
26	13328	13001	12677	12355	12035	11718	11403	11091	10780	10472	10166	09862
27	13322	12995	12671	12349	12030	11713	11398	11085	10775	10467	10161	09857
28	13317	12990	12666	12344	12025	11708	11393	11080	10770	10462	10156	09852
29	13311	12985	12660	12339	12019	11702	11387	11075	10765	10457	10151	09847
30	13306	12979	12655	12333	12014	11697	11382	11070	10760	10452	10146	09842
31	13300	12974	12650	12328	12009	11692	11377	11065	10754	10446	10141	09837
32	13295	12968	12644	12323	12003	11686	11372	11059	10749	10441	10136	09832
33	13289	12963	12639	12317	11998	11681	11367	11054	10744	10436	10131	09827
34	13284	12957	12634	12312	11993	11676	11361	11049	10739	10431	10125	09822
35	13278	12952	12628	12307	11987	11671	11356	11044	10734	10426	10120	09817
36	13273	12947	12623	12301	11982	11665	11351	11039	10729	10421	10115	09812
37	13267	12941	12617	12296	11977	11660	11346	11034	10724	10416	10110	09807
38	13262	12936	12612	12291	11972	11655	11340	11028	10718	10411	10105	09802
39	13257	12930	12607	12285	11966	11650	11335	11023	10713	10406	10100	09797
40	13251	12925	12601	12280	11961	11644	11330	11018	10708	10400	10095	09792
41	13246	12920	12596	12275	11956	11639	11325	11013	10703	10395	10090	09787
42	13240	12914	12590	12269	11950	11634	11320	11008	10698	10390	10085	09782
43	13235	12909	12585	12264	11945	11629	11314	11002	10693	10385	10080	09777
44	13229	12903	12580	12259	11940	11623	11309	10997	10688	10380	10075	09772
45	13224	12898	12574	12253	11935	11618	11304	10992	10682	10375	10070	09766
46	13218	12892	12569	12248	11929	11613	11299	10987	10677	10370	10065	09761
47	13213	12887	12564	12243	11924	11608	11294	10982	10672	10365	10059	09756
48	13207	12882	12558	12237	11919	11602	11288	10977	10667	10360	10054	09751
49	13202	12876	12553	12232	11913	11597	11283	10971	10662	10355	10049	09746
50	13197	12871	12548	12227	11908	11592	11278	10966	10657	10349	10044	09741
51	13191	12865	12542	12221	11903	11587	11273	10961	10652	10344	10039	09736
52	13186	12860	12537	12216	11897	11581	11267	10956	10646	10339	10034	09731
53	13180	12855	12531	12211	11892	11576	11262	10951	10641	10334	10029	09726
54	13175	12849	12526	12205	11887	11571	11257	10945	10636	10329	10024	09721
55	13169	12844	12521	12200	11882	11566	11252	10940	10631	10324	10019	09716
56	13164	12838	12515	12195	11876	11560	11247	10935	10626	10319	10014	09711
57	13158	12833	12510	12189	11871	11555	11241	10930	10621	10314	10009	09706
58	13153	12828	12505	12184	11866	11550	11236	10925	10616	10309	10004	09701
59	13148	12822	12499	12179	11860	11545	11231	10920	10610	10304	09999	09696

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	0	1	1	2	2	3	3	4	4

2 Degrees, or 2 Hours.

″ s	24′ m	25′ m	26′ m	27′ m	28′ m	29′ m	30′ m	31′ m	32′ m	33′ m	34′ m	35′ m
0	09691	09390	09092	08796	08501	08209	07918	07630	07343	07058	06775	06494
1	09686	09385	09087	08791	08496	08204	07913	07625	07338	07053	06770	06489
2	09681	09380	09082	08786	08491	08199	07908	07620	07333	07049	06766	06485
3	09676	09375	09077	08781	08486	08194	07904	07615	07329	07044	06761	06480
4	09671	09370	09072	08776	08482	08189	07899	07610	07324	07039	06756	06475
5	09666	09365	09067	08771	08477	08184	07894	07606	07319	07034	06752	06471
6	09661	09361	09062	08766	08472	08179	07889	07601	07314	07030	06747	06466
7	09656	09356	09057	08761	08467	08175	07884	07596	07310	07025	06742	06461
8	09651	09351	09052	08756	08462	08170	07880	07591	07305	07020	06738	06457
9	09646	09346	09047	08751	08457	08165	07875	07586	07300	07016	06733	06452
10	09641	09341	09042	08746	08452	08160	07870	07582	07295	07011	06728	06447
11	09636	09336	09037	08741	08447	08155	07865	07577	07291	07006	06724	06443
12	09631	09331	09033	08736	08442	08150	07860	07572	07286	07001	06719	06438
13	09626	09326	09028	08732	08438	08146	07855	07567	07281	06997	06714	06433
14	09621	09321	09023	08727	08433	08141	07851	07562	07276	06992	06709	06429
15	09616	09316	09018	08722	08428	08136	07846	07558	07272	06987	06705	06424
16	09611	09311	09013	08717	08423	08131	07841	07553	07267	06982	06700	06419
17	09606	09306	09008	08712	08418	08126	07836	07548	07262	06978	06695	06415
18	09601	09301	09003	08707	08413	08121	07831	07543	07257	06973	06691	06410
19	09596	09296	08998	08702	08408	08116	07827	07539	07253	06968	06686	06405
20	09591	09291	08993	08697	08403	08112	07822	07534	07248	06964	06681	06401
21	09586	09286	08988	08692	08398	08107	07817	07529	07243	06959	06677	06396
22	09581	09281	08983	08687	08394	08102	07812	07524	07238	06954	06672	06391
23	09576	09276	08978	08682	08389	08097	07807	07519	07234	06949	06667	06387
24	09571	09271	08973	08678	08384	08092	07802	07515	07229	06945	06663	06382
25	09566	09266	08968	08673	08379	08087	07798	07510	07224	06940	06658	06377
26	09561	09261	08963	08668	08374	08083	07793	07505	07219	06935	06653	06373
27	09555	09256	08958	08663	08369	08078	07788	07500	07215	06931	06648	06368
28	09550	09251	08953	08658	08364	08073	07783	07496	07210	06926	06644	06364
29	09545	09246	08948	08653	08359	08068	07778	07491	07205	06921	06639	06359
30	09540	09241	08943	08648	08355	08063	07774	07486	07200	06916	06634	06354
31	09535	09236	08939	08643	08350	08058	07769	07481	07196	06912	06630	06350
32	09530	09231	08934	08638	08345	08053	07764	07476	07191	06907	06625	06345
33	09525	09226	08929	08633	08340	08049	07759	07472	07186	06902	06620	06340
34	09520	09221	08924	08628	08335	08044	07754	07467	07181	06898	06616	06336
35	09515	09216	08919	08624	08330	08039	07750	07462	07177	06893	06611	06331
36	09510	09211	08914	08619	08325	08034	07745	07457	07172	06888	06606	06326
37	09505	09206	08909	08614	08320	08029	07740	07453	07167	06883	06602	06322
38	09500	09201	08904	08609	08316	08024	07735	07448	07162	06879	06597	06317
39	09495	09196	08899	08604	08311	08020	07730	07443	07158	06874	06592	06312
40	09490	09191	08894	08599	08306	08015	07726	07438	07153	06869	06588	06308
41	09485	09186	08889	08594	08301	08010	07721	07433	07148	06865	06583	06303
42	09480	09181	08884	08589	08296	08005	07716	07429	07143	06860	06578	06298
43	09475	09176	08879	08584	08291	08000	07711	07424	07139	06855	06574	06294
44	09470	09171	08874	08579	08286	07995	07706	07419	07134	06850	06569	06289
45	09465	09166	08869	08575	08282	07991	07702	07414	07129	06846	06564	06284
46	09460	09161	08865	08570	08277	07986	07697	07410	07124	06841	06560	06280
47	09455	09156	08860	08565	08272	07981	07692	07405	07120	06836	06555	06275
48	09450	09151	08855	08560	08267	07976	07687	07400	07115	06832	06550	06271
49	09445	09147	08850	08555	08262	07971	07682	07395	07110	06827	06545	06266
50	09440	09142	08845	08550	08257	07966	07678	07391	07105	06822	06541	06261
51	09435	09137	08840	08545	08252	07962	07673	07386	07101	06817	06536	06257
52	09430	09132	08835	08540	08248	07957	07668	07381	07096	06813	06531	06252
53	09425	09127	08830	08535	08243	07952	07663	07376	07091	06808	06527	06247
54	09420	09122	08825	08530	08238	07947	07658	07371	07087	06803	06522	06243
55	09415	09117	08820	08526	08233	07942	07654	07367	07082	06799	06517	06238
56	09410	09112	08815	08521	08228	07937	07649	07362	07077	06794	06513	06233
57	09405	09107	08810	08516	08223	07933	07644	07357	07072	06789	06508	06229
58	09400	09102	08805	08511	08218	07928	07639	07352	07068	06785	06503	06224
59	09395	09097	08800	08506	08213	07923	07634	07348	07063	06780	06499	06219

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	0	1	1	2	2	3	3	4	4

2 Degrees, or 2 Hours.

″ s	36′ m	37′ m	38′ m	39′ m	40′ m	41′ m	42′ m	43′ m	44′ m	45′ m	46′ m	47′ m
0	06215	05937	05662	05388	05115	04845	04576	04308	04043	03779	03516	03256
1	06210	05933	05657	05383	05111	04840	04571	04304	04038	03774	03512	03251
2	06206	05928	05652	05378	05106	04836	04567	04300	04034	03770	03508	03247
3	06201	05923	05648	05374	05102	04831	04562	04295	04030	03766	03503	03243
4	06196	05919	05643	05369	05097	04827	04558	04291	04025	03761	03499	03238
5	06192	05914	05639	05365	05093	04822	04553	04286	04021	03757	03495	03234
6	06187	05910	05634	05360	05088	04818	04549	04282	04016	03753	03490	03230
7	06182	05905	05630	05356	05084	04813	04544	04277	04012	03748	03486	03225
8	06178	05900	05625	05351	05079	04809	04540	04273	04008	03744	03482	03221
9	06173	05896	05620	05347	05075	04804	04536	04269	04003	03739	03477	03217
10	06168	05891	05616	05342	05070	04800	04531	04264	03999	03735	03473	03212
11	06164	05887	05611	05337	05066	04795	04527	04260	03994	03731	03469	03208
12	06159	05882	05607	05333	05061	04791	04522	04255	03990	03726	03464	03204
13	06155	05877	05602	05328	05056	04786	04518	04251	03986	03722	03460	03199
14	06150	05873	05597	05324	05052	04782	04513	04246	03981	03717	03455	03195
15	06145	05868	05593	05319	05047	04777	04509	04242	03977	03713	03451	03191
16	06141	05864	05588	05315	05043	04773	04504	04237	03972	03709	03447	03186
17	06136	05859	05584	05310	05038	04768	04500	04233	03968	03704	03442	03182
18	06131	05854	05579	05306	05034	04764	04495	04229	03963	03700	03438	03178
19	06127	05850	05575	05301	05029	04759	04491	04224	03959	03696	03434	03173
20	06122	05845	05570	05297	05025	04755	04486	04220	03955	03691	03429	03169
21	06117	05841	05565	05292	05020	04750	04482	04215	03950	03687	03425	03165
22	06113	05836	05561	05288	05016	04746	04478	04211	03946	03682	03421	03160
23	06108	05831	05556	05283	05011	04741	04473	04206	03941	03678	03416	03156
24	06104	05827	05552	05278	05007	04737	04469	04202	03937	03674	03412	03152
25	06099	05822	05547	05274	05002	04732	04464	04198	03933	03669	03408	03147
26	06094	05818	05543	05269	04998	04728	04460	04193	03928	03665	03403	03143
27	06090	05813	05538	05265	04993	04723	04455	04189	03924	03661	03399	03139
28	06085	05808	05533	05260	04989	04719	04451	04184	03919	03656	03395	03134
29	06080	05804	05529	05256	04984	04714	04446	04180	03915	03652	03390	03130
30	06076	05799	05524	05251	04980	04710	04442	04175	03911	03647	03386	03126
31	06071	05795	05520	05247	04975	04706	04437	04171	03906	03643	03381	03121
32	06067	05790	05515	05242	04971	04701	04433	04167	03902	03639	03377	03117
33	06062	05785	05511	05238	04966	04697	04429	04162	03897	03634	03373	03113
34	06057	05781	05506	05233	04962	04692	04424	04158	03893	03630	03368	03108
35	06053	05776	05501	05228	04957	04688	04420	04153	03889	03626	03364	03104
36	06048	05772	05497	05224	04953	04683	04415	04149	03884	03621	03360	03100
37	06043	05767	05492	05219	04948	04679	04411	04144	03880	03617	03355	03096
38	06039	05762	05488	05215	04944	04674	04406	04140	03875	03612	03351	03091
39	06034	05758	05483	05210	04939	04670	04402	04136	03871	03608	03347	03087
40	06030	05753	05479	05206	04935	04665	04397	04131	03867	03604	03342	03083
41	06025	05749	05474	05201	04930	04661	04393	04127	03862	03599	03338	03078
42	06020	05744	05470	05197	04926	04656	04388	04122	03858	03595	03334	03074
43	06016	05739	05465	05192	04921	04652	04384	04118	03853	03591	03329	03070
44	06011	05735	05460	05188	04917	04647	04380	04114	03849	03586	03325	03065
45	06006	05730	05456	05183	04912	04643	04375	04109	03845	03582	03321	03061
46	06002	05726	05451	05179	04908	04638	04371	04105	03840	03578	03316	03057
47	05997	05721	05447	05174	04903	04634	04366	04100	03836	03573	03312	03052
48	05993	05717	05442	05170	04899	04629	04362	04096	03832	03569	03308	03048
49	05988	05712	05438	05165	04894	04625	04357	04091	03827	03564	03303	03044
50	05983	05707	05433	05161	04890	04620	04353	04087	03823	03560	03299	03039
51	05979	05703	05429	05156	04885	04616	04348	04083	03818	03556	03295	03035
52	05974	05698	05424	05151	04881	04612	04344	04078	03814	03551	03290	03031
53	05970	05694	05419	05147	04876	04607	04340	04074	03810	03547	03286	03026
54	05965	05689	05415	05142	04872	04603	04335	04069	03805	03543	03282	03022
55	05960	05684	05410	05138	04867	04598	04331	04065	03801	03538	03277	03018
56	05956	05680	05406	05133	04863	04594	04326	04061	03796	03534	03273	03014
57	05951	05675	05401	05129	04858	04589	04322	04056	03792	03530	03269	03009
58	05947	05671	05397	05124	04854	04585	04317	04052	03788	03525	03264	03005
59	05942	05666	05392	05120	04849	04580	04313	04047	03783	03521	03260	03001

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	0	1	1	2	2	3	3	3	4

2 Degrees, or 2 Hours.

″ s	48′ m	49′ m	50′ m	51′ m	52′ m	53′ m	54′ m	55′ m	56′ m	57′ m	58′ m	59′ m
0	02996	02739	02482	02228	01974	01723	01472	01223	00976	00730	00486	00242
1	02992	02734	02478	02223	01970	01718	01468	01219	00972	00726	00481	00238
2	02988	02730	02474	02219	01966	01714	01464	01215	00968	00722	00477	00234
3	02983	02726	02470	02215	01962	01710	01460	01211	00964	00718	00473	00230
4	02979	02721	02465	02211	01958	01706	01456	01207	00960	00714	00469	00226
5	02975	02717	02461	02206	01953	01702	01452	01203	00955	00709	00465	00222
6	02970	02713	02457	02202	01949	01698	01447	01199	00951	00705	00461	00218
7	02966	02709	02453	02198	01945	01693	01443	01195	00947	00701	00457	00214
8	02962	02704	02448	02194	01941	01689	01439	01190	00943	00697	00453	00210
9	02958	02700	02444	02190	01937	01685	01435	01186	00939	00693	00449	00206
10	02953	02696	02440	02185	01932	01681	01431	01182	00935	00689	00445	00202
11	02949	02692	02436	02181	01928	01677	01427	01178	00931	00685	00441	00197
12	02945	02687	02431	02177	01924	01672	01422	01174	00927	00681	00436	00193
13	02940	02683	02427	02173	01920	01668	01418	01170	00923	00677	00432	00189
14	02936	02679	02423	02168	01916	01664	01414	01166	00918	00673	00428	00185
15	02932	02674	02419	02164	01911	01660	01410	01161	00914	00669	00424	00181
16	02927	02670	02414	02160	01907	01656	01406	01157	00910	00665	00420	00177
17	02923	02666	02410	02156	01903	01652	01402	01153	00906	00660	00416	00173
18	02919	02662	02406	02152	01899	01647	01398	01149	00902	00656	00412	00169
19	02915	02657	02402	02147	01895	01643	01393	01145	00898	00652	00408	00165
20	02910	02653	02397	02143	01890	01639	01389	01141	00894	00648	00404	00161
21	02906	02649	02393	02139	01886	01635	01385	01137	00890	00644	00400	00157
22	02902	02644	02389	02135	01882	01631	01381	01133	00886	00640	00396	00153
23	02897	02640	02385	02130	01878	01627	01377	01128	00881	00636	00392	00149
24	02893	02636	02380	02126	01874	01622	01373	01124	00877	00632	00388	00145
25	02889	02632	02376	02122	01869	01618	01368	01120	00873	00628	00384	00141
26	02884	02627	02372	02118	01865	01614	01364	01116	00869	00624	00380	00137
27	02880	02623	02368	02114	01861	01610	01360	01112	00865	00620	00376	00133
28	02876	02619	02363	02109	01857	01606	01356	01108	00861	00616	00372	00129
29	02872	02615	02359	02105	01853	01601	01352	01104	00857	00611	00367	00125
30	02867	02610	02355	02101	01848	01597	01348	01100	00853	00607	00363	00121
31	02863	02606	02351	02097	01844	01593	01344	01095	00849	00603	00359	00117
32	02859	02602	02346	02092	01840	01589	01339	01091	00845	00599	00355	00113
33	02854	02597	02342	02088	01836	01585	01335	01087	00840	00595	00351	00109
34	02850	02593	02338	02084	01832	01581	01331	01083	00836	00591	00347	00105
35	02846	02589	02334	02080	01827	01576	01327	01079	00832	00587	00343	00101
36	02841	02585	02329	02076	01823	01572	01323	01075	00828	00583	00339	00097
37	02837	02580	02325	02071	01819	01568	01319	01071	00824	00579	00335	00093
38	02833	02576	02321	02067	01815	01564	01315	01067	00820	00575	00331	00089
39	02829	02572	02317	02063	01811	01560	01310	01062	00816	00571	00327	00085
40	02824	02568	02312	02059	01806	01556	01306	01058	00812	00567	00323	00080
41	02820	02563	02308	02054	01802	01551	01302	01054	00808	00563	00319	00076
42	02816	02559	02304	02050	01798	01547	01298	01050	00804	00559	00315	00072
43	02811	02555	02300	02046	01794	01543	01294	01046	00799	00554	00311	00068
44	02807	02551	02295	02042	01790	01539	01290	01042	00795	00550	00307	00064
45	02803	02546	02291	02038	01785	01535	01286	01038	00791	00546	00303	00060
46	02799	02542	02287	02033	01781	01531	01281	01034	00787	00542	00299	00056
47	02794	02538	02283	02029	01777	01526	01277	01029	00783	00538	00295	00052
48	02790	02533	02278	02025	01773	01522	01273	01025	00779	00534	00290	00048
49	02786	02529	02274	02021	01769	01518	01269	01021	00775	00530	00286	00044
50	02781	02525	02270	02017	01764	01514	01265	01017	00771	00526	00282	00040
51	02777	02521	02266	02012	01760	01510	01261	01013	00767	00522	00278	00036
52	02773	02516	02262	02008	01756	01506	01257	01009	00763	00518	00274	00032
53	02769	02512	02257	02004	01752	01501	01252	01005	00759	00514	00270	00028
54	02764	02508	02253	02000	01748	01497	01248	01001	00754	00510	00266	00024
55	02760	02504	02249	01996	01744	01493	01244	00997	00750	00506	00262	00020
56	02756	02499	02245	01991	01739	01489	01240	00992	00746	00502	00258	00016
57	02751	02495	02240	01987	01735	01485	01236	00988	00742	00497	00254	00012
58	02747	02491	02236	01983	01731	01481	01232	00984	00738	00493	00250	00008
59	02743	02487	02232	01979	01727	01476	01228	00980	00734	00489	00246	00004

Proportional Part to tenths of ″ or s.	.1	.2	.3	.4	.5	.6	.7	.8	.9
	0	1	1	2	2	2	3	3	4

TABLE XI.
Depression or Dip of the Horizon.

H. of Eye.	Dip of Horiz.	Height of Eye.	Dip of Horiz.
Feet.	′ ″	Feet.	′ ″
1	1 0	16	3 58
1¼	1 7	16½	4 2
1½	1 13	17	4 5
1¾	1 19	17½	4 9
2	1 24	18	4 12
2¼	1 29	18½	4 16
2½	1 34	19	4 19
2¾	1 39	19½	4 23
3	1 43	20	4 26
3¼	1 47	21	4 33
3½	1 51	22	4 39
3¾	1 55	23	4 46
4	1 59	24	4 52
4¼	2 3	25	4 58
4½	2 6	26	5 4
4¾	2 10	27	5 10
5	2 13	28	5 16
5¼	2 17	29	5 22
5½	2 20	30	5 27
5¾	2 23	31	5 32
6	2 26	32	5 37
6½	2 32	33	5 42
7	2 38	34	5 47
7½	2 43	35	5 53
8	2 48	36	5 58
8½	2 53	37	6 2
9	2 58	38	6 7
9½	3 3	39	6 12
10	3 8	40	6 17
10½	3 12	42	6 26
11	3 17	44	6 35
11½	3 21	46	6 44
12	3 26	48	6 52
12½	3 31	50	7 1
13	3 35	55	7 21
13½	3 39	60	7 41
14	3 43	70	8 18
14½	3 47	80	8 53
15	3 51	90	9 25
15½	3 55	100	9 56

TABLE XII.
Dip at differ. Distances from the Observer.

Miles.	Height of the Eye in Feet.					
	5	10	15	20	25	30
¼	11′	23′	34′	45′	57′	68′
½	6	12	17	23	28	34
¾	4	8	12	15	19	23
1	3	6	9	12	15	17
1¼	3	5	7	10	12	14
1½	3	4	6	8	10	12
2	2	4	5	7	8	9
2½	2	3	4	6	7	8
3	2	3	4	5	6	7
3½	2	3	4	5	6	6
4	2	3	4	5	5	6
5	2	3	4	4	5	6
6	2	3	4	4	5	5

TABLE XIII.—Correction to be *added* to the Observed Altitude of the Sun's Lower Limb, when taken by a Fore Observation, to find the True Altitude.

Obs Alt.	Height of the Eye above the Sea in Feet.												
	6	8	10	12	14	16	18	20	22	24	26	28	30
°	′	′	′	′	′	′	′	′	′	′	′	′	′
5	3.8	3.5	3.1	2.8	2.5	2.3	2.1	1.8	1.6	1.4	1.2	1.0	0.8
6	5.3	4.9	4.6	4.3	4.0	3.7	3.5	3.3	3.0	2.8	2.6	2.4	2.2
7	6.4	6.0	5.7	5.4	5.1	4.8	4.6	4.4	4.1	3.9	3.7	3.5	3.3
8	7.2	6.8	6.5	6.2	5.9	5.7	5.4	5.3	5.0	4.8	4.6	4.4	4.2
9	7.9	7.5	7.2	6.9	6.6	6.4	6.1	5.9	5.7	5.5	5.3	5.1	4.9
10	8.5	8.1	7.8	7.5	7.2	6.9	6.7	6.5	6.2	6.0	5.8	5.6	5.4
11	8.9	8.6	8.2	7.9	7.6	7.4	7.2	6.9	6.7	6.5	6.3	6.1	5.9
12	9.3	9.0	8.7	8.3	8.0	7.8	7.6	7.3	7.1	6.9	6.7	6.5	6.3
14	9.9	9.6	9.2	8.9	8.7	8.4	8.2	7.9	7.7	7.5	7.3	7.1	6.9
16	10.4	10.1	9.7	9.4	9.1	8.9	8.7	8.4	8.2	8.0	7.8	7.6	7.4
18	10.8	10.4	10.1	9.8	9.5	9.3	9.0	8.8	8.6	8.4	8.2	8.0	7.8
20	11.1	10.7	10.4	10.1	9.8	9.6	9.3	9.1	8.9	8.7	8.5	8.2	8.1
22	11.4	11.0	10.7	10.4	10.1	9.8	9.6	9.4	9.1	8.9	8.7	8.5	8.3
26	11.7	11.4	11.0	10.7	10.5	10.2	10.0	9.7	9.5	9.3	9.1	8.9	8.7
30	12.0	11.7	11.3	11.0	10.8	10.5	10.3	10.0	9.8	9.6	9.4	9.2	9.0
35	12.3	11.9	11.6	11.3	11.0	10.7	10.6	10.3	10.1	9.9	9.7	9.4	9.2
40	12.5	12.2	11.8	11.5	11.3	11.0	10.8	10.5	10.3	10.1	9.9	9.7	9.5
45	12.7	12.4	12.0	11.7	11.5	11.2	11.0	10.7	10.5	10.2	10.1	9.8	9.7
50	12.8	12.5	12.2	11.9	11.6	11.3	11.1	10.9	10.6	10.4	10.3	10.0	9.8
55	13.0	12.6	12.3	12.0	11.7	11.5	11.2	11.0	10.7	10.5	10.3	10.1	9.9
60	13.1	12.7	12.4	12.1	11.8	11.6	11.3	11.1	10.9	10.6	10.4	10.2	10.1
65	13.2	12.8	12.5	12.2	11.9	11.7	11.4	11.2	11.0	10.7	10.5	10.3	10.1
70	13.3	12.9	12.6	12.3	12.0	11.8	11.5	11.3	11.0	10.8	10.6	10.4	10.2
80	13.4	13.1	12.7	12.4	12.1	11.9	11.7	11.4	11.2	11.0	10.8	10.6	10.4
90	13.6	13.2	12.9	12.6	12.3	12.0	11.8	11.6	11.3	11.1	10.9	10.7	10.5

Month,	Jan.	Feb.	Mar.	April,	May,	June,
Correction,	+ 0′.3	+ 0′.2	+ 0′.1	0′.0	— 0′.1	— 0′.2
Month,	July,	Aug.	Sept.	Oct.	Nov.	Dec.
Correction,	— 0′.3	— 0′.2	— 0′.1	+ 0′.1	+ 0′.2	+ 0′.3

TABLE XIV.—Correction to be *subtracted* from the Observed Altitude of a Fixed Star, to find the True Altitude.

Obs Alt.	Height of the Eye above the Sea in Feet.												
	6	8	10	12	14	16	18	20	22	24	26	28	30
°	′	′	′	′	′	′	′	′	′	′	′	′	′
5	12.3	12.7	13.0	13.3	13.6	13.8	14.1	14.3	14.6	14.8	15.0	15.2	15.3
6	10.9	11.3	11.6	11.9	12.2	12.4	12.7	12.9	13.2	13.4	13.6	13.8	13.9
7	9.8	10.2	10.5	10.8	11.1	11.3	11.6	11.8	12.1	12.3	12.5	12.7	12.8
8	8.9	9.3	9.6	9.9	10.2	10.4	10.7	10.9	11.2	11.4	11.6	11.8	11.9
9	8.2	8.6	8.9	9.2	9.5	9.7	10.0	10.2	10.5	10.7	10.9	11.1	11.2
10	7.7	8.1	8.4	8.7	9.0	9.2	9.5	9.7	10.0	10.2	10.4	10.6	10.7
11	7.2	7.6	7.9	8.2	8.5	8.7	9.0	9.2	9.5	9.7	9.9	10.1	10.2
12	6.8	7.2	7.5	7.8	8.1	8.3	8.6	8.8	9.1	9.3	9.5	9.7	9.8
14	6.2	6.6	6.9	7.2	7.5	7.7	8.0	8.2	8.5	8.7	8.9	9.1	9.2
16	5.7	6.1	6.4	6.7	7.0	7.2	7.5	7.7	8.0	8.2	8.4	8.6	8.7
18	5.3	5.7	6.0	6.3	6.6	6.8	7.1	7.3	7.6	7.8	8.0	8.2	8.3
20	5.0	5.4	5.7	6.0	6.3	6.5	6.8	7.0	7.3	7.5	7.7	7.9	8.0
22	4.8	5.2	5.5	5.8	6.1	6.3	6.6	6.8	7.1	7.3	7.5	7.7	7.8
26	4.4	4.8	5.1	5.4	5.7	5.9	6.2	6.4	6.7	6.9	7.1	7.3	7.4
30	4.1	4.5	4.8	5.1	5.4	5.6	5.9	6.1	6.4	6.6	6.8	7.0	7.1
35	3.8	4.2	4.5	4.8	5.1	5.3	5.6	5.8	6.1	6.3	6.5	6.7	6.8
40	3.6	4.0	4.3	4.6	4.9	5.1	5.4	5.6	5.9	6.1	6.3	6.5	6.6
45	3.4	3.8	4.1	4.4	4.7	4.9	5.2	5.4	5.7	5.9	6.1	6.3	6.5
50	3.2	3.6	3.9	4.2	4.5	4.7	5.0	5.2	5.5	5.7	5.9	6.1	6.3
55	3.1	3.5	3.8	4.1	4.4	4.6	4.9	5.1	5.4	5.6	5.8	6.0	6.2
60	3.0	3.4	3.7	4.0	4.3	4.5	4.8	5.0	5.3	5.5	5.7	5.9	6.1
65	2.9	3.3	3.6	3.9	4.2	4.4	4.7	4.9	5.2	5.4	5.6	5.8	6.0
70	2.8	3.2	3.5	3.8	4.1	4.3	4.6	4.8	5.1	5.3	5.5	5.7	5.9
80	2.6	3.0	3.3	3.7	3.9	4.1	4.4	4.6	4.9	5.1	5.3	5.5	5.7
90	2.4	2.8	3.1	3.4	3.7	3.9	4.2	4.4	4.7	4.9	5.1	5.3	5.5

TABLE XV.
Sun's Semi-diameter, &c.

Month.	Days.	Time of Sun's S.-diam. passing Merid.	Sun's Semi-diameter.	Sun's Hourly Motion.	Sun's Logarithm Distance.
		m. s.	′ ″	′ ″	
January.	1	1 10.8	16 17.8	2 32.9	9.992659
	7	1 10.5	16 17.7	2 32.9	9.992727
	13	1 10.1	16 17.4	2 32.8	9.992852
	19	1 9.5	16 16.9	2 32.6	9.993046
	25	1 8.9	16 16.3	2 32.4	9.993329
February.	1	1 8.1	16 15.3	2 32.2	9.993779
	7	1 7.4	16 14.3	2 31.8	9.994231
	13	1 6.7	16 13.2	2 31.5	9.994722
	19	1 6.1	16 11.9	2 31.1	9.995267
	25	1 5.5	16 10.5	2 30.7	9.995879
March.	1	1 5.2	16 9.6	2 30.4	9.996323
	7	1 4.8	16 8.1	2 29.9	9.997016
	13	1 4.5	16 6.5	2 29.4	9.997719
	19	1 4.3	16 4.9	2 28.9	9.998431
	25	1 4.2	16 3.2	2 28.4	9.999170
April.	1	1 4.2	16 1.3	2 27.8	0.000068
	7	1 4.4	15 59.6	2 27.3	0.000826
	13	1 4.6	15 58.0	2 26.8	0.001554
	19	1 4.9	15 56.4	2 26.3	0.002254
	25	1 5.4	15 54.9	2 25.8	0.002948
May.	1	1 5.8	15 53.4	2 25.4	0.003626
	7	1 6.3	15 52.1	2 25.0	0.004254
	13	1 6.8	15 50.8	2 24.6	0.004817
	19	1 7.2	15 49.6	2 24.2	0.005323
	25	1 7.7	15 48.6	2 23.9	0.005794
June.	1	1 8.1	15 47.6	2 23.6	0.006281
	7	1 8.3	15 46.9	2 23.4	0.006616
	13	1 8.5	15 46.3	2 23.2	0.006864
	19	1 8.6	15 45.9	2 23.1	0.007043
	25	1 8.6	15 45.6	2 23.0	0.007173
July.	1	1 8.5	15 45.5	2 23.0	0.007237
	7	1 8.3	15 45.5	2 23.0	0.007212
	13	1 8.0	15 45.8	2 23.1	0.007095
	19	1 7.5	15 46.1	2 23.2	0.006910
	25	1 7.0	15 46.7	2 23.3	0.006675
August.	1	1 6.5	15 47.5	2 23.6	0.006325
	7	1 6.0	15 48.3	2 23.9	0.005933
	13	1 5.5	15 49.3	2 24.1	0.005463
	19	1 5.0	15 50.4	2 24.5	0.004949
	25	1 4.6	15 51.6	2 24.9	0.004403
September.	1	1 4.2	15 53.2	2 25.3	0.003705
	7	1 3.9	15 54.6	2 25.8	0.003040
	13	1 3.8	15 56.2	2 26.2	0.002333
	19	1 3.8	15 57.7	2 26.7	0.001617
	25	1 3.9	15 59.4	2 27.2	0.000898
October.	1	1 4.1	16 1.0	2 27.7	0.000162
	7	1 4.4	16 2.7	2 28.2	9.999398
	13	1 4.9	16 4.4	2 28.7	9.998633
	19	1 5.4	16 6.0	2 29.3	9.997898
	25	1 6.0	16 7.6	2 29.7	9.997200
November.	1	1 6.8	16 9.4	2 30.3	9.996411
	7	1 7.5	16 10.8	2 30.7	9.995749
	13	1 8.3	16 12.2	2 31.2	9.995135
	19	1 8.9	16 13.4	2 31.6	9.994592
	25	1 9.6	16 14.5	2 31.9	9.994120
December.	1	1 10.0	16 15.5	2 32.2	9.993697
	7	1 10.5	16 16.3	2 32.4	9.993322
	13	1 10.8	16 16.9	2 32.6	9.993024
	19	1 10.9	16 17.4	2 32.8	9.992823
	25	1 11.0	16 17.7	2 32.9	9.992714

TABLE XVI.
Sun's Parallax in Altitude, &c.

Z. D.	Alt.	Jan.	Feb. Dec.	Mar. Nov.	April Oct.	May Sept.	June Aug.	July.
		Par.	Par.	Par.	Par.	Par.	Par.	Par.
°	°	″	″	″	″	″	″	″
0	90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	89	0.15	0.15	0.15	0.15	0.15	0.15	0.15
2	88	0.31	0.31	0.31	0.30	0.30	0.30	0.30
3	87	0.46	0.46	0.46	0.45	0.45	0.45	0.45
4	86	0.62	0.62	0.61	0.61	0.61	0.60	0.60
5	85	0.77	0.77	0.76	0.76	0.76	0.75	0.74
6	84	0.93	0.93	0.92	0.91	0.90	0.89	0.88
7	83	1.08	1.08	1.07	1.06	1.05	1.04	1.03
8	82	1.23	1.23	1.22	1.21	1.20	1.19	1.18
9	81	1.38	1.38	1.37	1.36	1.35	1.34	1.33
10	80	1.53	1.53	1.52	1.51	1.50	1.49	1.48
11	79	1.69	1.69	1.68	1.66	1.64	1.63	1.62
12	78	1.84	1.84	1.83	1.80	1.78	1.77	1.76
13	77	1.99	1.98	1.97	1.95	1.93	1.92	1.91
14	76	2.14	2.13	2.12	2.10	2.08	2.07	2.06
15	75	2.29	2.28	2.27	2.25	2.23	2.22	2.21
16	74	2.44	2.43	2.42	2.39	2.37	2.36	2.36
17	73	2.59	2.58	2.57	2.54	2.52	2.50	2.50
18	72	2.73	2.72	2.71	2.68	2.66	2.64	2.64
19	71	2.88	2.87	2.86	2.83	2.80	2.79	2.78
20	70	3.02	3.01	3.00	2.97	2.94	2.93	2.92
21	69	3.17	3.16	3.14	3.11	3.09	3.06	3.06
22	68	3.31	3.30	3.28	3.25	3.23	3.20	3.19
23	67	3.45	3.44	3.42	3.39	3.36	3.34	3.33
24	66	3.59	3.58	3.56	3.53	3.50	3.48	3.47
25	65	3.73	3.72	3.70	3.67	3.64	3.61	3.61
26	64	3.87	3.86	3.84	3.80	3.77	3.74	3.74
27	63	4.01	4.00	3.98	3.94	3.91	3.88	3.87
28	62	4.14	4.13	4.11	4.07	4.04	4.01	4.00
29	61	4.28	4.27	4.25	4.21	4.17	4.15	4.14
30	60	4.41	4.40	4.38	4.34	4.30	4.28	4.27
32	58	4.68	4.66	4.64	4.60	4.55	4.54	4.53
34	56	4.94	4.92	4.89	4.85	4.80	4.79	4.78
36	54	5.19	5.17	5.14	5.10	5.05	5.03	5.02
38	52	5.44	5.42	5.39	5.34	5.29	5.27	5.26
40	50	5.68	5.66	5.63	5.58	5.53	5.50	5.49
42	48	5.92	5.90	5.87	5.81	5.76	5.72	5.71
44	46	6.15	6.13	6.10	6.03	5.98	5.94	5.92
46	44	6.36	6.34	6.30	6.24	6.19	6.15	6.13
48	42	6.57	6.55	6.51	6.45	6.40	6.36	6.34
50	40	6.77	6.75	6.71	6.65	6.60	6.56	6.54
52	38	6.97	6.95	6.91	6.84	6.78	6.73	6.72
54	36	7.15	7.13	7.09	7.02	6.96	6.91	6.90
56	34	7.33	7.31	7.27	7.20	7.14	7.09	7.08
58	32	7.49	7.47	7.43	7.36	7.30	7.25	7.24
60	30	7.65	7.63	7.59	7.52	7.46	7.41	7.39
62	28	7.80	7.78	7.73	7.66	7.59	7.54	7.53
64	26	7.94	7.92	7.87	7.80	7.73	7.68	7.66
66	24	8.07	8.05	8.00	7.93	7.86	7.81	7.79
68	22	8.19	8.17	8.12	8.05	7.98	7.93	7.91
70	20	8.30	8.28	8.23	8.16	8.09	8.04	8.02
72	18	8.41	8.39	8.34	8.26	8.19	8.14	8.12
74	16	8.49	8.47	8.42	8.34	8.27	8.22	8.20
76	14	8.57	8.55	8.50	8.42	8.35	8.30	8.28
78	12	8.64	8.62	8.57	8.49	8.42	8.37	8.35
80	10	8.70	8.68	8.63	8.55	8.48	8.43	8.41
82	8	8.75	8.73	8.68	8.60	8.53	8.48	8.46
84	6	8.78	8.76	8.71	8.63	8.56	8.51	8.49
86	4	8.81	8.79	8.74	8.66	8.59	8.54	8.52
88	2	8.82	8.80	8.75	8.67	8.60	8.55	8.53
90	0	8.83	8.81	8.76	8.68	8.61	8.56	8.54

Fahrenheit's Thermometer 50°. English Barometer 30 Inches.

Z. D.	δr	Log. δr	Diff.	Z. D.	δr	Log. δr	Diff.	Z. D.	δr	Log. δr	Diff.
° '	' ''			° '	' ''			° '	' ''		
0 0	0 0.00	0.0000		10 0	0 10.30	1.0129	72	20 0	0 21.26	1.3277	38
10	0.17	9.2304	3011	10	10.47	1.0201	72	10	21.45	1.3315	39
20	0.34	9.5315	1761	20	10.65	1.0273	71	20	21.65	1.3354	39
30	0.51	9.7076	1249	30	10.82	1.0344	70	30	21.84	1.3393	38
40	0.68	9.8325	969	40	11.00	1.0414	69	40	22.03	1.3431	38
50	0.85	9.9294	791	50	11.17	1.0483	69	50	22.23	1.3469	38
1 0	0 1.02	0.0085	670	11 0	0 11.35	1.0552	66	21 0	0 22.42	1.3507	37
10	1.19	0.0755	580	10	11.53	1.0618	66	10	22.62	1.3544	38
20	1.36	0.1335	512	20	11.71	1.0684	66	20	22.81	1.3582	37
30	1.53	0.1847	457	30	11.89	1.0750	65	30	23.01	1.3619	37
40	1.70	0.2304	414	40	12.06	1.0815	64	40	23.21	1.3656	37
50	1.87	0.2718	379	50	12.24	1.0879	62	50	23.40	1.3693	36
2 0	0 2.04	0.3097	347	12 0	0 12.42	1.0941	62	22 0	0 23.60	1.3729	37
10	2.21	0.3444	322	10	12.60	1.1003	61	10	23.80	1.3766	36
20	2.38	0.3766	301	20	12.78	1.1064	60	20	24.00	1.3802	36
30	2.55	0.4067	280	30	12.95	1.1124	60	30	24.20	1.3838	36
40	2.72	0.4347	263	40	13.13	1.1184	58	40	24.40	1.3874	35
50	2.89	0.4610	250	50	13.31	1.1242	58	50	24.60	1.3909	36
3 0	0 3.06	0.4860	235	13 0	0 13.49	1.1300	57	23 0	0 24.80	1.3945	36
10	3.23	0.5095	224	10	13.67	1.1357	57	10	25.00	1.3981	34
20	3.40	0.5319	211	20	13.85	1.1414	55	20	25.20	1.4015	34
30	3.57	0.5530	203	30	14.02	1.1469	55	30	25.41	1.4049	35
40	3.74	0.5733	193	40	14.20	1.1524	54	40	25.61	1.4084	34
50	3.91	0.5926	186	50	14.38	1.1578	56	50	25.81	1.4118	33
4 0	0 4.08	0.6112	178	14 0	0 14.56	1.1634	52	24 0	0 26.01	1.4151	34
10	4.26	0.6290	171	10	14.74	1.1686	54	10	26.21	1.4185	34
20	4.43	0.6461	165	20	14.93	1.1740	53	20	26.42	1.4219	34
30	4.60	0.6626	158	30	15.11	1.1793	52	30	26.62	1.4253	33
40	4.77	0.6784	153	40	15.29	1.1845	52	40	26.83	1.4286	33
50	4.94	0.6937	149	50	15.48	1.1897	50	50	27.03	1.4319	33
5 0	0 5.11	0.7086	142	15 0	0 15.66	1.1947	51	25 0	0 27.24	1.4352	33
10	5.28	0.7228	139	10	15.84	1.1998	50	10	27.45	1.4385	33
20	5.45	0.7367	135	20	16.03	1.2048	50	20	27.66	1.4418	33
30	5.63	0.7502	131	30	16.21	1.2098	49	30	27.86	1.4451	32
40	5.80	0.7633	127	40	16.39	1.2147	48	40	28.07	1.4483	32
50	5.97	0.7760	122	50	16.58	1.2195	46	50	28.28	1.4515	32
6 0	0 6.14	0.7882	120	16 0	0 16.75	1.2241	46	26 0	0 28.49	1.4547	32
10	6.31	0.8002	116	10	16.93	1.2287	47	10	28.70	1.4579	32
20	6.48	0.8118	114	20	17.12	1.2334	46	20	28.91	1.4611	32
30	6.66	0.8232	111	30	17.30	1.2380	46	30	29.13	1.4643	31
40	6.83	0.8343	108	40	17.48	1.2426	46	40	29.34	1.4674	32
50	7.00	0.8451	106	50	17.67	1.2472	47	50	29.55	1.4706	30
7 0	0 7.17	0.8557	102	17 0	0 17.86	1.2519	45	27 0	0 29.76	1.4736	32
10	7.34	0.8659	101	10	18.05	1.2564	45	10	29.97	1.4768	31
20	7.52	0.8760	99	20	18.23	1.2609	44	20	30.19	1.4799	30
30	7.69	0.8859	97	30	18.42	1.2653	44	30	30.40	1.4829	31
40	7.86	0.8956	95	40	18.61	1.2697	43	40	30.62	1.4860	30
50	8.04	0.9051	93	50	18.79	1.2740	44	50	30.83	1.4890	31
8 0	0 8.21	0.9144	90	18 0	0 18.98	1.2784	42	28 0	0 31.05	1.4921	31
10	8.38	0.9234	89	10	19.17	1.2826	42	10	31.27	1.4952	30
20	8.56	0.9323	87	20	19.36	1.2868	42	20	31.49	1.4982	31
30	8.73	0.9410	85	30	19.55	1.2910	42	30	31.72	1.5013	30
40	8.90	0.9495	84	40	19.73	1.2952	42	40	31.94	1.5043	30
50	9.08	0.9579	84	50	19.92	1.2994	42	50	32.16	1.5073	29
9 0	0 9.25	0.9663	80	19 0	0 20.11	1.3036	39	29 0	0 32.38	1.5102	31
10	9.42	0.9743	80	10	20.30	1.3075	39	10	32.60	1.5133	29
20	9.60	0.9823	78	20	20.49	1.3116	41	20	32.83	1.5162	30
30	9.77	0.9901	77	30	20.69	1.3157	41	30	33.05	1.5192	29
40	9.95	0.9978	76	40	20.88	1.3197	40	40	33.27	1.5221	29
50	10.12	1.0054	75	50	21.07	1.3237	40	50	33.50	1.5250	29
10 0	10.30	1.0129	72	20 0	21.26	1.3277	38	30 0	33.72	1.5279	29

Fahrenheit's Thermometer 50°. English Barometer 30 Inches.

Z. D.	δθ	Log. δθ	Diff.	Z. D.	δθ	Log. δθ	Diff.	Z. D.	δθ	Log. δθ	Diff.
° ′	′ ″			° ′	′ ″			° ′	′ ″		
30 0	0 33.72	1.5279	29	40 0	0 48.99	1.69010	257	50 0	1 9.52	1.84208	256
10	33.95	1.5308	29	10	49.28	1.69267	256	10	9.94	1.84464	257
20	34.18	1.5337	29	20	49.58	1.69523	257	20	10.35	1.84721	256
30	34.40	1.5366	29	30	49.87	1.69780	257	30	10.77	1.84977	257
40	34.63	1.5395	28	40	50.16	1.70037	256	40	11.19	1.85234	256
50	34.86	1.5423	29	50	50.46	1.70293	257	50	11.60	1.85490	257
31 0	0 35.09	1.5452	29	41 0	0 50.75	1.70550	254	51 0	1 12.02	1.85747	258
10	35.32	1.5481	29	10	51.06	1.70804	254	10	12.46	1.86005	259
20	35.56	1.5510	28	20	51.36	1.71058	253	20	12.89	1.86264	258
30	35.79	1.5538	28	30	51.66	1.71311	253	30	13.33	1.86522	259
40	36.02	1.5566	28	40	51.96	1.71564	254	40	13.77	1.86781	258
50	36.26	1.5594	28	50	52.27	1.71818	252	50	14.20	1.87039	259
32 0	0 36.49	1.5622	28	42 0	0 52.57	1.72070	252	52 0	1 14.64	1.87298	260
10	36.73	1.5650	28	10	52.88	1.72322	252	10	15.10	1.87558	261
20	36.97	1.5678	29	20	53.19	1.72574	252	20	15.55	1.87819	261
30	37.21	1.5707	28	30	53.50	1.72826	252	30	16.01	1.88080	261
40	37.45	1.5735	27	40	53.81	1.73078	251	40	16.47	1.88341	260
50	37.69	1.5762	28	50	54.12	1.73329	251	50	16.92	1.88601	262
33 0	0 37.93	1.5790	28	43 0	0 54.43	1.73580	253	53 0	1 17.38	1.88863	262
10	38.17	1.5818	27	10	54.75	1.73833	254	10	17.86	1.89125	262
20	38.42	1.5845	28	20	55.07	1.74087	253	20	18.33	1.89387	263
30	38.66	1.5873	27	30	55.40	1.74340	253	30	18.81	1.89650	263
40	38.90	1.5900	27	40	55.72	1.74593	254	40	19.29	1.89913	263
50	39.15	1.5927	27	50	56.04	1.74847	253	50	19.76	1.90176	264
34 0	0 39.39	1.5954	27	44 0	0 56.35	1.75100	252	54 0	1 20.24	1.90440	265
10	39.64	1.5981	28	10	56.68	1.75352	252	10	20.74	1.90705	265
20	39.89	1.6009	27	20	57.02	1.75604	252	20	21.24	1.90970	266
30	40.14	1.6036	27	30	57.35	1.75856	252	30	21.75	1.91236	266
40	40.39	1.6063	27	40	57.69	1.76108	252	40	22.25	1.91502	267
50	40.64	1.6090	26	50	58.02	1.76360	251	50	22.75	1.91769	267
35 0	0 40.89	1.6116	27	45 0	0 58.36	1.76611	252	55 0	1 23.25	1.92036	268
10	41.14	1.6143	27	10	58.70	1.76863	252	10	23.78	1.92304	269
20	41.40	1.6170	27	20	59.05	1.77115	252	20	24.30	1.92573	268
30	41.65	1.6197	26	30	59.39	1.77367	252	30	24.83	1.92841	271
40	41.91	1.6223	27	40	59.74	1.77619	252	40	25.36	1.93112	270
50	42.16	1.6250	26	50	1 0.08	1.77871	252	50	25.88	1.93382	270
36 0	0 42.42	1.6276	27	46 0	1 0.43	1.78123	252	56 0	1 26.41	1.93653	271
10	42.68	1.6303	27	10	0.79	1.78375	253	10	26.96	1.93924	272
20	42.95	1.6330	26	20	1.15	1.78628	252	20	27.52	1.94196	273
30	43.21	1.6356	26	30	1.50	1.78880	252	30	28.07	1.94469	273
40	43.47	1.6382	26	40	1.86	1.79132	253	40	28.62	1.94742	274
50	43.74	1.6408	27	50	2.21	1.79385	252	50	29.18	1.95016	275
37 0	0 44.00	1.6435	26	47 0	1 2.57	1.79637	253	57 0	1 29.73	1.95291	275
10	44.27	1.6461	26	10	2.94	1.79890	253	10	30.31	1.95566	277
20	44.54	1.6487	26	20	3.31	1.80143	253	20	30.90	1.95843	277
30	44.80	1.6513	26	30	3 69	1.80396	253	30	31.48	1.96120	278
40	45.07	1.6539	26	40	4.06	1.80649	253	40	32.06	1.96397	279
50	45.34	1.6565	26	50	4.43	1.80902	253	50	32.65	1.96676	279
38 0	0 45.61	1.6591	26	48 0	1 4.80	1.81155	254	58 0	1 33.23	1.96955	280
10	45.89	1.6617	26	10	5.18	1.81409	254	10	33.85	1.97235	281
20	46.16	1.6643	26	20	5.57	1.81663	253	20	34.46	1.97516	281
30	46.44	1.6669	26	30	5.95	1.81916	254	30	35.08	1.97797	283
40	46.72	1.6695	25	40	6.34	1.82170	254	40	35.70	1.98080	282
50	46.99	1.6720	26	50	6.72	1.82424	254	50	36.31	1.98362	284
39 0	0 47.27	1.6746	26	49 0	1 7.11	1.82678	255	59 0	1 36.93	1.98646	285
10	47.56	1.6772	26	10	7.51	1.82933	255	10	37.58	1.98931	285
20	47.84	1.6798	26	20	7.91	1.83188	255	20	38.24	1.99216	287
30	48.13	1.6824	26	30	8.32	1.83443	255	30	38.89	1.99503	287
40	48.42	1.6850	26	40	8.72	1.83698	255	40	39.54	1.99790	289
50	48.70	1.6876	25	50	9.12	1.83953	255	50	40.20	2.00079	289
40 0	48.99	1.6901	26	50 0	9.52	1.84208	256	60 0	40.85	2.00368	290

Fahrenheit's Thermometer 50°. English Barometer 30 Inches.

Z.D.	$\delta\theta$	Log. $\delta\theta$	D.	Z.D.	$\delta\theta$	Log. $\delta\theta$	D.	$\frac{d\delta\theta}{d\tau}$	Z.D.	$\delta\theta$	Log. $\delta\theta$	Diff.	$\frac{d\delta\theta}{d\tau}$	$\frac{d\delta\theta}{dp}$
° ′	′ ″			° ′	′ ″				° ′	′ ″			—	
60 0	1 40.85	2.00368	290	70 0	2 39.16	2.20185	388		80 0	5 20.19	2.50541	696	0.030	0.04
10	41.52	2.00658	291	10	40.59	2.20573	390		10	25.36	2.51237	707	0.031	0.04
20	42.21	2.00949	292	20	42.04	2.20963	393		20	30.70	2.51944	716	0.033	0.04
30	42.90	2.01241	293	30	43.52	2.21356	396		30	36.20	2.52660	727	0.034	0.04
40	43.59	2.01535	294	40	45.02	2.21752	398		40	41.88	2.53387	738	0.036	0.05
50	44.30	2.01829	295	50	46.53	2.22150	402		50	47.74	2.54125	749	0.038	0.05
61 0	1 45.01	2.02124	296	71 0	2 48.08	2.22552	404		81 0	5 53.79	2.54874	759	0.040	0.05
10	45.73	2.02420	298	10	49.65	2.22956	407		10	6 0.04	2.55635	772	0.042	0.06
20	46.46	2.02718	299	20	51.25	2.23363	410		20	6.50	2.56407	785	0.044	0.06
30	47.18	2.03016	300	30	52.87	2.23773	413		30	13.18	2.57192	797	0.046	0.07
40	47.93	2.03316	301	40	54.53	2.24186	417		40	20.09	2.57989	811	0.049	0.07
50	48.68	2.03617	301	50	56.21	2.24603	419		50	27.26	2.58800	824	0.051	0.08
62 0	1 49.44	2.03918	303	72 0	2 57.92	2.25022	423		82 0	6 34.68	2.59624	838	0.053	0.08
10	50.21	2.04221	304	10	59.66	2.25445	425		10	42.37	2.60462	851	0.057	0.09
20	50.99	2.04525	305	20	3 1.43	2.25870	429		20	50.33	2.61313	866	0.060	0.09
30	51.77	2.04830	307	30	3.23	2.26299	433		30	58.59	2.62179	883	0.063	0.10
40	52.57	2.05137	308	40	5.06	2.26732	436		40	7 7.19	2.63062	899	0.067	0.10
50	53.36	2.05445	309	50	6.93	2.27168	440		50	16.13	2.63961	914	0.071	0.11
63 0	1 54.17	2.05754	310	73 0	3 8.83	2.27608	443	—	83 0	7 25.40	2.64875	931	0.074	0.11
10	54.99	2.06064	312	10	10.77	2.28051	447	0.003	10	35.05	2.65806	949	0.079	0.12
20	55.81	2.06376	312	20	12.74	2.28498	450	0.003	20	45.10	2.66755	967	0.084	0.12
30	56.66	2.06688	315	30	14.75	2.28948	454	0.004	30	55.58	2.67722	986	0.089	0.13
40	57.50	2.07003	315	40	16.80	2.29402	458	0.004	40	8 6.50	2.68708	1006	0.095	0.14
50	58.36	2.07318	317	50	18.88	2.29860	462	0.005	50	17.90	2.69714	1026	0.101	0.15
64 0	1 59.22	2.07635	318	74 0	3 21.01	2.30322	467	0.005	84 0	8 29.80	2.70740	1047	0.107	0.16
10	2 0.09	2.07953	320	10	23.18	2.30789	470	0.006	10	42.24	2.71787	1069	0.114	0.17
20	0.99	2.08273	321	20	25.39	2.31259	475	0.006	20	55.25	2.72856	1092	0.122	0.18
30	1.88	2.08594	323	30	27.66	2.31734	479	0.007	30	9 8.88	2.73948	1115	0.130	0.20
40	2.80	2.08917	324	40	29.95	2.32213	483	0.008	40	23.16	2.75063	1139	0.139	0.21
50	3.71	2.09241	326	50	32.30	2.32696	488	0.008	50	38.12	2.76202	1165	0.149	0.23
65 0	2 4.65	2.09567	327	75 0	3 34.70	2.33184	493	0.009	85 0	9 53.84	2.77367	1191	0.159	0.25
10	5.59	2.09894	330	10	37.16	2.33677	497	0.009	10	10 10.35	2.78558	1219	0.171	0.26
20	6.54	2.10224	330	20	39.65	2.34174	502	0.010	20	27.73	2.79777	1248	0.184	0.28
30	7.51	2.10554	332	30	42.21	2.34676	507	0.010	30	46.03	2.81025	1277	0.198	0.31
40	8.49	2.10886	334	40	44.82	2.35183	512	0.011	40	11 5.30	2.82302	1309	0.213	0.33
50	9.48	2.11220	335	50	47.48	2.35695	517	0.011	50	25.66	2.83611	1340	0.229	0.36
66 0	2 10.48	2.11555	337	76 0	3 50.21	2.36212	523	0.012	86 0	11 47.15	2.84951	1374	0.248	0.39
10	11.50	2.11892	339	10	53.00	2.36735	528	0.012	10	12 9.88	2.86325	1410	0.269	0.43
20	12.52	2.12231	340	20	55.85	2.37263	533	0.013	20	33.97	2.87735	1447	0.292	0.47
30	13.57	2.12571	342	30	58.76	2.37796	538	0.013	30	59.51	2.89182	1484	0.317	0.51
40	14.62	2.12913	345	40	4 1.74	2.38334	545	0.014	40	13 26.61	2.90666	1523	0.345	0.56
50	15.70	2.13258	345	50	4.79	2.38879	551	0.014	50	55.40	2.92189	1565	0.376	0.62
67 0	2 16.78	2.13603	348	77 0	4 7.91	2.39430	557	0.015	87 0	14 26.04	2.93754	1608	0.410	0.68
10	17.88	2.13951	349	10	11.11	2.39987	563	0.015	10	58.71	2.95362	1654	0.448	0.75
20	19.00	2.14300	352	20	14.39	2.40550	569	0.016	20	15 33.60	2.97016	1701	0.490	0.83
30	20.13	2.14652	354	30	17.74	2.41119	576	0.016	30	16 10.89	2.98717	1749	0.538	0.91
40	21.28	2.15006	355	40	21.19	2.41695	583	0.017	40	50.8	3.00466	1801	0.593	1.01
50	22.43	2.15361	358	50	24.72	2.42278	589	0.017	50	17 33.6	3.02267	1855	0.654	1.13
68 0	2 23.61	2.15719	359	78 0	4 28.33	2.42867	596	0.018	88 0	18 19.6	3.04122	1909	0.722	1.26
10	24.81	2.16078	362	10	32.04	2.43463	603	0.018	10	19 9.0	3.06031	1967	0.799	1.41
20	26.02	2.16440	364	20	35.84	2.44066	611	0.019	20	20 2.2	3.07998	2026	0.887	1.59
30	27.25	2.16804	366	30	39.75	2.44677	618	0.020	30	59.6	3.10024	2089	0.987	1.79
40	28.50	2.17171	368	40	43.76	2.45295	626	0.021	40	22 1.7	3.12113	2155	1.101	2.02
50	29.76	2.17539	371	50	47.88	2.45921	635	0.022	50	23 8.9	3.14268	2221	1.231	2.29
69 0	2 31.04	2.17910	373	79 0	4 52.12	2.46556	642	0.023	89 0	24 21.8	3.16489	2290	1.380	2.61
10	32.34	2.18283	375	10	56.47	2.47198	650	0.024	10	25 40.9	3.18779	2361	1.551	2.98
20	33.67	2.18658	378	20	5 0.94	2.47848	659	0.025	20	27 7.1	3.21140	2434	1.749	3.41
30	35.01	2.19036	381	30	5.54	2.48507	669	0.026	30	28 40.8	3.23574	2509	1.977	3.93
40	36.37	2.19417	383	40	10.28	2.49176	677	0.027	40	30 23.2	3.26083	2584	2.241	4.54
50	37.76	2.19800	385	50	15.16	2.49853	688	0.028	50	32 15.0	3.28667	2667	2.549	5.26
70 0	39.16	2.20185	388	80 0	20.19	2.50541	696	0.030	90 0	34 17.5	3.31334		2.909	6.12

TABLE XVIII.

Ex. Thermometer.

P. P.	Th.	Log.	P. P.	Th.	Log.
—	10°	0.03779	—	50°	0.00000
10	1	0.03680	9	1	9.99910
20	2	0.03582	18	2	9.99820
29	3	0.03484	27	3	9.99730
39	4	0.03386	36	4	9.99640
49	5	0.03288	45	5	9.99550
59	6	0.03191	54	6	9.99460
69	7	0.03094	63	7	9.99371
78	8	0.02997	72	8	9.99282
88	9	0.02900	81	9	9.99193
	20	0.02803		60	9.99104
10	1	0.02706	9	1	9.99016
19	2	0.02609	18	2	9.98927
29	3	0.02514	26	3	9.98839
38	4	0.02418	35	4	9.98751
48	5	0.02323	44	5	9.98663
58	6	0.02227	53	6	9.98575
67	7	0.02132	62	7	9.98488
77	8	0.02037	70	8	9.98401
86	9	0.01942	79	9	9.98314
	30	0.01848		70	9.98227
9	1	0.01754	9	1	9.98140
19	2	0.01660	17	2	9.98054
28	3	0.01566	26	3	9.97967
38	4	0.01472	34	4	9.97881
47	5	0.01379	43	5	9.97795
56	6	0.01285	52	6	9.97709
66	7	0.01192	60	7	9.97623
75	8	0.01099	69	8	9.97537
85	9	0.01006	77	9	9.97452
	40	0.00914		80	9.97367
9	1	0.00822	8	1	9.97282
18	2	0.00730	17	2	9.97197
28	3	0.00638	25	3	9.97112
37	4	0.00546	34	4	9.91027
46	5	0.00455	42	5	9.96943
55	6	0.00363	50	6	9.96859
64	7	0.00272	59	7	9.96775
74	8	0.00181	67	8	9.96691
83	9	0.00090	76	9	9.96607
	50	0.00000		90	9.96524

TABLE XIX.

Barometer.

P. P.	Bar.	Log.
	27.5	9.96221
	6	9.96379
	7	9.96536
	8	9.96692
	9	9.96848
+	28.0	9.97004
15	1	9.97158
30	2	9.97313
46	3	9.97466
61	4	9.97620
76	5	9.97772
91	6	9.97924
106	7	9.98076
122	8	9.98227
137	9	9.98378
	29.0	9.98528
15	1	9.98677
29	2	9.98826
44	3	9.98975
59	4	9.99123
73	5	9.99270
88	6	9.99417
103	7	9.99563
118	8	9.99709
132	9	9.99855
	30.0	0.00000
14	1	0.00145
29	2	0.00289
43	3	0.00432
57	4	0.00575
71	5	0.00718
86	6	0.00860
100	7	0.01002
114	8	0.01143
129	9	0.01284
	31.0	0.01424

TAB. XX. I. Thermometer.

Th.	Log.	Th.	Log.
10°	0.00173	50°	0.00000
11	0.00169	51	9.99996
12	0.00164	52	9.99991
13	0.00160	53	9.99987
14	0.00156	54	9.99983
15	0.00151	55	9.99978
16	0.00147	56	9.99974
17	0.00143	57	9.99970
18	0.00138	58	9.99965
19	0.00134	59	9.99961
20	0.00130	60	9.99957
21	0.00126	61	9.99953
22	0.00121	62	9.99948
23	0.00117	63	9.99944
24	0.00113	64	9.99940
25	0.00108	65	9.99935
26	0.00104	66	9.99931
27	0.00100	67	9.99927
28	0.00095	68	9.99922
29	0.00091	69	9.99918
30	0.00087	70	9.99913
31	0.00083	71	9.99909
32	0.00078	72	9.99904
33	0.00074	73	9.99900
34	0.00070	74	9.99896
35	0.00065	75	9.99891
36	0.00061	76	9.99887
37	0.00057	77	9.99883
38	0.00052	78	9.99878
39	0.00048	79	9.99874
40	0.00043	80	9.99870
41	0.00039	81	9.99866
42	0.00034	82	9.99861
43	0.00030	83	9.99857
44	0.00026	84	9.99853
45	0.00021	85	9.99848
46	0.00017	86	9.99844
47	0.00013	87	9.99840
48	0.00008	88	9.99835
49	0.00004	89	9.99831
50	0.00000	90	9.99827

P. P. to tenths of a Degree.

.1	.2	.3	.4	.5	.6	.7	.8	.9
—0	1	1	2	2	3	3	3	4

EXPLANATION.

The true refraction is computed by the following formula, viz. $r = \frac{1}{1+\beta(\tau-50)} \times \frac{p}{30} \times \delta + \frac{d\delta}{d\tau}(\tau-50°) - \frac{d\delta}{dp}(30-p)$; in which r denotes the true refraction, $\beta = .002084$ the expansion of a given volume of air at the surface of the earth for one degree of Fahrenheit's thermometer, p the height of the English barometer, τ the temperature in the open air by Fahrenheit's thermometer, δ the mean refraction for 30 inches and 50°; and $\frac{d\delta}{d\tau}$ and $\frac{d\delta}{dp}$ are expressions for determining the effects of changes in the temperature and barometric pressure respectively.

Table XVII. contains δ, the mean refractions, and the expressions for $\frac{d\delta}{d\tau}$ and $\frac{d\delta}{dp}$. Table XVIII. contains the logarithms of $\frac{1}{1+\beta.(\tau-50°)}$; Table XIX. the logarithms of $\frac{p}{30}$; and Table XX. the logarithms $-\frac{\tau-50}{10000} \times .484$.

TABLE XXI.

Augmentation of the Moon's Semi-diameter in Altitude and Zenith Dist.

Alt.	Z.D.	14′ 30″	15′ 0″	15′ 30″	16′ 0″	16′ 30″	17′ 0″
°	°	″	″	″	″	″	″
0	90	0.00	0.00	0.00	0.00	0.00	0 00
1	89	0.24	0.25	0.27	0.29	0.31	0.33
2	88	0.48	0.50	0.54	0.58	0.62	0.65
3	87	0.71	0.75	0.80	0.86	0.92	0.97
4	86	0.95	1.00	1.07	1.15	1.23	1.30
5	85	1.18	1.25	1.34	1.43	1.53	1.62
6	84	1.41	1.50	1.60	1.71	1.83	1.94
7	83	1.65	1.75	1.87	2.00	2.13	2.26
8	82	1.88	2.00	2.14	2.28	2.43	2.58
9	81	2.11	2.25	2.40	2.56	2.73	2.90
10	80	2.35	2.50	2.67	2.85	3.03	3.22
11	79	2.58	2.75	2.94	3.13	3.33	3.54
12	78	2.81	3.00	3.20	3.41	3 63	3.86
13	77	3.04	3.25	3.47	3.69	3.93	4.18
14	76	3.27	3.50	3.73	3.97	4.23	4.49
15	75	3.50	3.74	3.99	4.25	4.52	4.80
16	74	3.73	3.98	4.25	4.53	4.81	5.11
17	73	3.95	4.22	4.51	4.80	5.10	5.42
18	72	4.17	4.46	4.76	5.07	5.39	5.73
19	71	4.40	4.70	5.02	5.35	5.68	6.04
20	70	4.62	4.94	5.27	5.62	5.97	6.35
21	69	4.84	5.18	5.52	5.89	6.26	6.65
22	68	5.06	5.42	5.77	6.16	6.55	6.95
23	67	5 28	5.65	6.02	6.42	6.83	7.25
24	66	5.49	5.88	6.27	6.68	7.11	7.54
25	65	5.71	6.11	6.52	6.94	7.39	7.84
26	64	5.92	6.34	6.76	7.20	7.66	8.13
27	63	6.13	6.56	7.00	7.46	7.93	8.42
28	62	6.34	6.79	7.24	7.72	8.20	8.71
29	61	6.55	7.01	7.48	7.97	8.47	9.00
30	60	6.75	7.23	7.71	8.22	8.74	9.28
32	58	7.15	7.67	8.17	8.72	9.26	9.84
34	56	7.55	8.09	8.63	9.20	9.78	10.39
36	54	7.93	8.50	9.07	9.67	10.28	10.92
38	52	8.31	8.90	9.51	10.13	10.78	11.42
40	50	8.67	9.30	9.93	10.58	11.26	11.92
42	48	9.03	9.68	10.34	11.02	11.72	12.44
44	46	9.38	10.05	10.74	11.44	12.17	12.92
46	44	9.72	10.41	11.12	11.85	12.61	13.38
48	42	10.05	10.76	11.49	12.25	13.03	13.83
50	40	10.37	11.10	11.85	12.63	13.43	14.25
52	38	10.67	11.42	12.19	12.99	13.82	14.66
54	36	10.95	11.72	12.52	13.34	14.19	15.06
56	34	11.22	12.01	12.83	13.67	14.55	15.44
58	32	11.48	12.29	13.12	13.99	14.88	15.79
60	30	11.72	12.55	13.40	14.29	15.20	16.13
62	28	11.95	12.79	13.66	14.57	15.50	16.45
64	26	12.17	13.02	13.91	14.83	15.78	16.75
66	24	12.37	13.24	14.14	15.08	16.04	17.03
68	22	12.55	13.44	14.36	15.30	16.28	17.28
70	20	12.72	13.62	14.55	15.51	16.50	17.51
72	18	12.88	13.79	14.73	15.70	16.70	17.73
74	16	13.02	13.94	14.89	15.87	16.88	17.92
76	14	13.14	14.07	15.03	16.02	17.04	18.09
78	12	13.24	14.18	15.15	16.15	17.18	18.24
80	10	13.33	14.28	15.25	16.26	17.30	18.36
82	8	13.40	14.36	15.34	16.35	17.39	18.47
84	6	13.46	14.42	15.41	16.42	17.47	18.55
86	4	13.50	14.46	15.45	16.47	17.52	18.60
88	2	13.53	14.49	15.48	16.50	17.55	18.63
90	0	13.54	14.50	15.49	16.51	17.57	18.65

TABLE XXII.

Reduction of the Moon's Parallax in the Spheroid.

Lat.	54′	55′	56′	57′	58′	59′	60′	61′
°	″	″	″	″	″	″	″	″
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0 0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
6	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
7	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
8	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
9	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
10	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4
11	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
12	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5
13	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6
14	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7
15	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8
16	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9
17	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.0
18	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1
19	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2
20	1.2	1.2	1.3	1.3	1.3	1.3	1.4	1.4
21	1.3	1.4	1.4	1.4	1.4	1.5	1.5	1.5
22	1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.7
23	1.6	1.6	1.6	1.7	1.7	1.7	1.8	1.8
24	1.7	1.7	1.8	1.8	1.8	1.9	1.9	1.9
25	1.9	1.9	1.9	2.0	2 0	2.0	2.1	2.1
26	2.0	2.0	2.1	2.1	2.1	2.2	2.3	2.3
27	2.1	2.2	2.2	2.3	2.3	2.3	2.4	2.4
28	2.3	2.3	2.4	2.4	2.5	2.5	2.5	2.6
29	2.4	2.5	2.5	2.6	2 6	2.7	2.7	2.8
30	2.6	2.6	2.7	2.7	2.8	2.8	2.9	3.0
32	2.9	3.0	3.0	3.1	3.1	3.2	3.2	3.3
34	3.3	3.3	3.4	3.4	3.5	3.6	3.6	3.7
36	3.6	3.7	3.7	3.8	3.9	3.9	4.0	4.1
38	3.9	4.0	4.1	4.2	4.2	4.3	4.4	4.6
40	4.3	4.4	4.5	4.6	4.6	4.7	4.8	4.9
42	4.7	4.8	4.8	4.9	5.0	5.1	5.2	5.3
44	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7
46	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1
48	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5
50	6.1	6.3	6.4	6.5	6.6	6.7	6.8	6.9
52	6.5	6.6	6.7	6.9	7.0	7.1	7.2	7.3
54	6.9	7.0	7.1	7.2	7.4	7.5	7.6	7.7
56	7.2	7.3	7.5	7.6	7.7	7.9	8.0	8.1
58	7.5	7.7	7.8	7.9	8.1	8.2	8.4	8.5
60	7.9	8.0	8.2	8.3	8.4	8.6	8.7	8.9
62	8.2	8.3	8.5	8.6	8.8	8.9	9.1	9.2
64	8.5	8.6	8.8	8.9	9.1	9.3	9.4	9.6
66	8.8	8.9	9.1	9.2	9.4	9.6	9.7	9.9
68	9.0	9.2	9.4	9.5	9.7	9.9	10.0	10.2
70	9.3	9.4	9.6	9.8	10.0	10.1	10.3	10.5
72	9.5	9.7	9.9	10.0	10.2	10.4	10.6	10.7
74	9.7	9.9	10.1	10.2	10.4	10.6	10.8	11.0
76	9.9	10.1	10.3	10.4	10.6	10.8	11.0	11.2
78	10.0	10.2	10.4	10.6	10.8	11.0	11.2	11.4
80	10.2	10.4	10.6	10.7	10.9	11.1	11.3	11.5
82	10.3	10.5	10.7	10.9	11.1	11.3	11.4	11.6
84	10.4	10.6	10.8	11.0	11.2	11.4	11.5	11.7
86	10.5	10.6	10.8	11.0	11.2	11.4	11.6	11.8
88	10.5	10.7	10.9	11.1	11.3	11.5	11.7	11.9
90	10.5	10.7	10.9	11.1	11.3	11.5	11.7	11.9

TABLE XXIII

Logarithms of the Earth's Radii, in each Parallel of Latitude; the Equatorial Radius being Unity, and Compression $\frac{1}{300}$.

Lat.	Log. R	Lat.	Log. R	Lat.	Log. R
0°	0.0000000	30°	9.9996402	60°	9.9989151
1	9.9999995	31	96181	61	88932
2	9982	32	95957	62	88720
3	9960	33	95728	63	88512
4	9930	34	95496	64	88308
5	9890	35	95261	65	88111
6	9843	36	95023	66	87918
7	9786	37	94781	67	87732
8	9721	38	94537	68	87552
9	9648	39	94291	69	87378
10	9566	40	94044	70	87210
11	9.9999477	41	9.9993794	71	9.9987050
12	9379	42	93543	72	86896
13	9273	43	93291	73	86750
14	9158	44	93038	74	86611
15	9037	45	92786	75	86479
16	8908	46	92533	76	86356
17	8771	47	92280	77	86240
18	8627	48	92028	78	86131
19	8476	49	91776	79	86031
20	8318	50	91525	80	85940
21	9.9998153	51	9.9991277	81	9.9985857
22	7983	52	91030	82	85782
23	7805	53	90785	83	85716
24	7621	54	90542	84	85659
25	7431	55	90302	85	85610
26	7236	56	90065	86	85570
27	7035	57	89831	87	85539
28	6829	58	89600	88	85517
29	6618	59	89374	89	85504
30	6402	60	89151	90	85499

TABLE XXIV.

Angles of the Vertical with the Radius; or Reduction of the Latitude, in each Parallel, the Compression being $\frac{1}{300}$.

Lat.	Reduc.	Lat.	Reduc.	Lat.	Reduc.
°	′ ″	°	′ ″	°	′ ″
0	0 0.0	30	9 55.4	60	9 57.4
1	0 24.0	31	10 7.2	61	9 45.1
2	0 47.9	32	10 18.1	62	9 32.0
3	1 11.8	33	10 28.3	63	9 18.3
4	1 35.5	34	10 37.8	64	9 3.8
5	1 59.2	35	10 46.4	65	8 48.7
6	2 22.7	36	10 54.3	66	8 32.9
7	2 46.1	37	11 1.4	67	8 16.6
8	3 9.2	38	11 7.7	68	7 59.6
9	3 32.1	39	11 13.2	69	7 42.0
10	3 54.8	40	11 17.9	70	7 23.8
11	4 17.2	41	11 21.7	71	7 5.1
12	4 39.3	42	11 24.7	72	6 45.9
13	5 1.0	43	11 26.9	73	6 26.2
14	5 22.4	44	11 28.2	74	6 6.0
15	5 43.4	45	11 28.7	75	5 45.4
16	6 3.9	46	11 28.4	76	5 24.3
17	6 24.1	47	11 27.3	77	5 2.8
18	6 43.7	48	11 25.2	78	4 41.0
19	7 2.9	49	11 22.3	79	4 18.8
20	7 21.6	50	11 18.6	80	3 56.3
21	7 39.7	51	11 14.1	81	3 33.5
22	7 57.3	52	11 8.8	82	3 10.4
23	8 14.2	53	11 2.6	83	2 47.2
24	8 30.7	54	10 55.7	84	2 23.7
25	8 46.4	55	10 47.9	85	2 0.0
26	9 1.6	56	10 39.4	86	1 36.2
27	9 16.1	57	10 30.0	87	1 12.3
28	9 29.9	58	10 19.9	88	0 48.2
29	9 43.0	59	10 9.0	89	0 24.1
30	9 55.4	60	9 57.4	90	0 0.0

TABLE XXV.

For determining the Latitude, at *any time*, by the Pole Star.

θ	M	N	θ	M	N	θ	M	N	θ	M	N	θ	M	N	θ	M	N
h.m.	″	″	h.m.	″	″	h.m.	″	″	h.m.	″	″	h.m.	″	″	h.m.	″	″
0 0	0.00	0.00	1 0	5.85	0.11	2 0	21.82	0.37	3 0	43.63	0.60	4 0	65.45	0.63	5 0	81.42	0.41
10	0.17	0.00	10	7.89	0.14	10	25.19	0.41	10	47.44	0.62	10	68.66	0.61	10	83.18	0.35
20	0.66	0.01	20	10.20	0.19	20	28.71	0.46	20	51.21	0.64	20	71.68	0.59	20	84.64	0.28
30	1.49	0.03	30	12.78	0.23	30	32.34	0.50	30	54.93	0.65	30	74.49	0.55	30	85.78	0.22
40	2.63	0.05	40	15.59	0.27	40	36.06	0.53	40	58.56	0.65	40	77.06	0.51	40	86.60	0.15
50	4.09	0.08	50	18.61	0.32	50	39.83	0.57	50	62.07	0.64	50	79.38	0.46	50	87.10	0.07
1 0	5.85	0.11	2 0	21.82	0.37	3 0	43.63	0.60	4 0	65 45	0.63	5 0	81.42	0.41	6 0	87.26	0.00

$\psi = Z + p \cos. t - M \text{ cotan. } Z + N$; where ψ is = the Colatitude; Z = the Zenith Distance; $p = 1° 40'$, or $100'$; t = the Horary Angle; $\theta = t$ in the first Quadrant; $= 12^h - t$ in the second; $= t - 12^h$ in the third; and $= 24^h - t$ in the fourth; M and N being the Tabular Quantities. The quantity M is $= \frac{1}{2} p^2 \sin.^2 t$, and is always positive; but the quantity $N = \frac{1}{2} p^3 \sin.^2 t \cos. t$, becomes negative in the second and third Quadrants of t. When p (the Polar Distance) augments or diminishes 1′, the Tabular Quantity must also be augmented or diminished by 0.02M; and for any other quantity of variation in the same proportion.

TABLE XXVI.

To find the Augmentation of the Moon's Semidiameter by the Altitude of the Nonagesimal, and the Apparent Distance of the Moon therefrom.

PART I.

Alt. nona. + app. dis. Moon fr. nona. / Alt. nona. — app. dis. Moon fr. nona.	0. VI. (+ —)	I. VII. (+ —)	II. VIII. (+ —)	
0°	0″.00	4″.10	7″.10	30°
1	0.14	4.22	7.16	29
2	0.29	4.34	7.23	28
3	0.43	4.46	7.30	27
4	0.57	4.58	7.36	26
5	0.72	4.70	7.42	25
6	0.86	4.82	7.48	24
7	1.00	4.93	7.54	23
8	1.14	5.04	7.59	22
9	1.28	5.16	7.65	21
10	1.42	5.27	7.70	20
11	1.56	5.38	7.74	19
12	1.70	5.49	7.79	18
13	1.84	5.59	7.83	17
14	1.98	5.70	7.87	16
15	2.12	5.80	7.92	15
16	2.25	5.89	7.95	14
17	2.39	5.99	7.98	13
18	2.53	6.09	8.02	12
19	2.67	6.18	8.04	11
20	2.80	6.27	8.06	10
21	2.94	6.36	8.09	9
22	3.07	6.45	8.11	8
23	3.20	6.54	8.13	7
24	3.33	6.63	8.15	6
25	3.46	6.71	8.16	5
26	3.59	6.79	8.17	4
27	3.72	6.87	8.18	3
28	3.85	6.95	8.19	2
29	3.97	7.02	8.19	1
30	4.10	7.10	8.19	0
	XI. V. (— +)	X. IV. (— +)	IX. III. (— +)	

PART II.

Aggregate of No. from Part I.	Cor. +
1″	0″.00
2	0.00
3	0.01
4	0.02
5	0.03
6	0.04
7	0.05
8	0.06
8.5	0.07
9.0	0.08
9.5	0.09
10.0	0.10
10.5	0.11
11.0	0.12
11.5	0.13
12.0	0.14
12.3	0.15
12.7	0.16
13.0	0.17
13.3	0.18
13.7	0.19
14.0	0.20
14.3	0.21
14.7	0.22
15.0	0.23
15.3	0.24
15.7	0.25
16.0	0.26
16.3	0.27
16.7	0.28
17.0	0.29

PART III.

True Lat. of the Moon.	Parallax of the Moon in Lat. 0′	10′ —	20′ —	30′ —	40′ —	50′ —	60′ —
S. Lat.	″	″	″	″	″	″	″
6° 0′	0.00	0.29	0.59	0.90	1.22	1.54	1.88
5 40	0.00	0.27	0.56	0.85	1.16	1.46	1.78
5 20	0 00	0.26	0.53	0.80	1.09	1.38	1.69
5 0	0.00	0.24	0.49	0.76	1.02	1.30	1.59
4 40	0.00	0.23	0.46	0.71	0.96	1.22	1.49
4 20	0.00	0.21	0.43	0.66	0.90	1.14	1.40
4 0	0.00	0.19	0.40	0.61	0.83	1.06	1.30
3 40	0.00	0.18	0.37	0.56	0.77	0.98	1.20
3 20	0.00	0.16	0.33	0.52	0.70	0.90	1.11
3 0	0.00	0.14	0.30	0.47	0.64	0.82	1.01
2 40	0.00	0.13	0.27	0.42	0.58	0.74	0.91
2 20	0.00	0.11	0.24	0.37	0.51	0.66	0.82
2 0	0.00	0.10	0.21	0.33	0.45	0.58	0.72
1 40	0.00	0.08	0.17	0.28	0.39	0.50	0.63
1 20	0.00	0.06	0.14	0.23	0.32	0.42	0.53
1 0	0.00	0.05	0.11	0.18	0.26	0.34	0.43
0 50	0.00	0.04	0.09	0.16	0.23	0.30	0.39
0 40	0.00	0.03	0.08	0.13	0.19	0.26	0.34
0 30	0.00	0.02	0.06	0.11	0.16	0.22	0.29
0 20	0.00	0.02	0.05	0.08	0.13	0.18	0.24
0 10	0.00	0.01	0.03	0.06	0.10	0.14	0.19
0 0	0.00	+	0.02	0.04	0.06	0.10	0.14
N. Lat.							
0 10	0.00	0.01	+	0.01	0.03	0.06	0.10
				+			
0 20	0.00	0.02	0.02	0.01	+	0.02	0.05
						+	
0 30	0.00	0.02	0.03	0.04	0.03	0.02	+
0 40	0.00	0.03	0.05	0.06	0.06	0.06	0.06
0 50	0.00	0.04	0.07	0.09	0.10	0.10	0.10
1 0	0.00	0.05	0.08	0.11	0.13	0.14	0.15
1 20	0.00	0.06	0.11	0.15	0.19	0.22	0 24
1 40	0.00	0.08	0.14	0.20	0.26	0.30	0.34
2 0	0.00	0.10	0.18	0.25	0.32	0.38	0.44
2 20	0.00	0.11	0.21	0.30	0.39	0.46	0.53
2 40	0.00	0.13	0.24	0.35	0.45	0.54	0.63
3 0	0.00	0.14	0.27	0.39	0.51	0.62	0.72
3 20	0.00	0.16	0.30	0.44	0.58	0.70	0.82
3 40	0.00	0.18	0.34	0.49	0.64	0.78	0.92
4 0	0.00	0.19	0.37	0.54	0.70	0.86	1.01
4 20	0.00	0.21	0.40	0.59	0.77	0.94	1.11
4 40	0.00	0.23	0.43	0.64	0.83	1.02	1.20
5 0	0.00	0.24	0.46	0.68	0.90	1.10	1.30
5 20	0.00	0.26	0.50	0.73	0.96	1.18	1.40
5 40	0.00	0.27	0.53	0.78	1.02	1.26	1.49
6 0	0.00	0.29	0.56	0.83	1.09	1.34	1.59
	0′	+ 10′	+ 20′	+ 30′	+ 40′	+ 50′	+ 60′
	Parallax of the Moon in Lat.						

PART IV.

Sum of preced. Equation.	Semidiameter of the Moon. 14′ 40″ —	14′ 50″ —	15′ 0″ —	15′ 10″ —	15′ 20″ —	15′ 30″ —	15′ 40″ —	15′ 50″ —	16′ 0″
″	″	″	″	″	″	″	″	″	″
1	0.17	0.15	0.13	0.10	0.08	0.06	0.04	0.02	0
2	0.38	0.29	0.25	0.21	0.17	0.12	0.08	0.04	0
3	0.50	0.44	0.37	0.31	0.25	0.19	0.12	0.06	0
4	0.67	0.58	0.50	0.42	0.33	0.25	0.17	0.08	0
5	0.83	0.73	0.62	0.52	0.42	0.31	0.21	0.10	0
6	1.00	0.88	0.75	0.62	0.50	0.37	0.25	0.12	0
7	1.16	1.02	0.87	0.73	0.58	0.44	0.29	0.15	0
8	1.33	1.16	1.00	0.83	0.67	0.50	0.33	0.16	0
9	1.50	1.31	1.13	0.94	0.75	0.56	0.37	0.19	0
10	1.67	1.46	1.25	1.04	0.83	0.62	0.42	0.21	0
11	1.83	1.60	1.37	1.15	0.92	0.69	0.46	0.23	0
12	2.00	1.75	1.50	1.25	1.00	0.75	0.50	0.25	0
13	2.16	1.89	1.62	1.35	1.08	0.81	0.54	0.27	0
14	2.33	2.04	1.75	1.46	1.17	0.87	0.58	0.29	0
15	2.50	2.18	1.87	1.56	1.25	0.94	0.62	0.31	0
	+ 20″ 17′	+ 10″ 17′	+ 0″ 17′	+ 50″ 16′	+ 40″ 16′	+ 30″ 16′	+ 20″ 16′	+ 10″ 16′	0″ 16′

Time after Noon or Midnight.		Second Difference.											
		10′	20′	30′	1′	2′	3′	4′	5′	6′	7′	8′	9′
h. m.	h. m.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0 0	12 0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0	0 0.0
0 10	11 50	0 4.1	0 8.2	0 12.3	0 0.4	0 0.8	0 1.2	0 1.6	0 2.1	0 2.5	0 2.9	0 3.3	0 3.7
0 20	11 40	0 8.1	0 16.2	0 24.3	0 0.8	0 1.6	0 2.4	0 3.2	0 4.1	0 4.9	0 5.7	0 6.5	0 7.3
0 30	11 30	0 12.0	0 24.0	0 35.9	0 1.2	0 2.4	0 3.6	0 4 8	0 6.0	0 7.2	0 8.4	0 9.6	0 10 8
0 40	11 20	0 15.7	0 31.5	0 47.2	0 1.6	0 3.1	0 4.7	0 6.3	0 7.9	0 9.4	0 11.0	0 12.6	0 14.2
0 50	11 10	0 19.4	0 38.8	0 58.2	0 1.9	0 3.9	0 5.8	0 7.8	0 9.7	0 11.6	0 13.5	0 15 5	0 17.4
1 0	11 0	0 22.9	0 45.8	1 8 7	0 2.3	0 4.6	0 6.9	0 9.2	0 11.5	0 13.8	0 16.0	0 18.3	0 20.6
1 10	10 50	0 26.3	0 52.7	1 19 0	0 2.6	0 5 3	0 7.9	0 10.5	0 13.2	0 15.8	0 18.4	0 21.1	0 23.7
1 20	10 40	0 29.6	0 59.3	1 28 9	0 3.0	0 5.9	0 8.9	0 11.9	0 14.8	0 17.8	0 20.7	0 23.7	0 26.7
1 30	10 30	0 32.8	1 5.6	1 38.4	0 3.3	0 6 6	0 9.8	0 13.1	0 16.4	0 19.7	0 23.0	0 26.3	0 29.5
1 40	10 20	0 35.9	1 11.8	1 47.6	0 3.6	0 7.2	0 10.8	0 14.4	0 17.9	0 21.5	0 25.1	0 28.7	0 32.3
1 50	10 10	0 38.8	1 17.7	1 56.5	0 3.9	0 7.8	0 11.6	0 15 5	0 19.4	0 23.3	0 27.2	0 31.1	0 34.9
2 0	10 0	0 41.7	1 23.3	2 5.0	0 4.2	0 8.3	0 12.5	0 16.7	0 20.8	0 25.0	0 29.2	0 33.3	0 37.5
2 10	9 50	0 44.4	1 28.8	2 13.2	0 4.4	0 8.9	0 13.3	0 17.8	0 22.2	0 26.6	0 31.1	0 35.5	0 39.9
2 20	9 40	0 47.0	1 34.0	2 21.0	0 4.7	0 9.4	0 14.1	0 18.8	0 23.5	0 28.2	0 32.9	0 37.6	0 42.3
2 30	9 30	0 49.5	1 39.0	2 28.4	0 4.9	0 9.9	0 14.8	0 19.8	0 24.7	0 29.7	0 34.6	0 39.6	0 44.5
2 40	9 20	0 51.9	1 43.7	2 35.5	0 5.2	0 10.4	0 15.6	0 20.7	0 25.9	0 31.1	0 36.3	0 41.5	0 46.7
2 50	9 10	0 54.1	1 48.2	2 42.3	0 5.4	0 10 8	0 16.2	0 21.6	0 27.1	0 32.5	0 37.9	0 43.3	0 48.7
3 0	9 0	0 56.2	1 52.5	2 48.7	0 5.6	0 11.3	0 16.9	0 22.5	0 28.1	0 33.8	0 39.4	0 45.0	0 50.6
3 10	8 50	0 58.3	1 56.6	2 54.8	0 5.8	0 11.7	0 17.5	0 23.3	0 29.1	0 35.0	0 40.8	0 46.6	0 52.4
3 20	8 40	1 0.2	2 0.4	3 0.6	0 6.0	0 12.0	0 18.1	0 24.1	0 30.1	0 36.1	0 42.1	0 48.1	0 54.2
3 30	8 30	1 2.0	2 4.0	3 5.9	0 6.2	0 12.4	0 18.6	0 24.8	0 31.0	0 37.2	0 43.4	0 49.6	0 55.8
3 40	8 20	1 3.7	2 7.3	3 11.0	0 6.4	0 12.7	0 19.1	0 25.5	0 31.8	0 38.2	0 44.6	0 50.9	0 57.3
3 50	8 10	1 5.2	2 10.4	3 15.6	0 6.5	0 13.0	0 19.6	0 26.1	0 32.6	0 39.1	0 45.7	0 52.2	0 58.7
4 0	8 0	1 6.7	2 13.3	3 20.0	0 6.7	0 13.3	0 20.0	0 26.7	0 33.3	0 40.0	0 46.7	0 53.3	1 0.0
4 20	7 40	1 9.2	2 18.4	3 27.6	0 6.9	0 13.8	0 20.8	0 27.7	0 34.6	0 41.5	0 48.4	0 55.4	1 2.3
4 40	7 20	1 11.3	2 22.6	3 33.9	0 7.1	0 14.3	0 21.4	0 28.5	0 35.6	0 42.8	0 49.9	0 57.0	1 4.2
5 0	7 0	1 12.9	2 25.8	3 38.8	0 7.3	0 14.6	0 21.9	0 29.2	0 36.5	0 43.8	0 51.0	0 58.3	1 5.6
5 20	6 40	1 14.1	2 28.2	3 42.2	0 7.4	0 14.8	0 22.2	0 29.6	0 37.0	0 44.4	0 51.9	0 59.3	1 6.7
5 40	6 20	1 14.8	2 29.5	3 44.3	0 7.5	0 15.0	0 22.4	0 29.9	0 37.4	0 44.9	0 52.3	0 59.8	1 7.3
6 0	6 0	1 15.0	2 30.0	3 45 0	0 7.5	0 15.0	0 22.5	0 30.0	0 37.5	0 45.0	0 52.5	1 0.0	1 7.5

Time after Noon or Midnight.		Second Difference.														Equation for 24 h.	Time after Noon.
		10″	20″	30″	40″	50″	1″	2″	3″	4″	5″	6″	7″	8″	9″		
h. m.	h. m.	″	″	″	″	″	″	″	″	″	″	″	″	″	″	h. m.	h. m.
0 0	12 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0 0	24 0
0 10	11 50	0.1	0.1	0.2	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0 20	23 40
0 20	11 40	0.1	0.3	0.4	0.5	0.7	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0 40	23 20
0 30	11 30	0.2	0.4	0.6	0.8	1.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.2	0.2	1 0	23 0
0 40	11 20	0.3	0.5	0.8	1.0	1.3	0.0	0.1	0.1	0.1	0.1	0.2	0.2	0.2	0.2	1 20	22 40
0 50	11 10	0 3	0.6	1.0	1.3	1.6	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3	0.3	1 40	22 20
1 0	11 0	0.4	0.8	1.1	1.5	1.9	0.0	0.1	0.1	0.1	0.2	0 2	0.3	0.3	0.3	2 0	22 0
1 10	10 50	0.4	0.9	1.3	1.8	2.2	0 0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	2 20	21 40
1 20	10 40	0 5	1.0	1.5	2.0	2.5	0.0	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	2 40	21 20
1 30	10 30	0.5	1.1	1.6	2.2	2.7	0.0	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5	3 0	21 0
1 40	10 20	0.6	1.2	1.8	2.4	3.0	0.1	0.1	0.2	0.2	0.3	0.4	0.4	0.5	0.5	3 20	20 40
1 50	10 10	0 6	1.3	1.9	2.6	3.2	0.1	0.1	0.2	0.3	0.3	0.4	0.4	0.5	0.6	3 40	20 20
2 0	10 0	0.7	1.4	2.1	2.8	3.5	0.1	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.6	4 0	20 0
2 10	9 50	0.7	1.5	2.2	3.0	3.7	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.7	4 20	19 40
2 20	9 40	0.8	1.6	2.3	3.1	3.9	0.1	0.2	0.2	0.3	0.4	0.5	0 5	0.6	0.7	4 40	19 20
2 30	9 30	0.8	1.6	2.5	3.3	4.1	0.1	0.2	0.2	0.3	0.4	0.5	0.6	0.7	0.7	5 0	19 0
2 40	9 20	0.9	1.7	2.6	3.5	4.3	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.7	0.8	5 20	18 40
2 50	9 10	0.9	1.8	2.7	3.6	4.5	0.1	0.2	0.3	0.3	0.4	0.5	0.6	0.7	0.8	5 40	18 20
3 0	9 0	0.9	1.9	2.8	3.8	4.7	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7	0.8	6 0	18 0
3 10	8 50	1.0	1.9	2.9	3.9	4.9	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	6 20	17 40
3 20	8 40	1.0	2.0	3.0	4.0	5.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	6 40	17 20
3 30	8 30	1.0	2.1	3.1	4.1	5.2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	7 0	17 0
3 40	8 20	1.1	2.1	3.2	4.2	5.3	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0	7 20	16 40
3 50	8 10	1.1	2.2	3.3	4.3	5.4	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0	7 40	16 20
4 0	8 0	1.1	2.2	3.3	4.4	5.6	0.1	0.2	0.3	0.4	0.6	0.7	0.8	0.9	1.0	8 0	16 0
4 20	7 40	1.2	2.3	3.5	4.6	5.8	0.1	0.2	0.3	0.5	0.6	0.7	0.8	0.9	1.0	8 40	15 20
4 40	7 20	1.2	2.4	3.6	4.8	5.9	0.1	0.2	0.4	0.5	0.6	0.7	0.8	1.0	1.1	9 20	14 40
5 0	7 0	1.2	2.4	3.7	4.9	6.1	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	10 0	14 0
5 20	6 40	1.2	2.5	3.7	4.9	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	10 40	13 20
5 40	6 20	1.2	2.5	3.7	5.0	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	11 20	12 40
6 0	6 0	1.2	2.5	3.7	5.0	6.2	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1	12 0	12 0

TABLE XXVIII.

Reduction to the Meridian. Part I.

s	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m
	″	″	″	″	″	″	″	″	″	″	″	″	″
0	0.00	1.96	7.85	17.67	31.41	49.09	70.68	96.20	125.65	159.02	196.32	237.54	282.68
1	0.00	2.03	7.99	17.87	31.68	49.41	71.07	96.66	126.17	159.61	196.97	238.26	283.46
2	0.00	2.10	8.12	18.09	31.94	49.74	71.47	97.12	126.70	160.20	197.63	238.98	284.25
3	0.00	2.16	8.25	18.27	32.21	50.07	71.86	97.58	127.23	160.79	198.29	239.70	285.04
4	0.01	2.23	8.39	18.47	32.47	50.40	72.26	98.04	127.75	161.39	198.94	240.42	285.83
5	0.01	2.30	8.52	18.67	32.74	50.74	72.66	98.51	128.28	161.98	199.60	241.15	286.62
6	0.02	2.38	8.66	18.87	33.01	51.07	73.06	98.97	128.81	162.58	200.26	241.87	287.41
7	0.03	2.45	8.80	19.07	33.27	51.40	73.46	99.44	129.34	163.17	200.93	242.60	288.20
8	0.03	2.52	8.94	19.28	33.54	51.74	73.86	99.90	129.87	163.77	201.59	243.33	288.99
9	0.04	2.60	9.08	19.48	33.82	52.07	74.26	100.37	130.41	164.37	202.25	244.06	289.79
10	0.05	2.67	9.22	19.69	34.09	52.41	74.66	100.84	130.94	164.97	202.92	244.79	290.58
11	0.07	2.75	9.36	19.90	34.36	52.75	75.07	101.31	131.48	165.57	203.58	245.52	291.38
12	0.08	2.83	9.50	20.11	34.64	53.09	75.47	101.78	132.01	166.17	204.25	246.26	292.18
13	0.09	2.91	9.65	20.32	34.91	53.43	75.88	102.25	132.55	166.77	204.92	246.99	292.98
14	0.11	2.99	9.79	20.53	35.19	53.77	76.29	102.72	133.09	167.37	205.59	247.72	293.78
15	0.12	3.07	9.94	20.74	35.46	54.12	76.70	103.20	133.63	167.98	206.26	248.46	294.58
16	0.14	3.15	10.09	20.95	35.74	54.46	77.10	103.67	134.17	168.58	206.93	249.19	295.38
17	0.16	3.23	10.24	21.17	36.02	54.81	77.51	104.15	134.71	169.19	207.60	249.93	296.18
18	0.18	3.32	10.39	21.38	36.30	55.15	77.92	104.63	135.25	169.80	208.27	250.67	296.99
19	0.20	3.40	10.54	21.60	36.59	55.50	78.34	105.10	135.79	170.41	208.95	251.41	297.79
20	0.22	3.49	10.69	21.82	36.87	55.85	78.75	105.58	136.34	171.02	209.62	252.15	298.60
21	0.24	3.58	10.84	22.03	37.15	56.20	79.17	106.06	136.88	171.63	210.30	252.89	299.40
22	0.26	3.67	11.00	22.25	37.44	56.55	79.58	106.55	137.43	172.24	210.98	253.63	300.21
23	0.29	3.76	11.15	22.48	37.72	56.90	80.00	107.03	137.98	172.86	211.66	254.38	301.02
24	0.31	3.85	11.31	22.70	38.01	57.25	80.42	107.51	138.53	173.47	212.34	255.12	301.83
25	0.34	3.94	11.47	22.92	38.30	57.61	80.84	108.00	139.08	174.09	213.02	255.87	302.65
26	0.37	4.03	11.63	23.14	38.59	57.96	81.26	108.48	139.63	174.70	213.70	256.62	303.46
27	0.40	4.13	11.79	23.37	38.88	58.32	81.68	108.97	140.18	175.32	214.38	257.37	304.27
28	0.43	4.22	11.95	23.60	39.17	58.68	82.10	109.46	140.74	175.94	215.07	258.12	305.09
29	0.46	4.32	12.11	23.82	39.47	59.03	82.53	109.95	141.29	176.56	215.75	258.87	305.90
30	0.49	4.42	12.27	24.05	39.76	59.39	82.95	110.44	141.85	177.18	216.44	259.62	306.72
31	0.52	4.52	12.44	24.28	40.05	59.76	83.38	110.93	142.40	177.80	217.12	260.37	307.54
32	0.56	4.62	12.60	24.51	40.35	60.15	83.81	111.42	142.96	178.42	217.81	261.12	308.36
33	0.59	4.72	12.77	24.74	40.65	60.48	84.23	111.91	143.52	179.05	218.50	261.88	309.18
34	0.63	4.82	12.94	24.98	40.95	60.84	84.66	112.41	144.08	179.68	219.19	262.64	310.00
35	0.67	4.92	13.10	25.21	41.25	61.21	85.09	112.90	144.64	180.30	219.89	263.39	310.82
36	0.71	5.03	13.27	25.45	41.55	61.57	85.52	113.40	145.20	180.93	220.58	264.15	311.65
37	0.75	5.13	13.44	25.68	41.85	61.94	85.96	113.90	145.77	181.56	221.27	264.91	312.47
38	0.79	5.24	13.62	25.92	42.15	62.31	86.39	114.40	146.33	182.19	221.97	265.67	313.30
39	0.83	5.35	13.79	26.16	42.45	62.68	86.82	114.90	146.90	182.82	222.66	266.43	314.12
40	0.87	5.45	13.96	26.40	42.76	63.05	87.26	115.40	147.46	183.45	223.36	267.20	314.95
41	0.92	5.56	14.14	26.64	43.06	63.42	87.70	115.90	148.03	184.09	224.06	267.96	315.78
42	0.96	5.67	14.31	26.88	43.37	63.79	88.13	116.40	148.60	184.72	224.76	268.72	316.61
43	1.01	5.79	14.49	27.12	43.68	64.16	88.57	116.91	149.17	185.35	225.46	269.49	317.44
44	1.06	5.90	14.67	27.37	43.99	64.54	89.01	117.41	149.74	185.99	226.16	270.26	318.27
45	1.10	6.01	14.85	27.61	44.30	64.91	89.46	117.92	150.31	186.63	226.86	271.03	319.11
46	1.15	6.13	15.03	27.86	44.61	65.29	89.90	118.43	150.88	187.27	227.57	271.79	319.94
47	1.20	6.24	15.21	28.10	44.92	65.67	90.34	118.94	151.46	187.91	228.27	272.57	320.78
48	1.26	6.36	15.39	28.35	45.24	66.05	90.79	119.45	152.03	188.55	228.98	273.34	321.62
49	1.31	6.48	15.58	28.60	45.55	66.43	91.23	119.96	152.61	189.19	229.69	274.11	322.45
50	1.36	6.60	15.76	28.85	45.87	66.81	91.68	120.47	153.19	189.83	230.40	274.88	323.29
51	1.42	6.72	15.95	29.10	46.18	67.19	92.13	120.98	153.77	190.47	231.11	275.66	324.13
52	1.48	6.84	16.14	29.36	46.50	67.58	92.57	121.50	154.35	191.12	231.81	276.43	324.97
53	1.53	6.96	16.32	29.61	46.82	67.96	93.02	122.01	154.93	191.76	232.53	277.21	325.82
54	1.59	7.09	16.51	29.86	47.14	68.35	93.47	122.53	155.51	192.41	233.24	277.99	326.66
55	1.65	7.21	16.70	30.12	47.46	68.73	93.93	123.05	156.09	193.06	233.95	278.77	327.50
56	1.71	7.34	16.89	30.38	47.79	69.12	94.38	123.57	156.68	193.71	234.67	279.55	328.35
57	1.77	7.47	17.08	30.64	48.11	69.51	94.83	124.09	157.26	194.36	235.38	280.33	329.20
58	1.83	7.59	17.28	30.89	48.43	69.90	95.29	124.61	157.85	195.02	236.10	281.11	330.04
59	1.90	7.72	17.48	31.15	48.76	70.29	95.75	125.13	158.43	195.67	236.82	281.89	330.89
.2	0.01	0.02	0.03	0.05	0.06	0.07	0.09	0.10	0.11	0.12	0.14	0.15	0.16
.4	0.01	0.04	0.06	0.09	0.12	0.14	0.17	0.20	0.22	0.25	0.28	0.30	0.33
.6	0.02	0.06	0.10	0.14	0.18	0.21	0.26	0.30	0.34	0.37	0.41	0.45	0.49
.8	0.02	0.08	0.13	0.18	0.24	0.28	0.31	0.40	0.45	0.50	0.55	0.60	0.66

TABLE XXIX.—Reduction to either Solstice.

Obliquity of the Ecliptic 23° 27′ 40″.

T. XXVIII. Part II.		Arg.	Reduc.	Diff.	Variation in 100″ change of obliquity.	Arg.	Reduc.	Diff.	Variation in 100″ change of obliquity.	Arg.	Reduc.	Diff.	Variation in 100″ change of obliquity.
		m. s.	′ ″	″		m. s.	′ ″	″		m. s.	′ ″	″	
	+	0 0	0 0.00	0.02	0.0000	10 0	1 11.71	2.41	0.0651	20 0	4 46.83	4.80	0.2606
m. s.	″	10	0 0.02	0.06	0.0000	10	1 14.12	2.45	0.0673	10	4 51.63	4.84	0.2650
0 0	0.00	20	0 0.08	0.10	0.0001	20	1 16.57	2.49	0.0695	20	4 56 47	4.88	0.2694
0 30	0.00	30	0 0.18	0.14	0.0002	30	1 19.06	2.53	0.0717	30	5 1.35	4.92	0.2738
1 0	0.00	40	0 0.32	0.18	0.0003	40	1 21.59	2.57	0.0740	40	5 6.27	4.96	0.2783
1 30	0.00	50	0 0.50	0.22	0.0005	50	1 24.16	2.61	0.0763	50	5 11.23	5.00	0.2828
2 0	0.00	1 0	0 0.72	0.26	0.0007	11 0	1 26.77	2.65	0.0787	21 0	5 16.23	5.04	0.2873
2 30	0.00	10	0 0.98	0.29	0.0009	10	1 29.42	2.69	0.0811	10	5 21.27	5.07	0.2919
3 0	0.00	20	0 1.27	0.34	0.0011	20	1 32.11	2.72	0.0836	20	5 26.34	5.12	0.2965
3 30	0.00	30	0 1.61	0.38	0.0014	30	1 34.83	2.77	0.0860	30	5 31.46	5.16	0.3012
4 0	0.00	40	0 1.99	0.42	0.0018	40	1 37.60	2.81	0.0886	40	5 36.62	5.20	0.3059
4 30	0.01	50	0 2.41	0.46	0.0022	50	1 40.41	2.85	0.0911	50	5 41.82	5.24	0.3106
5 0	0.01	2 0	0 2.87	0.50	0.0026	12 0	1 43.26	2.89	0.0937	22 0	5 47.06	5.28	0.3154
10	0.01	10	0 3.37	0.54	0.0031	10	1 46.15	2.93	0.0963	10	5 52.34	5.32	0.3202
20	0.01	20	0 3.91	0.57	0.0036	20	1 49.08	2.96	0.0990	20	5 57.66	5.35	0.3250
30	0.01	30	0 4.48	0.62	0.0041	30	1 52.04	3.01	0.1017	30	6 3.01	5.40	0.3299
40	0.01	40	0 5.10	0.66	0.0046	40	1 55 05	3.05	0.1044	40	6 8.41	5.44	0.3349
50	0.01	50	0 5.76	0.69	0.0053	50	1 58.10	3.09	0.1072	50	6 13.85	5.48	0.3398
6 0	0.01	3 0	0 6.45	0.74	0.0059	13 0	2 1.19	3.12	0.1100	23 0	6 19.33	5.51	0.3448
10	0.01	10	0 7.19	0.78	0.0066	10	2 4.31	3.17	0.1128	10	6 24.84	5.56	0.3498
20	0.01	20	0 7.97	0.81	0.0073	20	2 7.48	3.21	0.1157	20	6 30.40	5.60	0.3549
30	0.02	30	0 8.78	0.86	0.0080	30	2 10.69	3.25	0.1186	30	6 36.00	5.64	0.3600
40	0.02	40	0 9.64	0.89	0.0088	40	2 13.94	3.28	0.1216	40	6 41.64	5.67	0.3651
50	0.02	50	0 10.53	0.94	0.0096	50	2 17.22	3.33	0.1246	50	6 47.31	5.72	0.3703
7 0	0.02	4 0	0 11.47	0.97	0.0104	14 0	2 20.55	3.36	0.1276	24 0	6 53.03	5.75	0.3755
10	0.02	10	0 12.44	1.02	0.0113	10	2 23.91	3.41	0.1307	10	6 58.78	5.80	0.3807
20	0.03	20	0 13.46	1.06	0.0122	20	2 27.32	3.45	0.1338	20	7 4.58	5.84	0.3860
30	0.03	30	0 14.52	1.09	0.0132	30	2 30.77	3.48	0.1369	30	7 10.42	5.87	0.3913
40	0.03	40	0 15.61	1.14	0.0142	40	2 34.25	3.53	0.1401	40	7 16.29	5.92	0.3967
50	0.04	50	0 16.75	1.18	0.0152	50	2 37.78	3.56	0.1433	50	7 22.21	5.95	0.4021
8 0	0 04	5 0	0 17.93	1.21	0.0163	15 0	2 41.34	3.61	0.1465	25 0	7 28.16	6.00	0.4075
10	0.04	10	0 19.14	1.26	0.0174	10	2 44.95	3.64	0.1498	10	7 34.16	6.03	0.4130
20	0.05	20	0 20.40	1.29	0.0185	20	2 48.59	3.69	0.1531	20	7 40.19	6.08	0.4185
30	0.05	30	0 21.69	1.34	0.0197	30	2 52.28	3.72	0.1564	30	7 46.27	6.11	0.4240
40	0.05	40	0 23.03	1.37	0 0209	40	2 56.00	3.77	0.1598	40	7 52.38	6.15	0.4296
50	0.06	50	0 24.40	1.42	0.0221	50	2 59.77	3.80	0.1632	50	7 58.53	6.20	0.4352
9 0	0.06	6 0	0 25.82	1.45	0.0234	16 0	3 3.57	3.85	0.1667	26 0	8 4 73	6.23	0.4408
10	0.07	10	0 27.27	1.49	0.0247	10	3 7.42	3.88	0.1702	10	8 10.96	6.28	0.4465
20	0.07	20	0 28.76	1.54	0.0261	20	3 11.30	3.92	0.1737	20	8 17.24	6.31	0.4522
30	0.08	30	0 30.30	1.57	0.0274	30	3 15.22	3.97	0.1773	30	8 23.55	6.35	0.4579
40	0.08	40	0 31.37	1.62	0.0289	40	3 19.19	4.00	0.1809	40	8 29.90	6.40	0.4637
50	0.09	50	0 33.49	1.65	0.0303	50	3 23.19	4.05	0.1845	50	8 36.30	6.43	0.4695
10 0	0.09	7 0	0 35.14	1.69	0.0318	17 0	3 27.24	4.08	0.1882	27 0	8 42.73	6.47	0.4754
10	0.10	10	0 36.83	1.73	0.0333	10	3 31.32	4.12	0.1919	10	8 49.20	6.52	0.4813
20	0.11	20	0 38.56	1.78	0.0349	20	3 35.44	4.16	0.1957	20	8 55.72	6.55	0.4873
30	0 11	30	0 40.34	1.81	0.0366	30	3 39.60	4.21	0.1994	30	9 2.27	6.59	0.4932
40	0.12	40	0 42.15	1.85	0.0381	40	3 43.81	4.24	0.2033	40	9 8.86	6.63	0.4993
50	0.13	50	0 44.00	1.89	0.0399	50	3 48 05	4.28	0.2071	50	9 15.49	6.68	0.5053
11 0	0.14	8 0	0 45.89	1.94	0.0416	18 0	3 52.33	4.33	0.2110	28 0	9 22.17	6.71	0.5114
10	0.15	10	0 47.83	1.97	0.0434	10	3 56.66	4.36	0.2149	10	9 28.88	6.75	0.5175
20	0.15	20	0 49.80	2.01	0.0452	20	4 1.02	4.40	0.2189	20	9 35.63	6.79	0.5237
30	0.16	30	0 51.81	2.05	0.0470	30	4 5.42	4.44	0.2229	30	9 42.42	6.83	0.5299
40	0 17	40	0 53.86	2.09	0.0489	40	4 9.86	4.48	0.2269	40	9 49.25	6.87	0.5361
50	0.18	50	0 55.95	2.13	0.0508	50	4 14.34	4.53	0.2310	50	9 56.12	6.91	0.5424
12 0	0.19	9 0	0 58.08	2.18	0.0527	19 0	4 18.87	4.56	0.2351	29 0	10 3.03	6.95	0.5487
10	0.21	10	1 0.26	2.21	0.0547	10	4 23.43	4.60	0.2393	10	10 9.98	6.99	0.5550
20	0.22	20	1 2.47	2.25	0.0567	20	4 28.03	4.64	0.2435	20	10 16.97	7.04	0.5614
30	0.23	30	1 4.72	2.29	0.0587	30	4 32.67	4.68	0.2477	30	10 24.01	7.07	0.5678
40	0.24	40	1 7.01	2.33	0.0608	40	4 37.35	4.72	0.2520	40	10 31.08	7.11	0.5743
50	0.25	50	1 9.34	2.37	0.0629	50	4 42.07	4.76	0.2563	50	10 38.19	7.15	0.5808
13 0	0.27	10 0	1 11.71		0.0651	20 0	4 46.83		0.2606	30 0	10 45.34		0.5873

TABLE XXX.

To change mean Solar into Sidereal Time.

Solar Days.	Add h. m. s.	Solar Min.	Add Seconds. s.	Solar Sec.	Add Parts of a Sec. s.
1	0 3 56.556	1	0.164	1	0.003
2	0 7 53.112	2	0.329	2	0.006
3	0 11 49.668	3	0.493	3	0.008
4	0 15 46.224	4	0.658	4	0.011
5	0 19 42.780	5	0.822	5	0.014
6	0 23 39.336	6	0.986	6	0.017
7	0 27 35.892	7	1.150	7	0.019
8	0 31 32.448	8	1.315	8	0.022
9	0 35 29.004	9	1.479	9	0.025
10	0 39 25.560	10	1.643	10	0.027
11	0 43 22.116	11	1.807	11	0.030
12	0 47 18.672	12	1.972	12	0.033
13	0 51 15.228	13	2.136	13	0.036
14	0 55 11.784	14	2.300	14	0.038
15	0 59 8.340	15	2.464	15	0.041
16	1 3 4.896	16	2.629	16	0.044
17	1 7 1.452	17	2.793	17	0.047
18	1 10 58.008	18	2.957	18	0.050
19	1 14 54.564	19	3.121	19	0.053
20	1 18 51.120	20	3.286	20	0.055
21	1 22 47.676	21	3.450	21	0.058
22	1 26 44.232	22	3.614	22	0.061
23	1 30 40.788	23	3.779	23	0.064
24	1 34 37.344	24	3.943	24	0.066
25	1 38 33.900	25	4.108	25	0.069
26	1 42 30.456	26	4.272	26	0.072
27	1 46 27.012	27	4.436	27	0.075
28	1 50 23.568	28	4.600	28	0.077
29	1 54 20.124	29	4.764	29	0.080
30	1 58 16.680	30	4.928	30	0.082
31	2 2 13.236	31	5.092	31	0.085
32	2 6 9.792	32	5.257	32	0.088
33	2 10 6.348	33	5.421	33	0.091
34	2 14 2.904	34	5.585	34	0.094
35	2 17 59.460	35	5.750	35	0.097
Sol. Hrs.	m. s.	36	5.914	36	0.100
1	0 9.8565	37	6.078	37	0.103
2	0 19.713	38	6.242	38	0.106
3	0 29.569	39	6.407	39	0.108
4	0 39.426	40	6.571	40	0.111
5	0 49.282	41	6.735	41	0.114
6	0 59.139	42	6.900	42	0.116
7	1 8.995	43	7.064	43	0.119
8	1 18.852	44	7.228	44	0.122
9	1 28.708	45	7.393	45	0.125
10	1 38.565	46	7.557	46	0.128
11	1 48.421	47	7.722	47	0.131
12	1 58.278	48	7.886	48	0.133
13	2 8.134	49	8.050	49	0.136
14	2 17.991	50	8.214	50	0.138
15	2 27.847	51	8.378	51	0.141
16	2 37.704	52	8.543	52	0.144
17	2 47.560	53	8.707	53	0.147
18	2 57.417	54	8.872	54	0.150
19	3 7.273	55	9.036	55	0.152
20	3 17.130	56	9.200	56	0.155
21	3 26.987	57	9.364	57	0.157
22	3 36.844	58	9.528	58	0.159
23	3 46.700	59	9.692	59	0.162
24	3 56.556	60	9.856	60	0.164

TABLE XXXI.

To change Sidereal into mean Solar Time.

Sidereal Days.	Subtract h. m. s.	Sider. Min.	Subtract Seconds. s.	Sider. Sec.	Subtr. Pts of a Sec. s.
1	0 3 55.908	1	0.164	1	0.003
2	0 7 51.816	2	0.328	2	0.005
3	0 11 47.724	3	0.491	3	0.008
4	0 15 43.632	4	0.655	4	0.011
5	0 19 39.540	5	0.819	5	0.014
6	0 23 35.448	6	0.983	6	0.016
7	0 27 31.356	7	1.147	7	0.019
8	0 31 27.264	8	1.311	8	0.022
9	0 35 23.172	9	1.474	9	0.025
10	0 39 19.080	10	1.636	10	0.027
11	0 43 14.988	11	1.802	11	0.030
12	0 47 10.896	12	1.966	12	0.032
13	0 51 6.804	13	2.130	13	0.035
14	0 55 2.712	14	2.294	14	0.038
15	0 58 58.620	15	2.457	15	0.041
16	1 2 54.528	16	2.621	16	0.044
17	1 6 50.436	17	2.785	17	0.046
18	1 10 46.344	18	2.949	18	0.049
19	1 14 42.252	19	3.113	19	0.052
20	1 18 38.160	20	3.277	20	0.055
21	1 22 34.068	21	3.440	21	0.057
22	1 26 29.976	22	3.604	22	0.060
23	1 30 25.884	23	3.768	23	0.063
24	1 34 21.792	24	3.932	24	0.066
25	1 38 17.700	25	4.096	25	0.068
26	1 42 13.608	26	4.259	26	0.071
27	1 46 9.516	27	4.423	27	0.074
28	1 50 5.424	28	4.587	28	0.076
29	1 54 1.332	29	4.751	29	0.079
30	1 57 57.240	30	4.915	30	0.082
31	2 1 53.148	31	5.079	31	0.085
32	2 5 49.056	32	5.242	32	0.087
33	2 9 44.964	33	5.406	33	0.090
34	2 13 40.872	34	5.570	34	0.093
35	2 17 36.780	35	5.734	35	0.096
Sid. Hrs.	m. s.	36	5.898	36	0.098
1	0 9.829	37	6.062	37	0.101
2	0 19.659	38	6.225	38	0.104
3	0 29.488	39	6.389	39	0.106
4	0 39.318	40	6.553	40	0.109
5	0 49.147	41	6.717	41	0.112
6	0 58.977	42	6.881	42	0.115
7	1 8.806	43	7.044	43	0.117
8	1 18.636	44	7.208	44	0.120
9	1 28.465	45	7.372	45	0.123
10	1 38.295	46	7.536	46	0.126
11	1 48.124	47	7.699	47	0.128
12	1 57.954	48	7.864	48	0.131
13	2 7.783	49	8.027	49	0.134
14	2 17.613	50	8.191	50	0.137
15	2 27.442	51	8.355	51	0.139
16	2 37.272	52	8.519	52	0.142
17	2 47.101	53	8.683	53	0.145
18	2 56.931	54	8.846	54	0.147
19	3 6.760	55	9.010	55	0.150
20	3 16.590	56	9.174	56	0.153
21	3 26.419	57	9.338	57	0.156
22	3 36.249	58	9.502	58	0.158
23	3 46.078	59	9.666	59	0.161
24	3 55.908	60	9.829	60	0.164

This Table may be used to shew the Sun's Right Ascension also, in Sidereal Time.

TABLE XXXII.

To convert Mean Time into Parts of the Equator.

TAB. XXXIII.

Lengths of Circular Arcs.

Mean Time.	Parts of the Equator.	Mean Time.	Parts of the Equator.	Mean Time.	Parts of the Equator.		Arc.
h.	° ′ ″	m.	° ′ ″	s.	′ ″		
1	15 2 27.847	1	0 15 2.464	1	0 15.041	1°	0.01745329
2	30 4 55.694	2	0 30 4.928	2	0 30.082	2	0.03490659
3	45 7 23.541	3	0 45 7.392	3	0 45.123	3	0.05235988
4	60 9 51.388	4	1 0 9.856	4	1 0.164	4	0.06981317
5	75 12 19.235	5	1 15 12.321	5	1 15.205	5	0.08726646
6	90 14 47.081	6	1 30 14.785	6	1 30.246	6	0.10471976
7	105 17 14.928	7	1 45 17.249	7	1 45.287	7	0.12217305
8	120 19 42.775	8	2 0 19.713	8	2 0.328	8	0.13962634
9	135 22 10.622	9	2 15 22.177	9	2 15.369	9	0.15707963
10	150 24 38.469	10	2 30 24.641	10	2 30.411	10	0.17453293
11	165 27 6.316	11	2 45 27.105	11	2 45.452	20	0.34906585
12	180 29 34.163	12	3 0 29.569	12	3 0.493	30	0.52359878
13	195 32 2.010	13	3 15 32.033	13	3 15.534	40	0.69813170
14	210 34 29.857	14	3 30 34.497	14	3 30.575	50	0.87266463
15	225 36 57.703	15	3 45 36.962	15	3 45.616	60	1.04719755
16	240 39 25.550	16	4 0 39.426	16	4 0.657	70	1.22173048
17	255 41 53.397	17	4 15 41.890	17	4 15.698	80	1.39626340
18	270 44 21.244	18	4 30 44.354	18	4 30.739	90	1.57079633
19	285 46 49.091	19	4 45 46.818	19	4 45.780	100	1.74532925
20	300 49 16.938	20	5 0 49.282	20	5 0.821	110	1.91986218
21	315 51 44.784	21	5 15 51.746	21	5 15.862	120	2.09439510
22	330 54 12.631	22	5 30 54.210	22	5 30.903	130	2.26892803
23	345 56 40.478	23	5 45 56.674	23	5 45.944	140	2.44346095
24	360 59 8.325	24	6 0 59.138	24	6 0.985	150	2.61799388
		25	6 16 1.603	25	6 16.027	160	2.79252680
		26	6 31 4.067	26	6 31.068	170	2.96705973
Decimals of Mean Time.	Parts of the Equator.	27	6 46 6.531	27	6 46.109	180	3.14159265
		28	7 1 8.995	28	7 1.150	210	3.66519143
		29	7 16 11.459	29	7 16.191	240	4.18879020
		30	7 31 13.923	30	7 31.232	270	4.71238898
s.	″	31	7 46 16.387	31	7 46.273	1′	0.00029089
0.1	1.504	32	8 1 18.851	32	8 1.314	2	0.00058178
0.2	3.008	33	8 16 21.315	33	8 16.355	3	0.00087266
0.3	4.512	34	8 31 23.779	34	8 31.396	4	0.00116355
0.4	6.016	35	8 46 26.244	35	8 46.437	5	0.00145444
0.5	7.521	36	9 1 28.708	36	9 1.478	6	0.00174533
0.6	9.025	37	9 16 31.172	37	9 16.519	7	0.00203622
0.7	10.529	38	9 31 33.636	38	9 31.560	8	0.00232711
0.8	12.033	39	9 46 36.100	39	9 46.601	9	0.00261799
0.9	13.537	40	10 1 38.565	40	10 1.643	10	0.00290888
s.	″	41	10 16 41.029	41	10 16.684	20	0.00581776
0.01	0.150	42	10 31 43.493	42	10 31.725	30	0.00872665
0.02	0.301	43	10 46 45.957	43	10 46.766	40	0.01163553
0.03	0.451	44	11 1 48.421	44	11 1.807	50	0.01454441
0.04	0.602	45	11 16 50.885	45	11 16.848	60	0.01745329
0.05	0.752	46	11 31 53.349	46	11 31.889	1″	0.00000485
0.06	0.903	47	11 46 55.813	47	11 46.930	2	0.00000970
0.07	1.053	48	12 1 58.277	48	12 1.971	3	0.00001454
0.08	1.203	49	12 17 0.741	49	12 17.012	4	0.00001939
0.09	1.354	50	12 32 3.206	50	12 32.053	5	0.00002424
s.	″	51	12 47 5.670	51	12 47.094	6	0.00002909
0.001	0.015	52	13 2 8.134	52	13 2.135	7	0.00003394
0.002	0.030	53	13 17 10.598	53	13 17.176	8	0.00003879
0.003	0.045	54	13 32 13.062	54	13 32.217	9	0.00004363
0.004	0.060	55	13 47 15.526	55	13 47.259	10	0.00004848
0.005	0.075	56	14 2 17.990	56	14 2.300	20	0.00009696
0.006	0.090	57	14 17 20.454	57	14 17.341	30	0.00014544
0.007	0.105	58	14 32 22.918	58	14 32.382	40	0.00019393
0.008	0.120	59	14 47 25.382	59	14 47.423	50	0.00024241
0.009	0.135	60	15 2 27.847	60	15 2.464	60	0.00029089

TABLE XXXIV.

Annual Precession of a Star in R. A. in Time.

Argument, R. A. of the Star in Time.

S.	—	+	—	+	—	+	—	+	—	+	—	+	S.
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
P. P.	m.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	m.
+	0	0.000	+	0.346	+	0.668	+	0.945	+	1.157	+	1.291	60
6	10	0.058	5	0.402	5	0.718	3	0.985	2	1.185	1	1.305	50
12	20	0.117	11	0.457	9	0.766	7	1.024	4	1.211	1	1.316	40
17	30	0.174	16	0.511	14	0.813	10	1.060	6	1.235	2	1.325	30
23	40	0.232	22	0.565	18	0.859	14	1.095	8	1.256	2	1.331	20
29	50	0.289	27	0.617	23	0.903	17	1.127	11	1.274	3	1.335	10
35	60	0.346	32	0.668	28	0.945	21	1.157	13	1.291	4	1.336	0
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	
S.	—	+	—	+	—	+	—	+	—	+	—	+	S.

Multiply the number found from the Table, with its proper sign, by the natural tangent of the Star's declination, to which add the constant quantity $3^s.068$ for the annual precession, $= a$ in the Synopsis.

TABLE XXXV.

Argument, R. A. of the Star in Time.

	—	+	—	+	—	+	—	+	—	+	—	+	
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
P. P.	m.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	m.
+	0	0.000	+	0.349	+	0.675	+	0.954	+	1.168	+	1.304	60
6	10	0.059	5	0.406	5	0.725	3	0.995	2	1.197	1	1.318	50
12	20	0.118	11	0.462	9	0.774	7	1.034	4	1.222	2	1.329	40
18	30	0.176	16	0.516	14	0.821	10	1.071	7	1.247	2	1.339	30
24	40	0.234	22	0.571	19	0.868	14	1.106	9	1.269	3	1.346	20
30	50	0.292	27	0.623	24	0.912	17	1.138	11	1.287	4	1.349	10
36	60	0.349	33	0.675	28	0.954	21	1.168	13	1.304	5	1.350	0
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	
	—	+	—	+	—	+	—	+	—	+	—	+	

The number from the Table $= p$, and $p \times$ sec. dec. $= b$.

TABLE XXXVI.

Argument, R. A. of the Star in Time.

	—	+	—	+	—	+	—	+	—	+	—	+	
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
P. P.	m.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	m.
—	0	1.239	—	1.196	—	1.073	—	0.876	—	0.619	—	0.321	60
0	10	1.237	2	1.181	3	1.045	4	0.837	5	0.572	5	0.268	50
0	20	1.234	4	1.164	7	1.016	9	0.796	10	0.523	11	0.215	40
1	30	1.228	6	1.144	10	0.983	13	0.754	15	0.474	16	0.162	30
1	40	1.220	8	1.123	14	0.949	18	0.710	20	0.424	22	0.108	20
1	50	1.209	10	1.099	17	0.913	22	0.666	25	0.373	27	0.054	10
1	60	1.196	13	1.073	20	0.876	26	0.619	30	0.321	32	0.000	0
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	18^h	6^h	18^h	
	+	—	+	—	+	—	+	—	+	—	+	—	

The number from the Table $= q$, and $q \times$ sec. dec. $= c$.

TABLE XXXVII.

Argument, R. A. of the Star in Time.

S.	+	—	+	—	+	—	+	—	+	—	+	—	S.
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h.	17^h	
N.	—	+	—	+	—	+	—	+	—	+	—	+	N.
P. P.	m.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	P. P.	s.	m.
—	0	0.643	—	0.621	—	0.557	—	0.455	—	0.322	—	0.166	60
0	10	0.643	1	0.613	2	0.542	2	0.435	3	0.297	3	0.139	50
1	20	0.641	2	0.604	4	0.527	5	0.413	5	0.272	6	0.112	40
1	30	0.638	3	0.594	6	0.510	7	0.392	8	0.246	8	0.084	30
2	40	0.633	4	0.583	8	0.493	9	0.369	10	0.220	11	0.056	20
2	50	0.628	5	0.571	10	0.474	11	0.346	13	0.193	14	0.028	10
3	60	0.621	7	0.557	12	0.455	14	0.322	16	0.166	17	0.000	0
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	
S.	—	+	—	+	—	+	—	+	—	+	—	+	S.

The number from the Table gives s, and $s \times$ tang. dec. $= d$.

TABLE XXXVIII.

Annual Precession of a Star in N. P. D.

Argument, R. A. of the Star in Time.

S.	—	+	—	+	—	+	—	+	—	+	—	+	S.
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
P. P.	m.	″	P. P.	″	P. P.	″	P. P.	″	P. P.	″	P. P.	″	m.
—	0	20.044	—	19.361	—	17.358	—	14.173	—	10.022	—	5.188	60
13	10	20.025	35	19.116	55	16.904	70	13.542	81	9.255	87	4.338	50
27	20	19.968	71	18.835	110	16.419	140	12.884	162	8.471	174	3.481	40
40	30	19.872	106	18.518	164	15.902	210	12.202	243	7.670	261	2.616	30
53	40	19.739	141	18.165	219	15.354	280	11.497	324	6.855	348	1.747	20
66	50	19.569	176	17.779	274	14.778	350	10.769	405	6.027	435	0.874	10
80	60	19.361	212	17.358	329	14.173	420	10.022	486	5.188	522	0.000	0
N.	—	+	—	+	—	+	—	+	—	+	—	+	N.
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	
S.	+	—	+	—	+	—	+	—	+	—	+	—	S.

The number from the Table $= a'$.

TABLE XXXIX.

Aberration in N. P. D. to find p'.

Argument, R. A. of the Star in Time.

	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
	—	+	—	+	—	+	—	+	—	+	—	+	
P. P.	m.	″	P. P.	″	P. P.	″	P. P.	″	P. P.	″	P. P.	″	m.
—	0	20.255	—	19.565	—	17.541	—	14.322	—	10.128	—	5.243	60
12	10	20.236	34	19.318	54	17.083	70	13.684	82	9.353	88	4.384	50
24	20	20.178	68	19.033	108	16.592	140	13.019	164	8.560	176	3.517	40
36	30	20.082	102	18.713	162	16.069	210	12.330	246	7.751	264	2.644	30
48	40	19.947	136	18.357	216	15.516	280	11.618	328	6.928	352	1.764	20
60	50	19.775	170	17.966	270	14.934	350	10.884	410	6.091	440	0.883	10
72	60	19.565	224	17.541	324	14.322	420	10.128	492	5.243	528	0.000	0
	+	—	+	—	+	—	+	—	+	—	+	—	
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	

Multiply the number found in the Table by the natural sine of the Star's declination; the result will give b'.

TABLE XL.

Aberration in N. P. D. to find q'.

Argument, R. A. of the Star in Time.													
	0^h +	12^h —	1^h +	13^h —	2^h +	14^h —	3^h +	15^h —	4^h +	16^h —	5^h +	17^h —	
P. P.	m.	"	P. P.	"	P. P.	"	P. P.	"	P. P.	"	P. P.	"	m.
+	0	0.000	+	4.809	+	9.290	+	13.138	+	16.090	+	17.947	60
80	10	0.810	75	5.588	64	9.984	50	13.700	30	16.480	10	18.140	50
160	20	1.619	150	6.355	128	10.657	100	14.233	60	16.839	20	18.298	40
240	30	2.426	225	7.110	192	11.311	150	14.740	90	17.166	30	18.420	30
320	40	3.226	300	7.852	256	11.943	200	15.220	120	17.460	40	18.509	20
400	50	4.022	375	8.579	320	12.552	250	15.669	150	17.720	50	18.562	10
480	60	4.809	450	9.290	384	13.138	300	16.090	180	17.947	60	18.580	0
	+ 11^h	— 23^h	+ 10^h	— 22^h	+ 9^h	— 21^h	+ 8^h	— 20^h	+ 7^h	— 19^h	+ 6^h	— 18^h	

The number from this Table, multiplied by the natural sine of the Star's declination, gives a product, to which r' being added, the result will be c'.

TABLE XLI.

Argument, Declination of the Star.

Dec. North — South +

D.	No.	D.	No.	D.	No.	D.	No.	D.	No.	D.	No.	D.	No.	D.	No.	D.	No.
°	"	°	"	°	"	°	"	°	"	°	"	°	"	°	"	°	"
0	8.066	10	7.944	20	7.579	30	6.985	40	6.179	50	5.185	60	4.033	70	2.759	80	1.401
1	8.065	11	7.918	21	7.530	31	6.913	41	6.087	51	5.076	61	3.910	71	2.626	81	1.262
2	8.061	12	7.890	22	7.479	32	6.840	42	5.994	52	4.966	62	3.787	72	2.493	82	1.123
3	8.055	13	7.859	23	7.425	33	6.765	43	5.900	53	4.854	63	3.662	73	2.358	83	0.983
4	8.046	14	7.826	24	7.369	34	6.687	44	5.803	54	4.741	64	3.536	74	2.223	84	0.843
5	8.035	15	7.791	25	7.311	35	6.607	45	5.703	55	4.626	65	3.409	75	2.088	85	0.703
6	8.021	16	7.753	26	7.250	36	6.526	46	5.603	56	4.510	66	3.281	76	1.951	86	0.563
7	8.005	17	7.713	27	7.187	37	6.442	47	5.501	57	4.393	67	3.152	77	1.814	87	0.423
8	7.987	18	7.671	28	7.122	38	6.356	48	5.397	58	4.274	68	3.022	78	1.677	88	0.282
9	7.967	19	7.626	29	7.055	39	6.268	49	5.292	59	4.154	69	2.891	79	1.539	89	0.141

The number from this Table is r'.

TABLE XLII.

Lunar Nutation in R. A. to find $s' = d'$.

Argument, R. A. of the Star in Time.													
S.	—	+	—	+	—	+	—	+	—	+	—	+	S.
	0^h	12^h	1^h	13^h	2^h	14^h	3^h	15^h	4^h	16^h	5^h	17^h	
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
P. P.	m.	"	P. P.	"	P. P.	"	P. P.	"	P. P.	"	P. P.	"	m.
+	0	0.000	+	2.497	+	4.824	+	6.823	+	8.355	+	9.319	60
42	10	0.421	40	2.901	34	5.185	26	7.113	17	8.558	4	9.419	50
84	20	0.841	80	3.300	68	5.534	52	7.391	34	8.744	9	9.501	40
126	30	1.260	120	3.693	102	5.874	78	7.655	51	8.914	14	9.566	30
168	40	1.675	160	4.077	136	6.202	104	7.903	68	9.066	18	9.611	20
210	50	2.088	200	4.455	170	6.518	130	8.137	85	9.202	22	9.639	10
252	60	2.497	240	4.824	204	6.823	156	8.355	102	9.319	27	9.648	0
N.	+	—	+	—	+	—	+	—	+	—	+	—	N.
	11^h	23^h	10^h	22^h	9^h	21^h	8^h	20^h	7^h	19^h	6^h	18^h	
S.	—	+	—	+	—	+	—	+	—	+	—	+	S.

FOR COMPUTING THE NUTATION OF A STAR IN RIGHT ASCENSION AND DECLINATION.

TABLE XLIII. OR TABLE I. OF NUTATION.

ARGUMENT.
For the Nutation in R. A.
R.A.Star.–Lon. Moon's Node.
For the Nutation in Declin.
R.A. Star + 3 signs — Lon. Moon's Node.

	S. S. O VI — +	S. S. I VII — +	S. S. II VIII — +	
°	"	"	"	°
0	8.77	7.60	4.39	30
1	8.77	7.52	4.25	29
2	8.77	7.44	4.12	28
3	8.76	7.36	3.98	27
4	8.75	7.27	3.84	26
5	8.74	7.18	3.71	25
6	8.72	7.10	3.57	24
7	8.71	7.00	3.43	23
8	8.69	6.91	3.29	22
9	8.66	6.82	3.14	21
10	8.64	6.72	3.00	20
11	8.61	6.62	2.86	19
12	8.58	6.52	2.71	18
13	8.55	6.41	2.56	17
14	8.51	6.31	2.42	16
15	8.47	6.20	2.27	15
16	8.43	6.09	2.12	14
17	8.39	5.98	1.97	13
18	8.34	5.87	1.82	12
19	8.29	5.75	1.67	11
20	8.24	5.64	1.52	10
21	8.19	5.52	1.37	9
22	8.13	5.39	1.22	8
23	8.07	5.28	1.07	7
24	8.01	5.16	0.92	6
25	7.94	5.03	0.76	5
26	7.88	4.90	0.61	4
27	7.81	4.78	0.46	3
28	7.74	4.65	0.31	2
29	7.67	4.52	0.15	1
30	7.60	4.39	0.00	0
	— + S. S. XI V	— + S. S. X IV	— + S. S. IX III	

TABLE XLIV. OR TABLE II. OF NUTATION.

ARGUMENT.
For the Nutation in R. A.
R.A.Star+Lon. Moon's Node.
For the Nutation in Declin.
R. A. Star+3 signs + Lon. Moon's Node.

	S. S. O VI — +	S. S. I VII — +	S. S. II VIII — +	
°	"	"	"	°
0	1.29	1.11	0.64	30
1	1.28	1.10	0.62	29
2	1.28	1.09	0.60	28
3	1.28	1.08	0.58	27
4	1.28	1.07	0.56	26
5	1.28	1.06	0.54	25
6	1.28	1.04	0.52	24
7	1.28	1.03	0.50	23
8	1.27	1.01	0.48	22
9	1.27	1.00	0.46	21
10	1.27	0.98	0.44	20
11	1.26	0.97	0.42	19
12	1.26	0.95	0.40	18
13	1.25	0.94	0.38	17
14	1.25	0.92	0.35	16
15	1.24	0.91	0.33	15
16	1.24	0.89	0.31	14
17	1.23	0.88	0.29	13
18	1.22	0.86	0.27	12
19	1.22	0.84	0.25	11
20	1.21	0.83	0.22	10
21	1.20	0.81	0.20	9
22	1.19	0.79	0.18	8
23	1.18	0.77	0.16	7
24	1.17	0.76	0.13	6
25	1.16	0.74	0.11	5
26	1.15	0.72	0.09	4
27	1.14	0.70	0.07	3
28	1.13	0.68	0.04	2
29	1.12	0.66	0.02	1
30	1.11	0.64	0.00	0
	— + S. S. XI V	— + S. S. X IV	— + S. S. IX III	

TABLE XLV. EQUATION OF EQUINOXES IN RIGHT ASCENSION.

ARGUMENT.
Longitude of the Moon's Node.

	S. S. O VI — +	S. S. I VII — +	S. S. II VIII — +	
°	"	"	"	°
0	0.00	8.62	14.93	30
1	0.30	8.88	15.08	29
2	0.60	9.14	15.23	28
3	0.90	9.39	15.36	27
4	1.20	9.64	15.50	26
5	1.50	9.89	15.63	25
6	1.80	10.14	15.75	24
7	2.10	10.38	15.87	23
8	2.40	10.62	15.99	22
9	2.70	10.85	16.10	21
10	2.99	11.08	16.20	20
11	3.29	11.31	16.30	19
12	3.59	11.54	16.40	18
13	3.88	11.76	16.49	17
14	4.17	11.98	16.58	16
15	4.46	12.19	16.66	15
16	4.75	12.40	16.73	14
17	5.04	12.61	16.88	13
18	5.33	12.81	16.87	12
19	5.61	13.01	16.93	11
20	5.90	13.21	16.98	10
21	6.18	13.40	17.03	9
22	6.46	13.59	17.08	8
23	6.74	13.77	17.12	7
24	7.01	13.95	17.15	6
25	7.29	14.13	17.18	5
26	7.56	14.30	17.20	4
27	7.83	14.46	17.22	3
28	8.10	14.62	17.23	2
20	8.36	14.78	17.24	1
30	8.62	14.93	17.24	0
	+ — S. S. XI V	+ — S. S. X IV	+ — S. S. IX III	

To find the Nutation of a Star in Right Ascension.

To the log of the sum or difference of the equations from Tables XLIII. XLIV. answering to their proper arguments, add the log tangent of the Star's declination; the sum will be the log of part first of nutation, and if the declination is south, change the sign—to which apply the equation from Table XLV. answering to the longitude of the Moon's node, and the sum or difference will be the nutation in right ascension.

To find the Nutation of a Star in Declination.

Increase the arguments of Tables XLIII. and XLIV. each by the three signs, and the sum or difference of the corresponding equations will be the nutation in declination. If the declination is south, change the sign of both equations.

FOR COMPUTING THE ABERRATION OF A STAR IN RIGHT ASCENSION AND DECLINATION.

TABLE XLVI. OR TABLE I. OF ABERRATION.

ARGUMENT.
For the Aberration of R.A.
R.A. Star—Lon. Sun.
For the Aberration in Decl.
R.Acen. Star+3 signs—Sun's Lon.

	S. S. O VI	S. S. I VII	S. S. II VIII	
	— +	— +	— +	
°	″	″	″	°
0	19.42	16.82	9.71	30
1	19.41	16.64	9.41	29
2	19.40	16.47	9.12	28
3	19.39	16.29	8.82	27
4	19.37	16.10	8.51	26
5	19.34	15.91	8.21	25
6	19.31	15.71	7.90	24
7	19.27	15.51	7.59	23
8	19.23	15.30	7.27	22
9	19.18	15.09	6.96	21
10	19.12	14.88	6.64	20
11	19.06	14.66	6.32	19
12	18.99	14.43	6.00	18
13	18.92	14.20	5.68	17
14	18.84	13.97	5.35	16
15	18.76	13.73	5.03	15
16	18.67	13.49	4.70	14
17	18.57	13.24	4.37	13
18	18.47	12.99	4.04	12
19	18.36	12.74	3.71	11
20	18.25	12.48	3.37	10
21	18.13	12.22	3.04	9
22	18.00	11.96	2.70	8
23	17.87	11.69	2.37	7
24	17.74	11.41	2.03	6
25	17.60	11.14	1.69	5
26	17.45	10.86	1.35	4
27	17.30	10.58	1.02	3
28	17.15	10.29	0.68	2
29	16.98	10.00	0.34	1
30	16.82	9.71	0.00	0
	— +	— +	— +	
	S. S. XI V	S. S. X IV	S. S. IX III	

TABLE XLVII. OR TABLE II. OF ABERRATION.

ARGUMENT.
For the Aberration in R.A.
R.A. Star+Sun's Lon.
For the Aberration in Decl.
R.Ascen. Star+3 signs+Sun's Decl.

	S. S. O VI	S. S. I VII	S. S. II VIII	
	+ —	+ —	+ —	
°	′	″	″	°
0	0.84	0.73	0.42	30
1	0.84	0.72	0.41	29
2	0.84	0.71	0.39	28
3	0.84	0.70	0.38	27
4	0.84	0.69	0.37	26
5	0.83	0.69	0.35	25
6	0.83	0.68	0.34	24
7	0.83	0.67	0.33	23
8	0.83	0.66	0.31	22
9	0.83	0.65	0.30	21
10	0.83	0.64	0.29	20
11	0.82	0.63	0.27	19
12	0.82	0.62	0.26	18
13	0.82	0.61	0.25	17
14	0.81	0.60	0.23	16
15	0.81	0.59	0.22	15
16	0.81	0.58	0.20	14
17	0.80	0.57	0.19	13
18	0.80	0.56	0.17	12
19	0.79	0.55	0.16	11
20	0.79	0.54	0.15	10
21	0.78	0.53	0.13	9
22	0.78	0.52	0.12	8
23	0.77	0.50	0.10	7
24	0.77	0.49	0.09	6
25	0.76	9.48	0.07	5
26	0.75	0.47	0.06	4
27	0.75	0.46	0.04	3
28	0.74	0.44	0.03	2
29	0.73	0.43	0.02	1
30	0.73	0.42	0.00	0
	+ —	+ —	+ —	
	S. S. XI V	S. S. X IV	S. S. IX III	

TABLE XLVIII. OR TABLE III. OF ABERRATION.

ARGUMENT.
For part 2d of Aber. in Decl.
Sun's Lon.+Star's Decl.
For part 3d Aber. in Decl.
Sun's Lon.—Star's Decl.

	S. S. O VI	S. S. I VII	S. S. II VIII	
	— +	— +	— +	
°	″	″	″	°
0	4.03	3.49	2.02	30
1	4.03	3.46	1.96	29
2	4.03	3.42	1.89	28
3	4.03	3.38	1.83	27
4	4.02	3.34	1.77	26
5	4.02	3.30	1.70	25
6	4.01	3.26	1.64	24
7	4.00	3.22	1.58	23
8	3.99	3.18	1.51	22
9	3.98	3.13	1.45	21
10	3.97	3.09	1.38	20
11	3.96	3.04	1.31	19
12	3.95	3.00	1.25	18
13	3.93	2.95	1.18	17
14	3.91	2.90	1.11	16
15	3.90	2.85	1.04	15
16	3.88	2.80	0.98	14
17	3.86	2.75	0.91	13
18	3.84	2.70	0.84	12
19	3.81	2.65	0.77	11
20	3.79	2.59	0.70	10
21	3.77	2.54	0.63	9
22	3.74	2.48	0.56	8
23	3.71	2.43	0.49	7
24	3.68	2.37	0.42	6
25	3.66	2.31	0,35	5
26	3.63	2.26	0.28	4
27	3.59	2.20	0.21	3
28	3.56	2.14	0.14	2
29	3.53	2.08	0.07	1
30	3.49	1.02	0.00	0
	— +	— +	— +	
	S. S. XI V	S. S. X IV	S. S. IX III	

To find the Aberration of a Star in Right Ascension.

To the log of the sum or difference of the equation from Tables XLVI. XLVII. answering to their arguments, add the log secant of the Star's declination, the sum will be the log of the aberration in right ascension.

To find the Aberration of a Star in Declination.

Find the sum or difference of the equations answering to the former arguments, increased by three signs, to the log of which add the log sine of the Star's declination; the sum will be the log of part 1st of the aberration. Take parts second and third of aberration from Table XLVIII. which, applied to the former, will give the aberration in declination. If the Star's declination is south, change the sign of parts 2d and 3d.

TABLE XLIX.

Mean Obliquity of the Ecliptic.

Year.	°	'	"
1800	23	27	56.24
1810	23	27	51.80
1820	23	27	47.35
1830	23	27	42.90

Annual Diminution.

0.4446"

Monthly Diminution.

		"
Jan.	1	0.000
Feb.	1	0.038
March	1	0.072
April	1	0.110
May	1	0.146
June	1	0.184
July	1	0.220
August	1	0.258
Sept.	1	0.296
October	1	0.333
Nov.	1	0.370
Dec.	1	0.407

370

TABLE LIII.

Sol. Nut. of the Equinoxes in R.A.

Sun's true Long.	— — O VI IX III + +	— — I VII X IV + +	Sun's true Long.
°	s	s	°
0	0.00	0.06	30
5	0.02	0.07	25
10	0.03	0.07	20
15	0.04	0.07	15
20	0.05	0.07	10
25	0.06	0.06	5
30	0.06	0.06	0
	— — II VIII XI V + +		

TABLE LII.

Lun. Nut. of the Equinoxes in R.A.

Long. Moon's Node.	— + O VI	— + I VII	— + II VIII	Long. Moon's Node.
°	s	s	s	°
0	0.00	0.55	0.97	30
2	0.04	0.59	0.98	28
4	0.08	0.62	1.00	26
6	0.11	0.65	1.01	24
8	0.15	0.69	1.02	22
10	0.19	0.71	1.04	20
12	0.23	0.75	1.05	18
14	0.26	0.78	1.06	16
16	0.30	0.81	1.07	14
18	0.34	0.83	1.08	12
20	0.37	0.86	1.09	10
22	0.41	0.88	1.09	8
24	0.44	0.90	1.10	6
26	0.48	0.92	1.10	4
28	0.51	0.95	1.10	2
30	0.56	0.97	1.10	0
	XI V + —	X IV + —	IX III + —	

TABLE L.

Solar Equation of the Obliquity.

Sun's true Long. s	°	s	Equation. "	Corresponding Days of the Month.			
O	0	VI	+0.43	Mar.	21	Sept.	23
	5		0.43		26		28
	10		0.41		31	Oct.	3
	15		0.37	April	5		9
	20		0.33		10		14
	25		0.29		15		19
I	0	VII	0.22		20		24
	5		0.15		25		29
	10		+0.08	May	1	Nov.	3
	15		0.00		6		8
	20		—0.08		11		13
	25		0.22		16		18
II	0	VIII	0.29		21		22
	5		0.33		27		27
	10		0.37	June	1	Dec.	2
	15		0.41		6		7
	20		0.43		11		12
	25		0.43		16		17
III	0	IX	0.43		22		22
	5		0.41		27		27
	10		0.37	July	2	Jan.	1
	15		0.33		7		6
	20		0.29		13		11
	25		0.22		18		16
IV	0	X	0.15		23		20
	5		—0.08		28		25
	10		0.00	Aug.	3		30
	15		+0.08		8	Feb.	4
	20		0.15		13		9
	25		0.22		18		14
V	0	XI	0.29		23		19
	5		0.33		29		24
	10		0.38	Sept.	3	Mar.	1
	15		0.41		8		6
	20		0.43		13		11
	25		0.43		18		16
VI	0	XII	0.43		23		21

TABLE LI.

Lunar Equation of the Obliquity.

Long. Moon's Node.	O VI + —	I VII + —	II VIII + —	Long. Moon's Node.
°	"	"	"	°
0	9.26	8.02	4.63	30
1	9.26	7.94	4.49	29
2	9.25	7.85	4.35	28
3	9.25	7.77	4.20	27
4	9.24	7.68	4.06	26
5	9.22	7.59	3.91	25
6	9.21	7.49	3.77	24
7	9.19	7.40	3.62	23
8	9.17	7.30	3.47	22
9	9.15	7.20	3.32	21
10	9.12	7.09	3.17	20
11	9.09	6.99	3.01	19
12	9.06	6.88	2.86	18
13	9.02	6.77	2.71	17
14	8.98	6.66	2.55	16
15	8.94	6.55	2.40	15
16	8.90	6.43	2.24	14
17	8.86	6.32	2.08	13
18	8.81	6.20	1.93	12
19	8.75	6.08	1.77	11
20	8.70	5.95	1.61	10
21	8.64	5.83	1.45	9
22	8.58	5.70	1.29	8
23	8.52	5.57	1.13	7
24	8.46	5.44	0.97	6
25	8.39	5.31	0.81	5
26	8.32	5.18	0.65	4
27	8.25	5.04	0.48	3
28	8.18	4.91	0.32	2
29	8.10	4.77	0.16	1
30	8.02	4.63	0.00	0
	+ — XI V	+ — X IV	+ — IX III	

TABLE LIV. Right Ascensions and Declinations of Stars for 1830.

No	Characters.	Constellations.	Proper Name.	Mag.	Mean Right Ascen. for 1830	Annual Variation.	Mean Declination for 1830.	Annual Variation.
					h m s	s	° ′ ″	″
1	γ	Pegasi		2	0 4 29.4	+ 3.08	14 14 18 N	+ 20.0
2	α	Phenicis		2	0 17 51.4	2.97	43 13 41 S	— 20.0
3	δ	Andromedæ		3	0 30 15.0	3.17	29 55 49 N	+ 19.9
4	α	Cassiopeæ		3	0 30 54.6	3.33	55 36 13 N	+ 19.9
5	β	Ceti		2	0 35 3.2	3.00	18 55 15 S	— 19.8
6	γ	Cassiopeæ		3	0 46 29.7	3.53	59 47 43 N	+ 19.6
7	α	Ursæ Minoris	Polaris	2.3	0 59 31.0	15.20	88 24 9 N	+ 19.4
8	β	Andromedæ		2	1 0 13.7	3.31	34 43 4 N	+ 19.4
9	θ	Ceti		3	1 15 31.5	3.00	9 3 45 S	— 19.0
10	α	Eridani	Achernar	1	1 31 22.0	2.24	58 6 12 S	— 18.5
11	β	Arietis		3	1 45 15.2	+ 3.28	19 58 27 N	+ 18.0
12	α	Piscis		3	1 53 15.5	3.09	1 56 24 N	+ 17.6
13	γ	Andromedæ		2	1 53 29.4	3.63	41 30 34 N	+ 17.6
14	α	Arietis		3	1 57 36.4	3.36	22 39 17 N	+ 17.5
15	δ	Ceti		3	2 30 46.6	3.06	0 24 35 S	— 15.9
16	ι	Ceti		3	2 31 20.6	2.88	12 35 51 S	— 15.8
17	γ	Ceti		3	2 34 29.8	3.10	2 30 52 N	+ 15.7
18	η	Eridani		3	2 48 7.8	2.92	9 34 40 S	— 14.9
19	α	Ceti		2.3	2 53 24.0	3.12	3 25 4 N	+ 14.6
20	α	Persei		2	3 12 13.6	4.22	49 14 55 N	+ 13.4
21	δ	Eridani		3	3 35 6.7	+ 2.87	10 20 36 S	— 11.2
22	η	Pleiadum		3	3 37 23.2	3.54	23 34 22 N	+ 11.7
23	γ	Eridani		3	3 50 6.0	2.79	13 59 50 S	— 10.8
24	γ	Tauri		3	4 10 7.4	3.39	15 12 37 N	+ 9.2
25	α	Tauri	Aldebaran	1	4 26 10.3	3.43	16 9 37 N	+ 8.0
26	β	Eridani		3	4 59 29.7	2.95	5 18 44 S	— 5.2
27	α	Aurigæ	Capella	1	5 4 8.5	4.40	45 48 55 N	+ 4.5
28	β	Orionis	Rigel	1	5 6 22.2	2.88	8 24 15 S	— 4.6
29	β	Tauri		2	5 15 33.0	3.78	28 27 18 N	+ 3.9
30	γ	Orionis	Bellatrix	2	5 16 1.0	3.20	6 11 17 N	+ 3.8
31	δ	Orionis		2	5 23 19.5	+ 3.06	0 25 55 S	— 3.2
32	α	Leporis		3	5 25 13.9	2.64	17 57 0 S	— 3.0
33	ζ	Tauri		3	5 27 28.7	3.58	21 1 52 N	+ 2.8
34	ι	Orionis		2	5 27 35.4	3.03	1 19 3 S	— 2.8
35	ζ	Orionis		2	5 32 11.1	3.01	2 2 21 S	— 2.4
36	α	Columbæ		2	5 33 29.3	2.17	34 10 8 S	— 2.3
37	κ	Orionis		2.3	5 39 41.5	2.84	9 44 8 S	— 1.8
38	β	Columbæ		3	5 44 58.1	2.10	35 50 23 S	— 1.3
39	α	Orionis	Betelguese	1	5 45 58.2	3.24	7 22 4 N	+ 1.2
40	β	Aurigæ		2.3	5 47 3.6	4.40	44 55 12 N	+ 1.1
41	η	Geminorum		2.3	6 4 36.6	+ 3.62	22 32 53 N	— 0.4
42	μ	Geminorum		3	6 12 40.2	3.62	22 35 35 N	— 1.1
43	ζ	Canis Majoris		3	6 13 47.3	2.30	29 59 42 S	+ 1.2
44	β	Canis Majoris		3	6 15 12.7	2.64	17 52 46 S	+ 1.3
45	α	Argus	Canopus	1	6 20 10.6	1.33	52 36 29 S	+ 1.7
46	γ	Geminorum		2.3	6 27 53.0	3.46	16 32 13 N	— 2.4
47	α	Canis Majoris	Sirius	1	6 37 39.2	2.64	16 29 20 S	+ 4.4
48	ε	Canis Majoris		3	6 51 56.6	2.35	28 44 43 S	+ 4.5
49	ζ	Geminorum		3	6 54 1.2	3.56	20 48 41 N	— 4.7
50	γ	Canis Majoris		2	6 56 3.8	2.71	15 23 14 S	+ 4.8
51	δ	Canis Majoris		2	7 1 28.6	+ 2.44	26 7 42 S	+ 5.3
52	δ	Geminorum		3	7 9 57.6	3.59	22 17 16 N	— 6.0
53	π	Argus		3	7 11 7.6	2.12	36 47 49 S	+ 6.1
54	η	Canis Majoris		2	7 17 21.6	2.37	28 58 36 S	+ 6.6
55	β	Canis Minoris		3	7 17 55.2	3.26	8 37 33 N	— 6.7
56	α	Geminorum	Castor	1.2	7 23 44.3	3.85	32 15 11 N	— 7.1
57	α	Canis Minoris	Procyon	1.2	7 30 23.9	3.15	5 39 16 N	— 8.7
58	β	Geminorum	Pollux	2.3	7 34 54.0	3.68	28 25 45 N	— 8.0
59	ζ	Argus		2	7 57 36.5	2.11	39 31 41 S	+ 9.8
60	β	Argus		2	9 11 19.0	0.73	69 1 16 S	+ 14.9
61	α	Hydræ		2	9 19 13.8	+ 2.94	7 55 32 S	+ 15.3
62	μ	Leonis		3	9 43 4.1	3.45	26 48 14 N	— 16.6
63	η	Leonis		3	9 58 2.9	3.28	17 35 18 N	— 17.3
64	α	Leonis	Regulus	1	9 59 18.5	3.21	12 47 43 N	— 17.3
65	λ	Ursæ Majoris		3.4	10 6 48.6	3.67	43 45 34 N	— 17.6

No	Characters.	Constellations.	Proper Name.	Mag.	Mean Right Ascen. for 1830	Annual Variation.	Mean Declination for 1830.	Annual Variation.
					h m s	s	° ′ ″	″
66	ζ	Leonis		3	10 7 13.0	+ 3.35	24 15 45 N	— 17.7
67	γ	Leonis		3	10 10 35.0	3.30	20 41 55 N	— 17.8
68	μ	Ursæ Majoris		3	10 12 10.2	3.62	42 21 7 N	— 17.9
69	β	Ursæ Majoris		2	10 51 31.8	3.68	57 17 31 N	— 19.2
70	α	Ursæ Majoris		2	10 53 9.8	3.80	62 40 1 N	— 19.2
71	δ	Leonis		2.3	11 5 3.1	+ 3.19	21 27 17 N	— 19.5
72	θ	Leonis		3	11 5 18.3	3.16	16 21 31 N	— 19.5
73	β	Leonis		2	11 40 22.8	3.06	15 31 21 N	— 20.0
74	β	Virginis		3	11 41 50.3	3.12	2 43 22 N	— 20.0
75	γ	Ursæ Majoris		2	11 44 50.9	3.20	54 38 23 N	— 20.0
76	α	Corvi		4	11 59 39.5	3.07	23 46 47 S	+ 20.0
77	α^2	Crucis		1	12 17 11.1	3.26	62 9 28 S	+ 20.0
78	γ	Crucis		2.3	12 21 48.0	3.26	56 9 25 S	+ 20.0
79	β	Corvi		3	12 25 27.8	3.13	22 27 20 S	+ 19.9
80	γ^1	Virginis		3	12 33 2.8	3.02	0 30 56 S	+ 19.8
81	β	Crucis		2	12 37 52.0	+ 3.43	58 45 22 S	+ 19.8
82	ε	Ursæ Majoris		2	12 46 31.0	2.66	56 52 59 N	— 19.6
83	δ	Virginis		3	12 47 2.4	3.00	4 19 25 N	— 19.6
84	ε	Virginis		3	12 53 42.8	3.00	11 52 33 N	— 19.5
85	γ	Hydræ con.		3	13 9 41.7	3.23	22 16 14 S	+ 19.1
86	ι	Centauri		3	13 11 4.9	3.36	35 48 41 S	+ 19.1
87	α	Virginis	Spica	1	13 16 14.8	3.15	10 16 14 S	+ 18.8
88	ζ	Ursæ Majoris		2	13 17 3.5	2.42	55 48 57 N	— 18.9
89	ζ	Virginis		3	13 26 2.3	3.07	0 16 39 N	— 18.6
90	η	Ursæ Majoris		2	13 40 49.9	2.36	50 9 52 N	— 18.1
91	η	Bootis		3	13 46 35.2	+ 2.86	19 15 16 N	— 17.9
92	β	Centauri		1	13 51 55.4	4.14	59 32 46 S	+ 17.7
93	θ	Centauri		2.3	13 56 43.1	3.49	35 31 47 S	+ 17.5
94	α	Bootis	Arcturus	1	14 7 54.5	2.73	20 4 17 N	— 19.0
95	α^2	Centauri		1	14 28 12.8	4.47	60 7 31 S	+ 16.0
96	ζ	Bootis		3	14 33 1.5	2.85	14 27 49 N	— 15.7
97	α^1	Libræ		3	14 41 17.8	3.30	15 17 5 S	+ 15.3
98	α^2	Libræ		2.3	14 41 29.3	+ 3.30	15 19 47 S	+ 15.3
99	β	Ursæ Minoris		3	14 51 17.4	— 0.30	74 51 1 N	— 14.7
100	β	Libræ		2.3	15 7 52.0	+ 3.22	8 44 56 S	+ 13.7
101	γ	Lupi		3	15 23 50.8	+ 3.96	40 35 10 S	+ 12.6
102	δ	Serpentis		3	15 26 40.9	2.86	11 6 51 N	— 12.4
103	α	Cor. Bor.		2.3	15 27 29.4	2.53	27 17 31 N	— 12.4
104	α	Serpentis		2.3	15 35 53.9	2.94	6 58 1 N	— 11.8
105	β	Serpentis		3	15 38 20.4	2.76	15 57 42 N	— 11.6
106	γ	Serpentis		3	15 48 36.3	2.74	16 13 23 N	— 12.2
107	β	Scorpionis		2	15 55 34.0	3.47	19 19 53 S	+ 10.4
108	δ	Ophiuchi		3	16 5 26.3	3.13	3 14 56 S	+ 9.6
109	α	Scorpionis	Antares	1	16 18 59.8	3.66	26 2 46 S	+ 8.6
110	β	Herculis		3	16 22 54.3	2.58	21 51 58 N	— 8.2
111	ζ	Ophiuchi		2.3	16 27 46.3	+ 3.29	10 12 51 S	+ 7.8
112	α	Trianguli A.		2	16 30 46.5	6.24	68 41 57 S	+ 7.6
113	ε	Scorpionis		3	16 39 10.1	3.87	33 58 32 S	+ 6.9
114	μ	Scorpionis		3	16 40 22.2	4.04	37 44 42 S	+ 6.8
115	η	Ophiuchi		2.3	17 0 38.1	3.43	15 30 16 S	+ 5.1
116	α	Herculis		3	17 6 53.9	2.73	14 35 26 N	— 4.6
117	δ	Herculis		3	17 8 2.1	2.46	25 2 49 N	— 4.5
118	λ	Scorpionis		3	17 22 4.6	4.06	36 58 3 S	+ 3.8
119	α	Ophiuchi		2	17 27 2.6	2.77	12 41 27 N	— 2.9
120	π	Scorpionis		3	17 30 44.0	4.14	38 55 53 S	+ 2.5
121	ι	Scorpionis		3	17 35 41.7	+ 4.18	40 2 59 S	+ 2.1
122	γ	Ophiuchi		3	17 39 22.0	3.00	2 46 46 N	— 1.8
123	ξ	Draconis		3	17 50 34.8	1.02	56 54 5 N	— 0.8
124	γ	Draconis	Rastaban	3	17 52 39.7	1.39	51 30 43 N	— 0.6
125	ε	Sagittarii		2.3	18 12 53.3	+ 3.98	34 27 7 S	+ 1.1
126	δ	Ursæ Minoris		3	18 27 8.0	— 19.16	86 35 6 N	+ 2.4
127	α	Lyræ	Vega	1	18 31 10.9	+ 2.02	38 37 48 N	+ 2.7
128	σ	Sagittarii		2.3	18 44 43.3	3.72	26 29 55 S	— 3.9
129	ζ	Sagittarii		3	18 51 47.4	3.82	30 6 50 S	— 4.5
130	λ	Aquilæ		3	18 57 13.1	3.18	5 7 42 S	— 4.9

TABLE LIV.—*Continued.*

No	Characters.	Constellation.	Proper Name.	Mag.	Mean Right Ascen. for 1830	Annual Variation.	Mean Declination for 1830.	Annual Variation.
					h m s	s	° ′ ″	″
131	π	Sagittarii		3	18 59 38.9	+ 3.57	21 17 4 S	— 5.2
132	δ	Draconis		3	19 12 29.1	0.02	67 21 45 N	+ 6.2
133	δ	Aquilæ		3	19 16 55.5	3.01	2 46 58 N	+ 6.6
134	β	Cygni		3	19 23 51.6	2.42	27 36 32 N	+ 7.2
135	γ	Aquilæ		3	19 38 10.6	2.85	10 12 18 N	+ 8.3
136	δ	Cygni		3	19 39 39.4	1.87	44 43 14 N	+ 8.4
137	α	Aquilæ	Altair	1.2	19 42 29.2	2.93	8 25 32 N	+ 8.7
138	η	Aquilæ		3	19 43 48.1	3.06	0 34 36 N	+ 8.8
139	β	Aquilæ		3	19 46 57.7	2.94	5 59 18 N	+ 7.5
140	α^1	Capricorni		3.4	20 8 13.1	3.33	13 1 38 S	— 10.6
141	α^2	Capricorni		3	20 8 36.9	+ 3.33	13 3 56 S	— 10.6
142	β	Capricorni		3	20 11 12.8	3.78	15 18 40 S	— 10.9
143	α	Pavonis		2	20 12 8.9	4.81	57 16 10 S	— 10.9
144	γ	Cygni		3	20 16 7.3	2.15	39 43 2 N	+ 11.2
145	α	Delphini		3	20 31 44.5	2.78	15 19 9 N	+ 12 3
146	α	Cygni		2	20 35 38.2	2.04	44 40 35 N	+ 12.6
147	α	Cephei		3	21 14 30.8	1.42	61 52 0 N	+ 15.0
148	β	Aquarii		3	21 22 36.2	3.15	6 18 51 S	— 15.5
149	ϵ	Pegasi		2.3	21 35 50.1	2.94	9 6 3 N	+ 16.2
150	δ	Capricorni		3	21 37 38.8	3.30	16 53 34 S	— 16.3
151	γ	Gruis		3	21 43 35.9	+ 3.66	38 9 31 S	— 16.6
152	α	Aquarii		3	21 57 2.8	3.08	1 8 32 S	— 17.2
153	α	Gruis		2	21 57 29.0	3.82	47 46 40 S	— 17.2
154	β	Piscis		3	22 21 49.4	3.43	33 12 51 S	— 18.2
155	ζ	Pegasi		3	22 32 58.9	2.98	9 56 53 N	+ 18.6
156	δ	Aquarii		3	22 45 37.1	3.20	16 43 15 S	— 19.0
157	α	Piscis Aust.	Fomalhaut	1	22 48 14.3	3.32	30 31 18 S	— 19.1
158	β	Pegasi		2	22 55 32.4	2.88	27 9 49 N	+ 19.3
159	α	Pegasi	Markab	2	22 56 17.8	2.97	14 17 32 N	+ 19.3
160	α	Andromedæ		2.3	23 59 36.8	+ 3.07	28 9 6 N	+ 20.0

TABLE LV. Decimal Numbers for each Day in the Year.

D.	Months. Jan.	Feb.	March	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	0.000	0.085	0.162	0.247	0.329	0.414	0.496	0.581	0.666	0.748	0.833	0.915
2	0.003	0.088	0.164	0.249	0.332	0.416	0.499	0.584	0.668	0.751	0.836	0.918
3	0.006	0.090	0.167	0.252	0.334	0.419	0.501	0.586	6.671	0.753	0.838	0.921
4	0.008	0.093	0.170	0.255	0.337	0.422	0.504	0.589	0.674	0.756	0.841	0.923
5	0.011	0.096	0.173	0.258	0.340	0.425	0.507	0.592	0.677	0.759	0.844	0.926
6	0.014	0.099	0.175	0.260	0.342	0.427	0.510	0.595	0.679	0.762	0.847	0.929
7	0.016	0.101	0.178	0.263	0.345	0.430	0.512	0.597	0.682	0.764	0.849	0.931
8	0.019	0.104	0.181	0.266	0.348	0.433	0.515	0.600	0.685	0.767	0.852	0.934
9	0.022	0.107	0.184	0.268	0.351	0.436	0.518	0.603	0.688	0.770	0.855	0.937
10	0.025	0.110	0.186	0.271	0.353	0.438	0.521	0.605	0.690	0.773	0.858	0.940
11	0.027	0.112	0.189	0.274	0.356	0.441	0.523	0.608	0.693	0.775	0.860	0.942
12	0.030	0.115	0.192	0.277	0.359	0.444	0.526	0.611	0.696	0.778	0.863	0.945
13	0.033	0.118	0.195	0.279	0.362	0.447	0.529	0.614	0.699	0.781	0.866	0.948
14	0.036	8.121	0.197	0.282	0.364	0.449	0.532	0.616	0.701	0.784	0.868	0.951
15	0.038	0.123	0.200	0.285	0.367	0.452	0.534	0.619	0.704	0.786	0.871	0.953
16	0.041	0.126	0.203	0.288	0.370	0.455	0.537	0.622	0.707	0.789	0.874	0.956
17	0.044	0.129	0.205	0.290	0.373	0.458	0.540	0.625	0.710	0.792	0.877	0.959
18	0.047	0.132	0.208	0.293	0.375	0.460	0.542	0.627	0.712	0.795	0.879	0.862
19	0.049	0.134	0.211	0.296	0.378	0.463	0.545	0.630	0.715	0.797	0.882	0.964
20	0.052	0.137	0.214	0.299	0.381	0.466	0.548	0.633	0.718	0.800	0.885	0.967
21	0.055	0.140	0.216	0.301	0.384	0.468	0.551	0.636	0.721	0.803	0.888	0.970
22	0.058	0.142	0.219	0.304	0.386	0.471	0.553	0.638	0.723	0.805	0.890	0.973
23	0.060	0.145	0.222	0.307	0.389	0.474	0.556	0.641	0.726	0.808	0.893	0.975
24	0.063	0.148	0.225	0.310	0.392	0.477	0.559	0.644	0.729	0.811	0.896	0.978
25	0.066	0.151	0.227	0.312	0.395	0.479	0.562	0.647	0.731	0.814	0.899	0.981
26	0.068	0.153	0.230	0.315	0.397	0.482	0.564	0.649	0.734	0.816	0.901	0.984
27	0.071	0.156	0.233	0.318	0.400	0.485	0.567	0.652	0.737	0.819	0.904	0.986
28	0.074	0.159	0.236	0.321	0.403	0.488	0.570	0.655	0.740	0.822	0.907	0.989
29	0.077	0.162	0.238	0.323	0.406	0.490	0.573	0.658	0.742	0.825	0.910	0.992
30	0.079		7.241	0.326	0.408	0.493	0.575	0.660	0.745	0.827	0.912	0.995
31	0.082		0.244		0.411		0.578	0.663		0.830		0.997

TABLE LVI.

Sun's Right Ascension for every Day in the Year 1828.

Days.	January.	February.	March.	April.	May.	June.
	h m s	h m s	h m s	h m s	h m s	h m s
1	18 44 5	20 56 36	22 49 40	0 43 11	2 34 27	4 37 12
2	18 48 30	21 0 41	22 53 24	0 46 49	2 38 16	4 41 18
3	18 52 55	21 4 45	22 57 7	0 50 27	2 42 6	4 45 24
4	18 57 19	21 8 47	23 0 51	0 54 6	2 45 56	4 49 31
5	19 1 43	21 12 50	23 4 33	0 57 45	2 49 47	4 53 38
6	19 6 7	21 16 51	23 8 16	1 1 24	2 53 38	4 57 45
7	19 10 30	21 20 51	23 11 57	1 5 3	2 57 30	5 1 52
8	19 14 52	21 24 51	23 15 39	1 8 42	3 1 23	5 6 0
9	19 19 14	21 28 50	23 19 20	1 12 22	2 5 16	5 10 8
10	19 23 36	21 32 48	23 23 1	1 16 2	3 9 10	5 14 17
11	19 27 57	21 36 45	23 26 41	1 19 42	3 13 4	5 18 25
12	19 32 17	21 40 42	23 30 22	1 23 23	3 16 59	5 22 34
13	19 36 37	21 44 38	23 34 1	1 27 3	3 20 55	5 26 43
14	19 40 57	21 48 33	23 37 41	1 30 45	3 24 51	5 30 52
15	19 45 15	21 52 27	23 41 21	1 34 26	3 28 48	5 35 2
16	19 49 33	21 56 21	23 45 0	1 38 8	3 32 45	5 39 11
17	19 53 50	22 0 14	23 48 39	1 41 50	3 36 43	5 43 20
18	19 58 7	22 4 6	23 52 18	1 45 33	3 40 42	5 49 30
19	20 2 23	22 7 57	23 55 56	1 49 16	3 44 40	5 51 39
20	20 6 38	22 11 48	23 59 35	1 52 59	3 48 40	5 55 49
21	20 10 52	22 15 38	0 3 13	1 56 43	3 52 40	5 59 59
22	20 15 6	22 19 27	0 6 51	2 0 27	3 66 41	6 4 8
23	20 19 19	22 23 16	0 10 29	2 4 12	4 0 42	6 8 18
24	20 23 31	22 27 4	0 14 7	2 7 57	4 4 43	6 12 27
25	20 27 42	22 30 52	0 17 45	2 11 43	4 8 45	6 16 36
26	20 31 52	22 34 38	0 21 23	2 15 29	4 12 48	6 20 45
27	20 36 1	22 38 25	0 25 1	2 19 15	4 16 51	6 24 54
28	20 40 10	22 42 10	0 28 39	2 23 2	4 20 54	6 29 3
29	20 44 18	22 45 55	0 32 17	2 26 50	4 24 58	6 33 11
30	20 48 25		0 35 55	2 30 38	4 29 2	6 37 20
31	20 52 31		0 39 33		4 33 7	

Days	July.	August.	September.	October.	November.	December.
	h m s	h m s	h m s	h m s	h m s	h m s
1	6 41 28	8 46 14	10 42 14	12 30 19	14 26 39	16 30 36
2	6 45 36	8 50 6	10 45 52	12 33 57	14 30 35	16 35 56
3	6 49 44	8 53 58	12 49 29	12 37 35	14 34 32	16 39 16
4	6 53 51	8 57 50	10 53 6	12 41 13	14 38 30	16 43 37
5	6 57 58	9 1 41	10 56 43	12 44 52	14 42 28	16 47 59
6	7 2 5	9 5 31	11 0 20	12 48 31	14 46 27	16 52 21
7	7 6 11	9 9 21	11 3 56	12 52 11	14 50 27	16 56 44
8	7 10 18	9 14 10	11 7 33	12 55 51	14 54 28	17 1 7
9	7 14 23	9 16 59	11 11 9	12 59 31	14 58 30	17 5 31
10	7 18 29	9 20 47	11 14 45	13 3 12	15 2 33	17 9 55
11	7 22 34	9 24 34	11 18 21	13 3 53	15 6 36	17 14 20
12	7 26 38	9 28 21	11 21 56	13 10 35	15 10 41	17 18 44
13	7 30 42	9 32 7	11 25 32	13 14 17	15 14 46	17 23 10
14	7 34 46	9 35 53	11 29 8	13 18 0	15 18 52	17 27 35
15	7 38 49	9 39 38	11 32 43	13 21 44	15 22 59	17 32 1
16	7 42 51	9 43 23	11 36 19	13 24 28	15 27 6	17 36 27
17	7 46 53	9 47 7	11 39 54	13 29 12	15 31 15	17 40 53
18	7 50 55	9 50 51	11 43 29	13 32 57	15 35 24	17 45 19
19	7 54 55	9 54 34	11 47 5	13 36 43	15 39 34	17 49 45
20	7 58 56	9 58 16	11 50 40	13 40 29	15 43 45	17 54 12
21	8 2 56	10 1 58	11 54 16	13 44 16	15 47 57	17 58 38
22	8 6 55	10 5 40	11 57 51	13 48 3	15 52 9	18 3 5
23	8 10 53	10 9 21	12 1 27	13 51 52	15 56 22	18 7 31
24	8 14 51	10 13 2	12 5 3	13 55 41	16 0 36	18 11 58
25	8 18 49	10 16 42	12 8 39	13 59 30	16 4 51	18 16 25
26	8 22 45	10 20 22	12 12 15	14 3 21	16 9 7	18 20 51
27	8 26 42	10 24 2	12 15 51	14 7 12	16 13 23	18 25 17
28	8 30 37	10 27 41	12 19 28	14 11 4	16 17 40	18 29 44
29	8 34 32	10 31 20	12 23 4	14 14 56	16 21 58	18 34 10
30	8 38 26	10 34 58	12 26 42	14 18 50	16 26 17	18 38 35
31	8 42 20	10 38 36		14 22 44		18 43 1

TABLE LVII.—Sun's Declination for every Day in the Year 1828.

Days.	January.	February.	March.	April.	May	June.
	South.	South.	South.	North.	North.	North.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	23 4 22	17 17 44	7 28 10	4 38 49	15 9 11	22 5 43
2	22 59 29	17 0 43	7 5 18	5 1 52	15 27 8	22 13 37
3	22 54 8	16 43 23	6 42 21	5 24 50	15 44 50	22 21 7
4	22 48 20	16 25 46	6 19 18	5 47 43	16 2 17	22 28 14
5	22 42 5	16 7 53	5 56 9	6 10 29	16 19 27	22 34 57
6	22 35 23	15 49 42	5 32 56	6 33 9	16 36 22	22 41 17
7	22 28 14	15 31 15	5 9 38	6 55 43	16 53 0	22 47 12
8	22 20 38	15 12 32	4 46 15	7 18 10	17 9 22	22 52 44
9	22 12 36	14 53 34	4 22 50	7 40 30	17 25 26	22 57 52
10	22 4 8	14 34 21	3 59 21	8 2 42	17 41 13	23 2 36
11	21 55 14	14 14 53	3 35 48	8 24 46	17 56 43	23 6 55
12	21 45 54	13 55 10	3 12 13	8 46 42	18 11 54	23 10 50
13	21 36 9	13 35 14	2 48 36	9 8 29	18 26 48	23 14 20
14	21 25 59	13 15 5	2 34 57	9 30 6	18 41 22	23 17 26
15	21 15 24	12 54 48	2 1 16	9 51 35	18 55 38	23 20 7
16	21 4 24	12 34 8	1 37 35	10 12 54	19 9 35	23 22 24
17	20 53 0	12 13 21	1 13 52	10 34 2	29 23 12	23 24 16
18	21 41 13	11 52 22	0 50 10	10 55 1	19 36 29	23 25 43
19	20 29 2	11 31 13	0 26 27	11 15 48	19 49 27	23 26 45
20	20 16 27	11 9 52	0 2 45 S	11 36 24	20 2 4	23 27 22
21	20 3 30	10 48 22	0 20 56 N	11 56 49	20 14 20	23 27 35
22	19 50 11	10 26 41	0 44 36	12 17 1	20 26 16	23 27 22
23	19 36 29	10 4 51	1 8 14	12 37 2	20 37 51	22 26 45
24	19 22 26	9 42 52	1 31 50	22 56 50	20 49 5	23 25 44
25	19 8 1	9 20 44	1 55 24	13 16 25	20 59 57	23 24 17
26	18 53 16	8 58 28	2 18 56	13 35 48	21 10 28	23 22 26
27	18 38 10	8 36 5	2 42 24	13 54 56	21 20 36	23 20 10
28	18 22 43	8 13 33	3 5 49	14 13 51	21 30 23	23 17 30
29	18 6 57	7 50 55	3 29 10	14 32 32	21 39 47	23 14 25
30	17 50 52		3 52 27	14 50 59	21 48 49	23 10 56
31	17 34 27		4 15 40		21 57 28	

Days.	July.	August.	September.	October.	November.	December.
	North.	North.	North.	South.	South.	South.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1	23 7 2	17 59 28	8 13 9	3 16 33	14 31 39	21 52 8
2	23 2 44	17 44 9	7 51 16	3 39 51	14 50 44	22 1 9
3	22 58 2	17 28 33	7 29 15	4 3 7	15 9 35	22 9 44
4	22 52 56	17 12 39	7 7 6	4 26 20	15 28 11	22 17 53
5	22 47 26	16 56 29	6 44 51	4 49 30	15 46 32	22 25 37
6	22 41 32	16 40 2	6 22 28	5 12 36	16 4 37	22 32 54
7	22 35 14	16 23 19	5 59 59	5 35 39	16 22 26	22 39 45
8	22 28 33	16 6 21	5 37 25	5 58 37	16 39 59	22 46 9
9	22 21 29	15 49 6	5 14 44	6 21 31	16 57 14	22 52 6
10	22 14 1	15 31 36	4 51 59	6 44 19	17 14 12	22 57 36
11	22 6 11	15 13 52	4 29 8	7 7 2	17 30 52	23 2 38
12	21 57 58	14 55 53	4 6 12	7 29 40	17 47 14	23 7 13
13	21 49 22	14 37 39	3 43 13	7 52 11	18 3 18	23 11 21
14	21 40 23	14 19 11	3 20 9	8 14 35	18 19 2	23 15 0
15	21 31 3	14 0 30	2 57 2	8 36 52	18 34 27	23 18 12
16	21 21 20	13 41 35	2 33 51	8 59 2	18 49 32	23 20 56
17	21 11 16	13 22 27	2 10 38	9 21 4	19 4 17	23 23 12
18	21 0 50	13 3 7	1 47 22	9 42 58	19 18 41	23 24 59
19	20 50 3	12 43 35	1 24 4	10 4 43	19 32 44	23 26 19
20	20 38 55	12 23 50	1 0 43	10 26 19	19 46 27	23 27 10
21	20 27 26	12 3 54	0 37 22	10 47 46	19 59 47	23 27 33
22	20 15 36	11 43 46	0 13 59 N	11 9 3	20 12 46	23 27 27
23	20 3 27	11 23 28	0 9 25 S	11 30 10	20 25 22	23 26 53
24	19 50 57	11 2 58	0 32 50	11 51 7	20 37 35	23 25 51
25	19 38 7	10 42 18	0 56 15	12 11 53	20 49 26	23 24 21
26	19 24 58	10 21 28	1 19 39	12 32 28	21 0 53	23 22 22
27	19 11 30	10 0 28	1 43 4	12 52 51	21 11 57	23 19 55
28	18 57 42	9 39 18	2 6 28	13 13 2	21 22 36	23 13 37
29	18 43 36	9 17 59	2 29 51	13 33 1	21 32 52	23 17 0
30	18 29 11	8 56 31	2 53 13	13 52 47	21 42 43	23 9 46
31	18 14 28	8 34 54		14 12 20		23 5 27

TABLE LVIII.—Equation of Time for every Day in the Year 1828.

Days.	Jan.		Feb.		Mar.		April.		May.		June.		July.		Aug.		Sept.		Oct.		Nov.		Dec.	
	Add		Add		Add		Add		Sub.		Sub.		Add		Add		Sub.		Sub.		Sub.		Sub.	
	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s	m	s
1	3	35	13	52	12	35	3	54	3	5	2	33	3	25	5	57	0	15	10	25	16	17	10	38
2	4	4	14	0	12	23	3	36	3	13	2	24	3	37	5	53	0	34	10	43	16	17	10	14
3	4	32	14	7	12	10	3	18	3	19	2	14	3	48	5	49	0	53	11	2	16	17	9	50
4	4	59	14	13	11	56	3	0	3	26	2	4	4	58	5	44	1	12	11	20	16	16	9	26
5	5	27	14	18	11	42	2	42	3	31	1	54	4	9	5	38	1	32	11	38	16	14	9	1
6	5	54	14	23	11	28	2	25	3	37	1	43	4	19	5	32	1	51	11	55	16	11	8	35
7	6	20	14	27	11	14	2	7	3	41	1	33	4	29	5	25	2	11	12	12	16	8	8	9
8	6	46	14	30	10	59	1	50	3	45	1	21	4	39	5	18	2	32	12	29	16	3	7	43
9	7	11	14	32	10	43	1	33	3	48	1	10	4	48	5	10	2	52	12	45	15	58	7	15
10	7	36	14	34	10	28	1	17	3	51	0	58	5	57	5	1	3	12	13	0	15	52	6	48
11	8	1	14	35	10	12	1	1	3	53	0	46	5	5	4	52	3	33	13	15	15	45	6	20
12	8	25	14	35	9	55	0	45	3	55	0	34	5	13	4	43	3	54	13	30	15	37	5	52
13	8	48	14	34	9	39	0	29	3	55	0	21	5	20	4	32	4	15	13	44	15	29	5	23
14	9	10	14	33	9	22	0	14	3	56	0	9	5	27	4	22	4	36	13	58	15	19	4	55
							Sub.				Add													
15	9	32	14	30	9	5	0	1	3	56	0	4	5	34	4	10	4	57	14	11	15	9	4	25
16	9	54	14	28	8	47	0	16	3	55	0	17	5	40	3	58	5	18	14	24	14	58	3	56
17	10	15	14	24	8	30	0	30	3	54	0	30	5	45	3	46	5	39	14	36	14	46	3	27
18	10	35	14	19	8	12	0	44	3	52	0	43	5	50	3	33	6	0	14	47	14	34	2	57
19	10	54	14	14	7	54	0	58	3	49	0	56	5	54	3	20	6	21	14	58	14	20	2	27
20	11	12	14	9	7	36	1	11	3	46	1	9	5	58	3	6	6	42	15	8	14	6	1	58
21	11	30	14	2	7	18	1	24	3	43	1	22	6	1	2	51	7	3	15	18	13	51	1	28
22	11	47	13	55	7	0	1	36	3	39	1	35	6	4	2	37	7	24	15	27	13	35	0	58
23	12	3	13	47	6	41	1	48	3	34	1	47	6	6	2	21	7	45	15	35	13	18	0	28
																							Add	
24	12	18	13	39	6	23	1	59	3	29	2	0	6	7	2	5	8	5	15	43	13	1	0	2
25	12	33	13	29	6	4	2	10	3	24	2	13	6	8	1	49	8	26	15	50	12	42	0	32
26	12	47	13	20	5	46	2	20	3	18	2	25	6	8	1	33	8	46	15	56	12	23	1	2
27	13	0	13	9	5	27	2	30	3	12	2	38	6	8	1	16	9	7	16	1	11	4	1	32
28	13	12	12	59	5	8	2	40	3	5	2	50	6	7	0	58	9	26	16	6	11	43	2	1
29	13	23	12	47	4	50	2	49	2	57	3	2	6	5	0	41	9	46	16	10	11	22	2	30
30	13	33			4	31	2	57	2	50	3	14	6	3	0	23	10	6	16	13	11	0	3	0
31	13	43			4	13			2	41			6	0	0	4			16	15			3	29

TABLE LIX.—Correction of the Longitude by Chronometers.

Days.	Log.	Days.	Log.	Days.	Log.	Days.	Log.
1	0.00000	31	2.69548	61	3.27669	91	3.62180
2	0.47712	32	2.72263	62	3.29070	92	3.63124
3	0.77815	33	2.74896	63	3.30449	93	3.64058
4	1.00000	34	2.77452	64	3.31806	94	3.64982
5	1.17609	35	2.79934	65	3.33143	95	3.65896
6	1.32222	36	2.82347	66	3.34459	96	3.66801
7	1.44716	37	2.84696	67	3.35755	97	3.67697
8	1.55630	38	2.86982	68	3.37033	98	3.68583
9	1.65321	39	2.89209	69	3.38292	99	3.69461
10	1.74036	40	2.91381	70	3.39533	100	3.70329
11	1.81954	41	2.93500	71	3.40756	101	3.71189
12	1.89209	42	2.95569	72	3.41963	102	3.72041
13	1.95904	43	2.97589	73	3.43152	103	3.72884
14	2.02119	44	2.99564	74	3.44326	104	3.73719
15	2.07918	45	3.01494	75	3.45484	105	3.74547
16	2.13354	46	3.03383	76	3.46627	106	3.75366
17	2.18469	47	3.05231	77	3.47756	107	3.76178
18	2.23300	48	3.07041	78	3.48869	108	3.76982
19	2.27875	49	3.08814	79	3.49969	109	3.77779
20	2.32222	50	3.10551	80	3.51055	110	3.78569
21	2.36361	51	3.12254	81	3.52127	111	3.79351
22	3.40312	52	3.13925	82	3.53186	112	3.80127
23	2.44091	53	3.15564	83	3.54233	113	3.80895
24	2.47712	54	3.17173	84	3.55267	114	3.81657
25	2.51188	55	3.18752	85	3.56289	115	3.82413
26	2.54531	56	3.20303	86	3.57299	116	3.83161
27	2.57749	57	3.21827	87	3.58297	117	3.83904
28	2.60853	58	3.23325	88	3.59284	118	3.84640
29	2.63849	59	3.24797	89	3.60260	119	3.85370
30	2.66745	60	3.26245	90	3.61225	120	3.86094

Names of Places.	Latitude. ° ′ ″	Longitude. In Degrees. ° ′ ″	Longitude. In Time. h m s	High Water. Time. h m	Spring. feet.	Neap. feet.
Aberdeen (Obs. Mar. Coll.)	57 8 56 N	2 5 30 W	0 8 22 W	0 45	19	14
Altona (Obs.)	53 32 51 N	9 57 30 E	0 39 50 E			
Amsterdam	52 22 17 N	4 53 15 E	0 19 33 E	3 0	17	
Archangel	64 34 0 N	40 43 0 E	2 42 52 E	6 20		
Barbadoes (Bridgetown)	13 5 0 N	59 41 15 W	3 58 45 W			
Batavia	6 9 0 S	106 51 45 E	7 7 27 E			
Berlin (Obs.)	52 31 45 N	13 22 15 E	0 53 29 E			
Berwick	55 46 10 N	2 0 20 W	0 8 1 W	2 15	16	10
Bombay (Church)	18 57 44 N	72 54 43 E	4 51 39 E	11 10	17	
Bremen	53 4 38 N	8 48 0 E	0 35 12 E	6 0		
Brest	48 23 14 N	4 28 45 W	0 17 55 W	4 20	18	
Brighton	50 49 32 N	0 7 40 W	0 0 30 W	10 10	16	
Bristol	51 27 6 N	2 35 29 W	0 10 22 W	6 50	42	
Cadiz	36 32 0 N	6 17 22 W	0 25 9 W	4 20		
Calais	50 57 32 N	1 51 16 E	0 7 25 E	11 40	18	
Calcutta	22 34 16 N	88 25 30 E	5 53 42 E	3 10		
Cambridge	52 12 43 N	0 7 34 E	0 0 30 E			
Canton	23 8 9 N	113 17 39 E	7 33 11 E			
Coimbra	40 12 30 N	8 24 42 W	0 33 39 W			
Constantinople	41 1 27 N	28 55 15 E	1 55 42 E			
Copenhagen	55 41 4 N	12 35 6 E	0 50 20 E			
Cork	51 51 50 N	8 16 30 W	0 33 6 W	5 30	18	
Dantzic	54 20 48 N	18 38 5 E	1 14 32 E			
Dorpat (Obs.)	58 22 47 N	26 42 0 E	1 46 48 E			
Dublin (Obs.)	53 23 13 N	6 20 30 W	0 25 22 W	9 20	12	
Dundee	56 28 0 N	2 58 0 W	0 11 52 W	2 10	16	
Edinburgh (Observatory)	55 57 20 N	3 10 30 W	0 12 42 W	2 20	16	9
Florence	43 46 41 N	11 15 45 E	0 45 3 E			
Genoa	44 25 0 N	8 58 0 E	0 35 52 E			
Glasgow	55 51 32 N	4 16 0 W	0 17 4 W	0 0		
Gotha (Obs.)	50 56 8 N	10 44 0 E	0 42 56 E			
Gottingen (Obs.)	51 31 50 N	9 56 30 E	0 39 46 E			
Greenwich (Obs.)	51 28 38½ N	0 0 0	0 0 0			
Heligoland	54 11 34 N	7 53 13 E	0 31 33 E	11 0		
Hull	53 48 0 N	0 33 0 W	0 2 12 W	6 0	18	
Hieres	43 7 2 N	6 7 55 E	0 24 32 E			
Jamaica (Port Royal)	17 58 0 N	76 52 30 W	5 7 30 W		1	
Konigsberg (Obs.)	54 42 12 N	20 29 15 E	1 21 57 E			
Lisbon	38 42 24 N	9 8 30 W	0 36 34 W	2 15		
Liverpool	53 24 40 N	2 58 55 W	0 11 56 W	11 10		
Lizard (Light)	49 57 44 N	5 11 5 W	0 20 44 W	5 0	18	
London (St Paul's)	51 30 49 N	0 5 47 W	0 0 23 W	2 50	19	
Madras (Obs.)	13 4 9 N	80 17 21 E	5 21 9 E			
Madrid	40 24 57 N	3 42 15 W	0 14 49 W			
Malta	35 53 0 N	14 30 35 E	0 58 2 E			
May (Light)	56 11 22 N	2 32 47 W	0 10 11 W	1 50		
Montpelier	43 36 16 N	3 52 40 E	0 15 31 E			
Moscow	55 45 45 N	37 33 0 E	2 30 12 E			
Naples	40 50 15 N	14 15 45 E	0 57 3 E			
Oxford (Obs.)	51 45 39 N	1 35 22 W	0 5 1 W			
Palermo (Obs.)	38 6 44 N	13 22 0 E	0 53 28 E			
Paris (Obs.)	[illegible] 50 14 N	2 20 22 E	0 9 21½ E			
Pekin (Obs.)	[illegible] 13 N	116 26 45 E	7 45 51 E			
Petersburgh	59 56 23 N	30 18 45 E	2 1 15 E			
Philadelphia	39 56 55 N	75 11 30 W	5 0 46 W	2 30		
Plymouth	50 22 20 N	4 8 5 W	0 16 32 W	5 30	18	
Portsmouth	50 48 3 N	1 6 12 W	0 4 25 W	11 30	18	
Rome	41 53 54 N	12 29 47 E	0 49 59 E			
Rotterdam	51 55 22 N	4 29 11 E	0 17 56 E	3 30	7	
Slough (Obs.)	51 30 20 N	0 36 0 W	0 2 24 W			
Stockholm	59 20 31 N	18 3 30 E	1 12 14 E			
Toulon	43 7 9 N	5 55 41 E	0 23 43 E			
Upsal	59 51 50 N	17 39 0 E	1 10 36 E			
Venice	45 25 32 N	12 20 59 E	0 49 24 E			
Vienna	48 12 36 N	17 22 45 E	1 5 31 E			
Yarmouth	52 36 40 N	1 43 35 E	0 6 54 E	8 50	8	
York, New (Battery)	40 42 6 N	73 59 0 W	4 55 56 W	9 0		

TABLE LXI.

Space into Time.

To convert Degrees and parts of the Equator into Sidereal Time.

°	h m	″	m s	″	s
1	0 4	1	0 4	1	0.067
2	0 8	2	0 8	2	0.133
3	0 12	3	0 12	3	0.200
4	0 16	4	0 16	4	0.267
5	0 20	5	0 20	5	0.333
6	0 24	6	0 24	6	0.400
7	0 28	7	0 28	7	0.467
8	0 32	8	0 32	8	0.533
9	0 36	9	0 36	9	0.600
10	0 40	10	0 40	10	0.667
11	0 44	11	0 44	11	0.733
12	0 48	12	0 48	12	0.800
13	0 52	13	0 52	13	0.867
14	0 56	14	0 56	14	0.933
15	1 0	15	1 0	15	1.000
16	1 4	16	1 4	16	1.067
17	1 8	17	1 8	17	1.133
18	1 12	18	1 12	18	1.200
19	1 16	19	1 16	19	1.267
20	1 20	20	1 20	20	1.333
25	1 40	21	1 24	21	1.400
30	2 0	22	1 28	22	1.467
35	2 20	23	1 32	23	1.533
40	2 40	24	1 36	24	1.600
45	3 0	25	1 40	25	1.667
50	3 20	26	1 44	26	1.733
55	3 40	27	1 48	27	1.800
60	4 0	28	1 52	28	1.867
65	4 20	29	1 56	29	1.933
70	4 40	30	2 0	30	2.000
75	5 0	31	2 4	31	2.067
80	5 20	32	2 8	32	2.133
90	6 0	33	2 12	33	2.200
100	6 40	34	2 16	34	2.267
110	7 20	35	2 20	35	2.333
120	8 0	36	2 24	36	2.400
130	8 40	37	2 28	37	2.467
140	9 20	38	2 32	38	2.533
150	10 0	39	2 36	39	2.600
160	10 40	40	2 40	40	2.667
170	11 20	41	2 44	41	2.733
180	12 0	42	2 48	42	2.800
190	12 40	43	2 52	43	2.867
200	13 20	44	2 56	44	2.933
210	14 0	45	3 0	45	3.000
220	14 40	46	3 4	46	3.067
230	15 20	47	3 8	47	3.133
240	16 0	48	3 12	48	3.200
250	16 40	49	3 16	49	3.267
260	17 20	50	3 20	50	3.333
270	18 0	51	3 24	51	3.400
280	18 40	52	3 28	52	3.467
290	19 20	53	3 32	53	3.533
300	20 0	54	3 36	54	3.600
310	20 40	55	3 40	55	3.667
320	21 20	56	3 44	56	3.733
330	22 0	57	3 48	57	3.800
340	22 40	58	3 52	58	3.867
350	23 20	59	3 56	59	3.933
360	24 0	60	4 0	60	4.000

Or to convert Degrees and parts of Terrestrial Longitude into Time.

TABLE LXII.

Time into Space.

To convert Sidereal Time into Degrees and parts of the Equator.

h	°
1	15
2	30
3	45
4	60
5	75
6	90
7	105
8	120
9	135
10	150
11	165
12	180
13	195
14	210
15	225
16	240
17	255
18	270
19	285
20	300
21	315
22	330
23	345
24	360

Tenths.

s	″
0.1	1.5
0.2	3.0
0.3	4.5
0.4	6.0
0.5	7.5
0.6	9.0
0.7	10.5
0.8	12.0
0.9	13.5
1.0	15.0

Hundredths.

s	″
0.01	0.15
0.02	0.30
0.03	0.45
0.04	0.60
0.05	0.75
0.06	0.90
0.07	1.05
0.08	1.20
0.09	1.35
0.10	1.50

Thousandths.

s	″
0.0[illegible]	0.015
0.002	0.030
0.003	0.045
0.004	0.060
0,005	0.075
0.006	0.090
9.007	0.105
0.008	0.120
0.009	0.135
0.010	0.150

m	° ′	s	′ ″
1	0 15	1	0 15
2	0 30	2	0 30
3	0 45	3	0 45
4	1 0	4	1 0
5	1 15	5	1 15
6	1 30	6	1 30
7	1 45	7	1 45
8	2 0	8	2 0
9	2 15	9	2 15
10	2 30	10	2 30
11	2 45	11	2 45
12	3 0	12	3 0
13	3 15	13	3 15
14	3 30	14	3 30
15	3 45	15	3 45
16	4 0	16	4 0
17	4 15	17	4 15
18	4 30	18	4 30
19	4 45	19	4 45
20	5 0	20	5 0
21	5 15	21	5 15
22	5 30	22	5 30
23	5 45	23	5 45
24	6 0	24	6 0
25	6 15	25	6 15
26	6 30	26	6 30
27	6 45	27	6 45
28	7 0	28	7 0
29	7 15	29	7 15
30	7 30	30	7 30
31	7 45	31	7 45
32	8 0	32	8 0
33	8 15	33	8 15
34	8 30	34	8 30
35	8 45	35	8 45
36	9 0	36	9 0
37	9 15	37	9 15
38	9 30	38	9 30
39	9 45	39	9 45
40	10 0	40	10 0
41	10 15	41	10 15
42	10 30	42	10 30
43	10 45	43	10 45
44	11 0	44	11 0
45	11 15	45	11 15
46	11 30	46	11 30
47	11 45	47	11 45
48	12 0	48	12 0
49	12 15	49	12 15
50	12 30	50	12 30
51	12 45	51	12 45
52	13 0	52	13 0
53	13 15	53	13 15
54	13 30	54	13 30
55	13 45	55	13 45
56	14 0	56	14 0
57	14 15	57	14 15
58	14 30	58	14 30
59	14 45	59	14 45
60	15 0	60	15 0

Or to convert Time into Degrees and Parts of Terrestrial Longitude.

No	Characters.	Natural Numbers.	Logarithms.	Arith. Comp. Logarithms.
1	π	3.1415926536=circum. of a circle to D^r=1	0.4971499	9.5028501
2	π^2	9.86960440=square of the former	0.9942997	9.0057003
3	$\frac{1}{4}\pi$	0.78539816, this × square of D^r=area of C.	9.8950899	0.1049101
4	$\frac{1}{6}\pi$	0.52359878, this × cube of D^r=solidity of sph.	9.7189986	0.2810014
5	$\frac{1}{4\pi}$	0.07957747, this × circum. ² of a cir. =area	8.9007902	1.0992098
6	A = R°	57°.29577951 the length of an arc=rad. in deg.	1.7581226	8.2418774
7	A = R′	3437′.74677 . . . minutes	3.5362739	6.4637261
8	A = R″	206264″.81 . . . . seconds	5.3144251	4.6855749
9	A = 1°	0.01745329=length of an arc of one degree	8.2418773	1.7581227
10	A = 1′	0.0002908882 . . . minute	6.4637261	3.5362739
11	A = 1″	0.0000048481368 . . second	4.6855749	5.3144251
12	Sin 1″	0.0000048481368	4.6855749	5.3144251
13	Sin 2″	0.0000096962736	4.9866049	5.0133951
14	Sin 3″	0.0000145444104	5.1626961	4.8373039
15	Circle	1296000″=seconds in a circle . .	6.1126050	3.8873950
16	Day	86400ˢ=seconds of time in 24 hours .	4.9365137	5.0634863
17	Half Day	43200ˢ=———————— 12 hours .	4.6354837	5.3645163
18	Eighth Day	10800ˢ=———————— 3 hours .	4.0334238	5.9665762
19	Hour	3600ˢ=———————— 1 hour .	3.5563025	6.4436975
20	E. Rot.	86164ˢ.0908=time of the earth's rotation	4.9353264	5.0646736
21	Trop. Year	365ᵈ 5ʰ 48ᵐ 50ˢ=31556930ˢ length of tr. yr.	7.4990948	2.5009052
22	————	365.242245=length of the trop. yr. in days	2.5625810	7.4374190
23	Sid. Year	365.256389=———— sid. year in days	2.5625978	7.4374022
24	S. T.	Log. to convert sidereal into mean solar time	9.9988126	0.0011874
25	Aberration	20″.36=coefficient of aberration .	1.3087778	8.6912222
26	Nutation	9″.26=coefficient of nutation . .	0.9666110	9.0333890
27	Earth	24856 miles=mean circumference of earth	4.3954312	5.6045688
28	————	7912 —— = —— diameter of the earth	3.8982863	6.1017137
29	————	41775360 feet=————————	7.6209202	2.3790798
30	————	20887680 feet=—— radius of the earth	7.3198902	2.6801098
31	a	20921180 feet=the radius of the equator	7.3205862	2.6794138
32	$a\,(1-\epsilon)$	20853180 feet=the polar semiaxis .	7.3191723	2.6808277
33	ϵ	0.003245=the ellipticity of the earth	7.5112147	2.4887853
34	$d°$	365144 feet=a degree on the equator	5.5624636	4.4375364
35	Mile Geog.	6075.6 feet=length of geogr. or naut. mile	3.7835892	6.2164108
36	—— English	5280 feet=———— English mile .	3.7226339	6.2773661
37	R. of N. & E.	1.15068 to 1=ratio of naut. to English mile	0.0609553	9.9390447
38	Metre	39.37079 English inches, at 62° F. .	1.5951741	8.4048259
39	————	3.2808992 ——— feet . . .	0.5159929	9.4840071
40	Mil. Met.	Log. to convert millimetres into Eng. inches	8.5951741	1.4048259
41	Toise	1.065825 English fathoms, at 56°.3 F.	0.0276859	9.9723141
42	————	6.39495 ——— feet . . .	0.8058371	9.1941629
43	Pied	1.065825 ——— foot . . .	0.0276859	9.9723141
44	Myriametre	6.213824 English miles . . .	0.7933590	9.2066410
45	Hectare	2.47114 ——— acres . . .	0.3928979	9.6071021
46	Cubic Metre	35.3166 ——— cubic feet . .	1.5479787	8.4520213
47	Litre	61.0271 ——— cubic inches .	1.7855223	8.2144777
48	Gramme	15.434 grains Troy	1.1884785	8.8115215
49	Kilogramme	2.20486 pounds avoirdupois .	0.3433805	9.6566195
50	M.	0.4342944819=log. modulus . .	9.6377843	0.3622157
51	H. L. 10	2.3025851=hyp. log. 10 . .	0.3622157	9.6377843
52	R. H. L.	2.7182818=radix of the hyp. logs. .	0.4342945	9.5657055
53	P.	39.1393 in.=length of pend. at London	1.5926130	8.4073870
54	P.	39.1555 — = ———————— Edinburgh	1.5927928	8.4072072
55	g.	32.1908 feet=force of grav. at London	1.5077315	8.4922685
56	g.	32.2041 — =———————— Edinburgh	1.5079113	8.4920887
57	$\frac{1}{2}$ g.	16.0954 — =half the above at London	1.2067015	8.7932985
58	$\frac{1}{2}$ g.	16.10205 — =———————— Edinburgh	1.2068813	8.7931187
59	Thermometer	$x°$ Centigrade = $(32° + \frac{9}{5}\,x°)$ Fahrenheit = $\frac{4}{5}\,x°$ Reaumur		
60	————	$x°$ Fahrenheit = $(x° - 32°)\,\frac{4}{9}$ Reaumur = $(x° - 32°)\,\frac{5}{9}$ Centig.		
61	————	$x°$ Reaumur = $(32° + \frac{9}{4}\,x°)$ Fahrenheit = $\frac{5}{4}\,x°$ Centigrade		

TO FIND THE TIME OF HIGH WATER.

Months. — Days. | Years. — Correction to be added to the time of High Water at Full and Change.

Jan.	Feb. Apr.	Mar.	May.	June.	July.	Aug.	Sep. Oct.	Nov. Dec.	1800 1819 1838 1857 1876	1801 1820 1839 1858 1877	1802 1821 1840 1859 1878	1803 1822 1841 1860 1879	1804 1823 1842 1861 1880	1805 1824 1843 1862 1881	1806 1825 1844 1863 1882	1807 1826 1845 1864 1883	1808 1827 1846 1865 1884	1809 1828 1847 1866 1885	1810 1829 1848 1867 1886	1811 1830 1849 1868 1887	1812 1831 1850 1869 1888	1813 1832 1851 1870 1889	1814 1833 1852 1871 1890	1815 1834 1853 1872 1891	1816 1835 1854 1873 1892	1817 1836 1855 1874 1893	1818 1837 1856 1875 1894
									h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
1	29	30	28	27	26	25	23	21	3 1	12 45	22 21	5 40	14 30	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18
2	30	1	29	28	27	26	24	22	3 44	13 19	23 3	6 58	15 11	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31
3	1	2	30	29	28	27	25	23	4 37	13 54	23 42	8 14	15 56	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31
4	2	3	1	30	29	28	26	24	5 40	14 30	0 0	9 17	16 51	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21
5	3	4	2	1	30	29	27	25	6 58	15 11	0 36	10 9	18 0	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3
6	4	5	3	2	1	30	28	26	8 14	15 56	1 11	10 53	19 18	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42
7	5	6	4	3	2	1	29	27	9 17	16 51	1 46	11 33	20 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0
8	6	7	5	4	3	2	30	28	10 9	18 0	2 21	12 8	21 31	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36
9	7	8	6	5	4	3	1	29	10 53	19 18	3 1	12 45	22 21	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11
10	8	9	7	6	5	4	2	30	11 33	20 31	3 44	13 19	23 3	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46
11	9	10	8	7	6	5	3	1	12 8	21 31	4 37	13 54	23 42	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21
12	10	11	9	8	7	6	4	2	12 45	22 21	5 40	14 30	0 0	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1
13	11	12	10	9	8	7	5	3	13 19	23 3	6 58	15 11	0 36	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44
14	12	13	11	10	9	8	6	4	13 54	23 42	8 14	15 56	1 11	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37
15	13	14	12	11	10	9	7	5	14 30	0 0	9 17	16 51	1 46	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40
16	14	15	13	12	11	10	8	6	15 11	0 36	10 9	18 0	2 21	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58
17	15	16	14	13	12	11	9	7	15 56	1 11	10 53	19 18	3 1	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14
18	16	17	15	14	13	12	10	8	16 51	1 46	11 33	20 31	3 44	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17
19	17	18	16	15	14	13	11	9	18 0	2 21	12 8	21 31	4 37	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9
20	18	19	17	16	15	14	12	10	19 18	3 1	12 45	22 21	5 40	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53
21	19	20	18	17	16	15	13	11	20 31	3 44	13 19	23 3	6 58	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33
22	20	21	19	18	17	16	14	12	21 31	4 37	13 54	23 42	8 14	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8
23	21	22	20	19	18	17	15	13	22 21	5 40	14 30	0 0	9 17	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45
24	22	23	21	20	19	18	16	14	23 3	6 58	15 11	0 36	10 9	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19
25	23	24	22	21	20	19	17	15	23 42	8 14	15 56	1 11	10 53	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54
26	24	25	23	22	21	20	18	16	0 0	9 17	16 51	1 46	11 33	21 31	4 37	13 54	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30
27	25	26	24	23	22	21	19	17	0 36	10 9	18 0	2 21	12 8	22 21	5 40	14 30	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11
28	26	27	25	24	23	22	20	18	1 11	10 53	19 18	3 1	12 45	23 3	6 58	15 11	0 36	10 9	18 0	2 21	12 8	21 31	4 37	13 54	23 42	8 14	15 56
29	27	28	26	25	24	23	21	19	1 46	11 33	20 31	3 44	13 19	23 42	8 14	15 56	1 11	10 53	19 18	3 1	12 45	22 21	5 40	14 30	0 0	9 17	16 51
30	28	29	27	26	25	24	22	20	2 21	12 8	21 31	4 37	13 54	0 0	9 17	16 51	1 46	11 33	20 31	3 44	13 19	23 3	6 58	15 11	0 36	10 9	18 0

TABLE LXV.—For Determining the Time of High Water.

Moon's Transit	Moon's Horizontal Parallax. 54′	55′	56′	57′	58′	59′	60′	Moon's Transit	Moon's Transit	Moon's Horizontal Parallax. 54′	55′	56′	57′	58′	59′	60′	Moon's Transit
h m	m	m	m	m	m	m	m	h m	h m	m	m	m	m	m	m	m	h m
0 0	−4	−3	−2	−0	+2	+4	+6	12 0	6 0	−56	−58	−60	−62	−65	−69	−72	18 0
10	6	5	4	3	+0	+1	+2	10	10	52	54	56	59	62	65	68	10
20	8	7	6	5	−0	−0	−1	20	20	49	51	53	55	58	60	63	20
30	10	10	9	8	7	6	5	30	30	46	48	50	51	54	56	58	30
40	12	12	11	10	10	9	8	40	40	43	44	45	47	49	51	53	40
50	15	14	13	13	12	12	11	50	50	38	39	40	41	43	44	45	50
1 0	17	17	16	16	16	15	15	13 0	7 0	32	33	33	34	35	36	37	19 0
10	20	20	19	19	19	19	18	10	10	27	27	28	28	29	30	30	10
20	22	22	22	22	22	22	22	20	20	22	22	22	22	22	22	22	20
30	24	24	25	25	25	25	25	30	30	18	18	17	16	16	15	14	30
40	27	27	28	28	28	29	29	40	40	11	11	10	10	8	−7	−6	40
50	29	30	31	31	31	32	33	50	50	6	−6	−5	−4	−2	0	+1	50
2 0	31	32	33	33	34	35	36	14 0	8 0	−1	+1	+2	+3	+5	+7	+9	20 0
10	34	35	36	36	37	38	39	10	10	+2	4	5	7	9	12	14	10
20	36	37	38	39	39	41	43	20	20	5	7	9	11	14	16	19	20
30	38	39	40	41	42	44	46	30	30	8	10	12	15	18	21	24	30
40	40	41	43	44	46	48	50	40	40	11	13	16	18	21	25	28	40
50	42	43	45	46	48	50	52	50	50	13	16	18	20	23	27	30	50
3 0	44	45	47	49	51	53	55	15 0	9 0	14	17	19	21	24	28	32	21 0
10	46	47	49	51	54	56	58	10	10	15	18	20	23	26	30	34	10
20	48	49	51	53	56	58	61	20	20	17	19	22	25	28	32	36	20
30	50	52	54	56	58	61	64	30	30	16	18	21	24	27	31	35	30
40	52	54	56	58	61	64	67	40	40	16	18	21	24	27	31	35	40
50	53	55	57	60	63	66	69	50	50	16	18	21	23	27	30	34	50
4 0	55	57	59	62	65	69	72	16 0	10 0	15	17	20	23	27	30	34	22 0
10	56	58	61	63	66	70	73	10	10	14	17	20	22	25	29	32	10
20	57	60	63	65	68	72	75	20	20	13	16	18	20	23	27	31	20
30	58	61	64	66	69	73	76	30	30	12	15	17	19	22	26	30	30
40	59	62	65	67	70	74	78	40	40	11	13	16	18	21	25	28	40
50	60	62	65	67	70	75	79	50	50	9	11	14	16	19	22	25	50
5 0	60	63	66	68	71	75	79	17 0	11 0	7	9	12	14	17	20	23	23 0
10	60	63	66	68	72	76	80	10	10	6	8	10	12	15	17	20	10
20	60	63	66	68	71	75	80	20	20	4	6	7	9	11	14	17	20
30	59	62	65	67	70	74	78	30	30	+2	4	6	7	9	12	14	30
40	58	61	63	65	68	72	76	40	40	0	+2	4	5	7	9	11	40
50	57	60	62	65	68	71	74	50	50	−2	−1	1	2	4	6	8	50
6 0	56	58	61	64	67	69	72	18 0	12 0	−1	0	+0	+0	+2	+4	+6	24 0

(PART I.) TABLE LXVI.—For Finding the Height of the Tide. (PART II.)

Time of Transit.		Moon's Hor. Par. 60′ Multipliers.	Moon's Hor. Par. 57′ Multipliers.	Moon's Hor. Par. 54′ Multipliers.	Time from H. W.	Mult.	Time from H. W.	Mult.
h m	h m				h m		h m	
0 0	12 0	0.995a+0.149b	0.883a+0.117b	0.795a+0.082b	0 0	1.000	3 10	0.510
0 40	12 40	1.104a+0.038b	0.970a+0.030b	0.874a+0.021b	0 10	0.998	3 20	0.460
1 20	13 20	1.138a+0.000b	1.000a+0.000b	0.901a+0.000b	0 20	0.993	3 30	0.429
2 0	14 0	1.104a+0.038b	0.970a+0.030b	0.874a+0.021b	0 30	0.985	3 40	0.389
2 40	14 40	0.995a+0.149b	0.883a+0.117b	0.795a+0.082b	0 40	0.974	3 50	0.349
3 20	15 20	0.853a+0.319b	0.750a+0.250b	0.676a+0.176b	0 50	0.959	4 0	0.311
4 0	16 0	0.668a+0.527b	0.587a+0.413b	0.529a+0.290b	1 0	0.941	4 10	0.274
4 40	16 40	0.460a+0.749b	0.413a+0.587b	0.372a+0.412b	1 10	0.921	4 20	0.238
5 20	17 20	0.284a+0.958b	0.250a+0.750b	0.225a+0.527b	1 20	0.897	4 30	0.204
6 0	18 0	0.133a+1.127b	0.117a+0.883b	0.105a+0.621b	1 30	0.871	4 40	0.173
6 40	18 40	0.034a+1.238b	0.030a+0.970b	0.027a+0.682b	1 40	0.843	4 50	0.143
7 20	19 20	0.000a+1.277b	0.000a+1.000b	0.000a+0.703b	1 50	0.812	5 0	0.116
8 0	20 0	0.034a+1.238b	0.030a+0.970b	0.027a+0.682b	2 0	0.779	5 10	0.091
8 40	20 40	0.133a+1.127b	0.117a+0.883b	0.105a+0.621b	2 10	0.774	5 20	0.069
9 20	21 20	0.284a+0.958b	0.250a+0.750b	0.225a+0.527b	2 20	0.708	5 30	0.050
10 0	22 0	0.460a+0.749b	0.413a+0.587b	0.372a+0.412b	2 30	0.670	5 40	0.033
10 40	22 40	0.668a+0.527b	0.587a+0.413b	0.529a+0.290b	2 40	0.631	5 50	0.020
11 20	23 20	0.853a+0.319b	0.750a+0.250b	0.676a+0.176b	2 50	0.591	6 0	0.010
12 0	24 0	0.995a+0.149b	0.883a+0.117b	0.795a+0.082b	3 0	0.551	6 20	0.000

TABLE LXVII.

Diminution of the Semidiameters of the Sun and Moon on account of their Declinations.

Dec.	0′	10′	20′	30′	40′	50′	Dec.	0′	10′	20′	30′	40′	50′	Dec.	0′	10′	20′	30′	40′	50′
°	″	″	″	″	″	″	°	″	″	″	″	″	″	°	″	″	″	″	″	″
1	0	0	0	0	0	0	11	12	12	12	13	13	13	21	42	43	44	45	45	46
2	0	0	1	1	1	1	12	14	14	15	15	15	16	22	47	47	48	49	49	50
3	1	1	1	1	1	1	13	16	17	17	18	18	18	23	51	52	52	53	54	55
4	2	2	2	2	2	2	14	19	19	20	20	21	21	24	55	56	57	58	59	59
5	2	3	3	3	3	3	15	22	22	23	23	24	24	25	60	61	62	63	63	64
6	3	4	4	4	4	5	16	25	25	26	26	27	27	26	65	66	67	68	68	69
7	5	5	5	5	6	6	17	28	28	29	30	30	31	27	70	71	72	73	74	75
8	6	6	7	7	7	8	18	31	32	32	33	34	34	28	75	76	77	78	79	80
9	8	8	8	9	9	9	19	35	35	36	37	37	38	29	81	82	83	84	85	86
10	10	10	10	11	11	11	20	39	39	40	40	41	42	30	87	88	89	90	91	92

TABLE LXVIII.—Difference of the Diminished Semidiameters.

Moon's Transit. Subtract	−3′	−2′ 40″	−2′ 20″	−2′	−1′ 40″	−1′ 20″	−1′	−0′ 40″	−0′ 20″	0″	+0′ 20″	+0′ 40″	+1′	+1′ 20″	+1′ 40″	+2′	+2′ 30″	Moon's Transit. Add.
h m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	h m
1 20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1 20
30	4	4	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2	10
40	8	7	7	7	6	6	6	5	5	5	5	4	4	4	4	4	3	1 0
50	11	11	10	10	9	9	8	8	8	7	7	7	6	6	6	6	5	50
2 0	15	14	14	13	12	12	11	11	10	10	9	9	8	8	8	7	7	40
10	19	18	17	16	15	15	14	13	13	12	12	11	11	10	10	9	9	30
20	22	21	20	19	18	18	17	16	15	14	14	13	13	12	11	11	10	20
30	26	25	24	23	21	20	19	18	18	17	16	15	15	14	13	13	12	10
40	30	28	27	26	24	23	22	21	20	19	18	17	16	16	15	14	13	12 0
50	33	32	30	29	27	26	25	23	22	21	20	19	18	17	17	16	15	50
3 0	37	35	33	32	30	29	27	26	24	23	22	21	20	19	18	17	16	40
10	40	38	37	35	33	31	30	28	27	25	24	23	22	21	20	19	17	30
20	44	42	40	37	35	34	32	30	29	27	26	25	23	22	21	20	19	20
30	47	45	42	40	38	36	34	32	31	29	28	26	25	24	22	21	20	10
40	51	48	45	43	40	38	36	34	32	31	29	28	26	25	24	23	21	11 0
50	54	51	48	45	43	40	38	36	34	32	31	29	28	26	25	24	22	50
4 0	57	54	51	48	45	42	40	38	36	34	32	30	29	27	26	24	23	40
10	60	56	53	50	47	44	42	39	37	35	33	31	30	28	27	25	23	30
20	62	59	55	52	49	46	43	41	38	36	34	32	30	29	27	26	24	20
30	65	61	57	54	50	47	44	42	39	37	35	33	31	29	28	26	24	10
40	67	63	59	55	52	48	45	42	40	38	35	33	31	30	28	26	24	10 0
50	70	65	61	56	53	49	46	43	40	38	36	33	31	30	28	26	24	50
5 0	71	67	62	57	53	50	46	43	41	38	36	33	31	30	28	26	24	40
10	73	68	63	58	54	50	46	43	40	38	35	33	31	29	27	26	24	30
20	74	69	63	58	54	50	46	43	40	37	35	32	30	29	27	25	23	20
30	75	69	63	58	53	49	45	42	39	36	34	32	30	28	26	24	22	10
40	75	69	63	57	52	48	44	41	38	35	33	30	28	26	25	23	21	9 0
50	74	68	61	56	50	46	42	39	36	33	31	29	27	25	23	22	20	50
6 0	72	66	59	53	48	44	40	37	34	31	29	27	25	23	22	20	18	40
10	69	63	56	50	45	41	37	34	31	29	26	24	23	21	20	18	17	30
20	65	59	52	45	41	37	33	30	28	26	24	22	20	19	17	16	15	20
30	59	53	47	40	36	32	29	26	24	22	20	19	17	16	15	14	13	10
40	51	46	40	34	30	27	24	22	20	18	17	15	14	13	12	11	10	8 0
50	41	37	32	27	24	21	19	17	15	14	13	12	11	10	9	9	8	50
7 0	29	26	22	18	16	14	13	12	10	9	9	8	7	7	6	6	5	40
10	15	13	11	9	8	7	6	6	5	5	4	4	4	3	3	3	3	30
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7 20
Subtract	−3′	−2′ 40″	−2′ 20″	−2′	−1′ 40″	−1′ 20″	−1′	−0′ 40″	−0′ 20″	0″	+0′ 20″	+0′ 40″	+1′	+1′ 20″	+1′ 40″	+2′	+2′ 30″	Add.
	Numbers to be subtracted always.																	
	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	m	
	30	28	27	26	24	23	22	21	20	19	18	17	16	16	15	14	13	

TABLE LXIX.

To Compute the Longitude of the Moon's Node.

Years.	Long. ☊	Day.		Motion. ☊ —	Day.	Motion. ☊ —
1820	6°.375	Jan.	0	0°.000	*1	0°.053
1821	347 .033		10	0 .530	2	0 .106
1822	327 .692		20	1 .059	3	0 .159
1823	308 .350		30	1 .589	4	0 .212
1824	289 .008	Feb.	9	2 .118	5	0 .265
1825	269 .666		19	2 .648	6	0 .318
1826	250 .324	Mar.	1	3 .177	7	0 .371
1827	230 .983		11	3 .707	8	0 .424
1828	211 .641		21	4 .236	9	0 .477
1829	192 .299		31	4 .766		
1830	172 .957	Apr.	10	5 .296	1h	0°.002
1831	153 .616		20	5 .825	2	0 .004
1832	134 .274		30	6 .355	3	0 .007
1833	114 .932	May.	10	6 .884	4	0 .009
1834	95 .590		20	7 .414	5	0 .011
1835	76 .248		30	7 .943	6	0 .013
1836	56 .907	June.	9	8 .473	7	0 .015
1837	37 .565		19	9 .002	8	0 .018
1838	18 .223		29	9 .532	9	0 .020
1839	358 .881	July.	9	10 .062	10	0 .022
1840	339 .539		19	10 .591	11	0 .024
1841	320 .198		29	11 .121	12	0 .026
1842	300 .856	Aug.	8	11 .650	13	0 .029
1843	281 .514		18	12 .180	14	0 .031
1844	262 .172		28	12 .709	15	0 .033
1845	242 .831	Sept.	7	13 .239	16	0 .035
1846	223 .489		17	13 .768	17	0 .037
1847	204 .147		27	14 .298	18	0 .040
1848	184 .805	Oct.	7	14 .829	19	0 .042
1849	165 .463		17	15 .357	20	0 .044
1850	146 .122		27	15 .887	21	0 .046
		Nov.	6	16 .416	22	0 .049
			16	16 .946	23	0 .051
			26	17 .475		
		Dec.	6	18 .005		
			16	18 .534		
			26	19 .064		
			31	19 .324		

TABLE LXX.—Correction of Moon's Transit. + W. — E.

Long.	Daily Retardation.					
	m 40	m 45	m 50	m 55	m 60	m 65
°	m	m	m	m	m	m
0	0	0	0	0	0	0
10	1	1	1	1	2	2
20	2	2	3	3	3	4
30	3	4	4	4	5	5
40	4	5	6	6	7	7
50	6	6	7	7	8	9
60	7	7	8	9	10	11
70	8	9	10	10	12	13
80	9	10	11	12	13	14
90	10	11	12	13	15	16
100	11	12	14	15	17	18
110	12	13	15	16	18	20
120	13	15	16	18	20	21
130	14	16	17	19	22	23
140	16	17	19	21	23	25
150	17	18	20	22	25	27
160	18	20	21	24	27	28
170	19	21	23	26	28	30
180	20	22	25	27	30	32

TABLE LXXI.—Contraction of the Semidiameters of Sun and Moon.

A	Inclination to Horizon.								
	10°	20°	30°	40°	50°	60°	70°	80°	90°
°	"	"	"	"	"	"	"	"	"
5	1	3	7	11	15	19	22	24	25
6	1	3	5	8	11	14	16	18	19
7	0	2	4	6	8	11	12	14	14
8	0	2	3	5	6	8	10	11	11
9	0	1	2	4	5	6	8	9	9
10	0	1	2	4	4	5	7	7	8
12	0	1	1	2	4	4	5	5	5
14	0	1	1	2	2	3	4	4	4
16	0	0	1	1	2	2	3	3	3
20	0	0	0	1	1	2	2	2	2
30	0	0	0	0	1	1	1	1	1
90	0	0	0	0	0	0	0	0	0

TABLE LXXII.—Solar Nutation of a Star in Right Ascension and Declination.

Deg.	Right Ascension.		Declination.	Deg.
	2 ☉	2 ☉ — R.A.	2 ☉ — R.A.	
°	"	"	"	°
0	—0.00+	—0.47—	—0.00 +	360
10	0.18	0.46	0.08	350
20	0.35	0.44	0.16	340
30	0.51	0.41	0.24	330
40	0.66	0.36	0.30	320
50	0.79	0.30	0.36	310
60	0.89	0.24	0.41	300
70	0.96	0.16	0.44	290
80	1.01	—0.08—	0.46	280
90	1.03	0.00	0.47	270
100	1.01	+0.08+	0.46	260
110	0.96	0.16	0.44	250
120	0.89	0.24	0.41	240
130	0.79	0.30	0.36	230
140	0.66	0.36	0.30	220
150	0.51	0.41	0.24	210
160	0.35	0.44	0.16	200
170	0.18	0.46	0.08	190
180	—0.00+	+ 0.47+	—0.00+	180

TABLE LXXIII.

Degrees of Latitude, Longitude, Length of Pendulum, and Increase of Vibrations.

Lat.	Degree of Latitude.	Degree of Longitude.	Length of Pendulum.	Increase of Vibration.
°				s
0	1.00000	1.00000	1.00000	0.00
5	1.00008	0.99622	1.00004	1.77
10	1.00030	0.98490	1.00016	7.02
15	1.00067	0.96614	1.00036	15.60
20	1.00117	0.94006	1.00063	27.24
25	1.00178	0.90685	1.00096	41.59
30	1.00250	0.86675	1.00135	58.21
35	1.00328	0.82005	1.00177	76.60
40	1.00413	0.76710	1.00223	96.21
45	1.00499	0.70828	1.00269	116.42
50	1.00586	0.64404	1.00316	136.64
55	1.00670	0.57485	1.00362	156.25
60	1.00749	0.50126	1.00404	174.63
65	1.00820	0.42377	1.00443	191.26
70	1.00882	0.34302	1.00476	205.61
75	1.00932	0.25960	4.00503	217.25
80	1.00968	0.17421	1.00523	225.82
85	1.00991	0.08764	1.00535	231.08
90	1.00998	0.00000	1.00539	232.85

* In bissextile years subtract one day in the months of January and February from the number of days.

Days.	January.	February.	March.	April.	May.	June.
	s ° ′	s ° ′	s ° ′	s ° ′	s ° ′	s ° ′
1	9 10 8	10 11 41	11 10 56	0 11 44	1 11 2	2 10 53
2	9 11 9	10 12 42	11 11 57	0 12 44	1 12 0	2 11 50
3	9 12 10	10 13 43	11 12 57	0 13 43	1 12 59	2 12 48
4	9 13 11	10 14 44	11 13 57	0 14 42	1 13 57	2 13 45
5	9 14 12	10 15 44	11 14 57	0 15 41	1 14 55	2 14 43
6	9 15 14	10 16 45	11 15 57	0 16 39	1 15 53	2 15 40
7	9 16 15	10 17 46	11 16 57	0 17 38	1 16 51	2 16 37
8	9 17 16	10 18 47	11 17 56	0 18 37	1 17 46	2 17 35
9	9 18 17	10 19 47	11 18 56	0 19 36	1 18 47	2 18 32
10	9 19 18	10 20 48	11 19 56	0 20 35	1 19 45	2 19 29
11	9 20 19	10 21 49	11 20 56	0 21 34	1 20 42	2 20 27
12	9 21 21	10 22 49	11 21 56	0 22 32	1 21 40	2 21 24
13	9 22 22	10 23 50	11 22 56	0 23 31	1 22 38	2 22 21
14	9 23 23	10 24 51	11 23 55	0 24 30	1 23 36	2 23 19
15	9 24 24	10 25 51	11 24 55	0 25 29	1 24 34	2 24 16
16	9 25 25	10 26 52	11 25 55	0 26 27	1 25 32	2 25 13
17	9 26 26	10 27 52	11 26 54	0 27 26	1 26 29	2 26 11
18	9 27 27	10 28 53	11 27 54	0 28 24	1 27 27	2 27 8
19	9 28 29	10 29 53	11 28 54	0 29 23	1 28 25	2 28 5
20	9 29 30	11 0 54	11 29 53	1 0 21	1 29 23	2 29 2
21	10 0 31	11 1 54	0 0 53	1 1 20	2 0 20	3 0 0
22	10 1 32	11 2 55	0 1 52	1 2 18	2 1 18	3 0 57
23	10 2 33	11 3 55	0 2 51	1 3 17	2 2 16	3 1 54
24	10 3 34	11 4 55	0 3 51	1 4 15	2 3 13	3 2 51
25	10 4 35	11 5 56	0 4 50	1 5 13	2 4 11	3 3 48
26	10 5 36	11 6 56	0 5 49	1 6 12	2 5 8	3 4 46
27	10 6 37	11 7 56	0 6 49	1 7 10	2 6 6	3 5 43
28	10 7 38	11 8 56	0 7 48	1 8 8	2 7 3	3 6 40
29	10 8 39	11 9 55	0 8 47	1 9 6	2 8 0	3 7 37
30	10 9 39		0 9 47	1 10 4	2 8 58	3 8 34
31	10 10 40		0 10 45		2 9 56	

Days.	July.	August.	September.	October.	November.	December.
	s ° ′	s ° ′	s ° ′	s ° ′	s ° ′	s ° ′
1	3 9 32	4 9 7	5 8 57	6 8 15	7 9 3	8 9 20
2	3 10 29	4 10 5	5 9 55	6 9 14	7 10 4	8 10 21
3	3 11 26	4 11 2	5 10 54	6 10 13	7 11 4	8 11 22
4	3 12 23	4 11 59	5 11 52	6 11 13	7 12 4	8 12 22
5	3 13 20	4 12 57	5 12 50	6 12 12	7 13 4	8 13 24
6	3 14 18	4 13 54	5 13 48	6 13 11	7 14 5	8 14 25
7	3 15 15	4 14 52	5 14 47	6 14 10	7 15 5	8 15 26
8	3 16 12	4 15 50	5 15 45	6 15 10	7 16 5	8 16 27
9	3 17 9	4 16 47	5 16 43	6 16 9	7 17 6	8 17 28
10	3 18 6	4 17 45	5 17 42	6 17 9	7 18 6	8 18 29
11	3 19 4	4 18 42	5 18 40	6 18 8	7 19 7	8 19 30
12	3 20 0	4 19 40	5 19 39	6 19 7	7 20 7	8 20 31
13	3 20 58	4 20 38	5 20 37	6 20 7	7 21 8	8 21 32
14	3 21 55	4 21 35	5 21 36	6 21 7	7 22 8	8 22 33
15	3 22 53	4 22 33	5 22 34	6 22 6	7 23 9	8 23 35
16	3 23 50	4 23 31	5 23 33	6 23 6	7 24 9	8 24 36
17	3 24 47	4 24 29	5 24 31	6 24 5	7 25 10	8 25 37
18	3 25 45	4 25 26	5 25 30	6 25 5	7 26 10	8 26 38
19	3 26 42	4 26 24	5 26 29	6 26 5	7 27 11	8 27 39
20	3 27 39	4 27 22	5 27 27	6 27 4	7 28 11	8 28 40
21	3 28 36	4 28 20	5 28 26	6 28 4	7 29 12	8 29 41
22	3 29 34	4 29 18	5 29 25	6 29 4	8 0 13	9 0 42
23	4 0 31	5 0 15	6 0 24	7 0 4	8 1 14	9 1 44
24	4 1 28	5 1 13	6 1 22	7 1 3	8 2 14	9 2 45
25	4 2 26	5 2 11	6 2 21	7 2 3	8 3 15	9 3 46
26	4 3 23	5 3 9	6 3 20	7 3 3	8 4 16	9 4 47
27	4 4 20	5 4 7	6 4 19	7 4 3	8 5 17	9 5 48
28	4 5 18	5 5 5	6 5 18	7 5 3	8 6 17	9 6 49
29	4 6 15	5 6 3	6 6 17	7 6 3	8 7 18	9 7 51
30	4 7 12	5 7 1	6 7 16	7 7 3	8 8 19	9 8 52
31	4 8 10	5 7 59		7 8 3		9 9 53

TABLE LXXV.—Reduced Logarithmic Versines, or Log. Sin² ½ P.

°	′	m	0° 0 h	Diff. to 25′ or 100s.	15° 1 h	Diff. to 25′ or 100s.	30° 2 h	Diff. to 25′ or 100s.	45° 3 h	Diff. to 25′ or 100s.	m	°	′
0	0	0	0.000000		8.231395	23790	8.825992	11737	9.165679	7602	60	15	0
0	15	1	4.677574		245669	23398	833034	11635	170240	7555	59	14	45
0	30	2	5.279632		259708	23018	840015	11535	174773	7509	58	14	30
0	45	3	5.631811		273519	22653	846936	11437	179278	7463	57	14	15
1	0	4	5.881684		287111	22295	853798	11340	183756	7418	56	14	0
1	15	5	6.075498		300488	21952	860602	11245	188207	7373	55	13	45
1	30	6	6.233852		313659	21617	867349	11152	192631	7329	54	13	30
1	45	7	6.367737		326629	21292	874040	11060	197028	7286	53	13	15
2	0	8	6.483711		339404	20977	880676	10970	201399	7243	52	13	0
2	15	9	6.586004		351990	20670	887258	10880	205745	7198	51	12	45
2	30	10	6.677506		8.364392	20372	8.893785	10793	9.210064	7157	50	12	30
2	45	11	6.760277		376615	20083	900261	10705	214358	7115	49	12	15
3	0	12	6.835838		388665	19802	906684	10618	218627	7072	48	12	0
3	15	13	6.905345		400546	19527	913055	10536	222870	7032	47	11	45
3	30	14	6.969696		412262	19260	919377	10452	227089	6992	46	11	30
3	45	15	7.029602		423818	19000	925648	10372	231284	6950	45	11	15
4	0	16	7.085638		435218	18748	931871	10290	235454	6910	44	11	0
4	15	17	7.138273		446467	18502	938045	10210	239600	6870	43	10	45
4	30	18	7.187897		457568	18260	944171	10132	243722	6832	42	10	30
4	45	19	7.234833		468524	18027	950251	10055	247821	6793	41	10	15
5	0	20	7.279359	70585	8.479340	17798	8.956284	9978	9.251897	6753	40	10	0
5	15	21	321710	67295	490019	17575	962271	9903	255949	6715	39	9	45
5	30	22	362087	64298	500564	17358	968213	9830	259978	6678	38	9	30
5	45	23	400666	61557	510979	17145	974111	9756	263985	6640	37	9	15
6	0	24	437600	59040	521266	16938	979965	9683	267969	6602	36	9	0
6	15	25	473024	56720	531429	16735	985775	9613	271930	6567	35	8	45
6	30	26	507056	54573	541470	16537	991543	9543	275870	6530	34	8	30
6	45	27	539800	52585	551392	16343	997269	9473	279788	6493	33	8	15
7	0	28	571351	50733	561198	16153	9.002953	9405	283684	6457	32	8	0
7	15	29	601791	49010	570890	15968	008596	9337	287558	6423	31	7	45
7	30	30	7.631197	47398	8.580471	15788	9.014198	9273	9.291412	6387	30	7	30
7	45	31	659636	45888	589944	15612	019761	9205	295244	6352	29	7	15
8	0	32	687169	44472	599311	15437	025284	9140	299055	6317	28	7	0
8	15	33	713852	43140	608573	15268	030768	9075	302845	6283	27	6	45
8	30	34	739736	41885	617734	15102	036213	9013	306615	6249	26	6	30
8	45	35	764867	40700	626795	14938	041621	8950	310364	6216	25	6	15
9	0	36	789287	39580	635758	14778	046991	8887	314094	6182	24	6	0
9	15	37	813035	38520	644625	14623	052323	8827	317803	6148	23	5	45
9	30	38	836147	37515	653399	14470	057619	8767	321492	6115	22	5	30
9	45	39	858656	36560	662081	14322	062879	8707	325161	6083	21	5	15
10	0	40	7.880592	35653	8.670674	14172	9.068103	8648	9.328811	6052	20	5	0
10	15	41	901984	34790	679177	14030	073292	8590	332442	6019	19	4	45
10	30	42	922858	33965	687595	13887	078446	8532	336053	5987	18	4	30
10	45	43	943237	33182	695927	13748	083565	8477	339645	5956	17	4	15
11	0	44	963146	32430	704176	13612	088651	8418	343219	5924	16	4	0
11	15	45	982604	31713	712343	13480	093702	8363	346773	5893	15	3	45
11	30	46	8.001632	31027	720431	13347	098720	8310	350309	5863	14	3	30
11	45	47	020248	30368	728439	13220	103706	8255	353827	5832	13	3	15
12	0	48	038469	29738	736371	13092	108658	8202	357326	5802	12	3	0
12	15	49	056312	29133	744226	12968	113579	8148	360807	5772	11	2	45
12	30	50	8.073792	28550	8.752007	12847	9.118468	8095	9.364270	5742	10	2	30
12	45	51	090922	27993	759715	12725	123325	8045	367715	5712	9	2	15
13	0	52	107718	27453	767350	12610	128151	7992	371142	5683	8	2	0
13	15	53	124190	26937	774916	12492	132946	7942	374552	5655	7	1	45
13	30	54	140352	26438	782411	12380	137711	7892	377945	5625	6	1	30
13	45	55	156215	25957	789839	12267	142446	7842	381320	5597	5	1	15
14	0	56	171789	25493	797199	12158	147151	7792	384678	5567	4	1	0
14	15	57	187085	25045	804494	12048	151826	7744	388018	5540	3	0	45
14	30	58	202112	24612	811723	11943	156473	7695	391342	5513	2	0	30
14	45	59	216879	24193	818889	11838	161090	7648	394650	5483	1	0	15
15	0	60	8.231395		8.825992		9.165679		9.397940		0	0	0
°	′	m	345° 23 h		330° 22 h		315° 21 h		300° 20 h		m	°	′

° ′	m	60° 4 h	Diff. to 25′ or 100s.	75° 5 h	Diff. to 25′ or 100s.	90° 6 h	Diff. to 25′ or 100s.	105° 7 h	Diff. to 25′ or 100s.	m	° ′
0 0	0	9.397940	5457	9.568894	4107	9.698970	3152	9.798933	2418	60	15 0
0 15	1	401214	5428	571358	4088	[illegible]	3138	800384	2407	59	14 45
0 30	2	404471	5403	573811	[illegible]	702743	3125	801828	2397	58	14 30
0 45	3	407713	5375	[illegible]	4052	704618	3110	803266	2385	57	14 15
1 0	4	410938	5348	578684	4033	706484	3097	804697	2375	56	14 0
1 15	5	[illegible]	5322	581104	4015	708342	3083	806122	2363	55	13 45
1 30	6	417340	5295	583513	3997	710192	3070	807540	2353	54	13 30
1 45	7	420517	5270	585911	3980	712034	3057	808952	2342	53	13 15
2 0	8	423679	5243	588299	3962	713868	3043	810357	2332	52	13 0
2 15	9	426825	5217	590676	3943	715694	3030	811756	2322	51	12 45
2 30	10	9.429955	5192	9.593042	3927	9.717512	3016	9.813149	2310	50	12 30
2 45	11	433070	5167	595398	3910	719322	3004	814535	2300	49	12 15
3 0	12	436170	5142	597744	3892	721124	2992	815915	2290	48	12 0
3 15	13	439255	5117	600078	3875	722919	2977	817289	2278	47	11 45
3 30	14	442325	5090	602403	3857	724705	2965	818656	2268	46	11 30
3 45	15	445379	5067	604717	3840	726484	2952	820017	2258	45	11 15
4 0	16	448419	5043	607021	3823	728255	2938	821372	2248	44	11 0
4 15	17	451445	5017	609315	3805	730018	2927	822721	2237	43	10 45
4 30	18	454455	4993	611598	3789	731774	2912	824063	2227	42	10 30
4 45	19	457451	4970	613872	3772	733522	2900	825399	2217	41	10 15
5 0	20	9.460433	4946	9.616135	3755	9.735262	2887	9.826729	2206	40	10 0
5 15	21	463400	4922	618388	3740	736994	2875	828053	2196	39	9 45
5 30	22	466354	4898	620632	3722	738719	2863	829370	2186	38	9 30
5 45	23	469293	4875	622865	3707	740437	2850	830682	2175	37	9 15
6 0	24	472218	4852	625089	3690	742147	2837	831987	2167	36	9 0
6 15	25	475129	4828	627303	3673	743849	2825	833287	2155	35	8 45
6 30	26	478026	4805	629507	3657	745544	2813	834580	2145	34	8 30
6 45	27	480909	4783	631701	3642	747232	2800	835867	2136	33	8 15
7 0	28	483779	4760	633886	3625	748912	2787	837148	2126	32	8 0
7 15	29	486635	4738	636061	3610	750585	2777	838424	2115	31	7 45
7 30	30	9.489478	4715	9.638227	3593	9.752251	2763	9.839693	2105	30	7 30
7 45	31	492307	4693	640383	3577	753909	2751	840956	2095	29	7 15
8 0	32	495123	4672	642529	3562	755560	2738	842213	2085	28	7 0
8 15	33	497926	4650	644666	3547	757203	2727	843464	2076	27	6 45
8 30	34	500716	4628	646794	3532	758840	2715	844710	2065	26	6 30
8 45	35	503492	4607	648913	3516	760469	2703	845949	2056	25	6 15
9 0	36	506256	4585	651022	3500	762091	2692	847183	2045	24	6 0
9 15	37	509007	4563	653122	3485	763706	2680	848410	2036	23	5 45
9 30	38	511745	4542	655213	3470	765314	2668	849632	2026	22	5 30
9 45	39	514470	4522	657294	3455	766914	2656	850848	2017	21	5 15
10 0	40	9.517183	4500	9.659367	3439	9.768508	2643	9.852058	2008	20	5 0
10 15	41	519883	4478	661430	3424	770094	2633	853263	1997	19	4 45
10 30	42	522570	4458	663485	3408	771674	2621	854461	1988	18	4 30
10 45	43	525245	4438	665530	3395	773247	2609	855654	1978	17	4 15
11 0	44	527908	4418	667567	3379	774812	2598	856841	1969	16	4 0
11 15	45	530559	4397	669594	3365	776371	2586	858022	1960	15	3 45
11 30	46	533197	4377	671613	3350	777922	2575	859198	1950	14	3 30
11 45	47	535823	4357	673623	3335	779467	2563	860367	1941	13	3 15
12 0	48	538437	4338	675624	3321	781005	2552	861532	1930	12	3 0
12 15	49	541040	4318	677617	3307	782536	2542	862690	1921	11	2 45
12 30	50	9.543630	4297	9.679601	3292	9.784061	2528	9.863843	1912	10	2 30
12 45	51	546208	4278	681576	3277	785578	2518	864990	1902	9	2 15
13 0	52	548775	4258	683543	3262	787089	2507	866131	1893	8	2 0
13 15	53	551330	4240	685501	3248	788593	2495	867267	1884	7	1 45
13 30	54	553874	4220	687450	3235	790090	2483	868397	1875	6	1 30
13 45	55	556406	4200	689391	3222	791580	2473	869522	1865	5	1 15
14 0	56	558926	4182	691324	3207	793064	2462	870641	1855	4	1 0
14 15	57	561435	4163	693248	3192	794541	2452	871754	1846	3	0 45
14 30	58	563933	4143	695163	3179	796012	2440	872862	1837	2	0 30
14 45	59	566419	4124	697071	3165	797476	2428	873964	1828	1	0 15
15 0	60	9.568894		9.698970		9.798933		9.875061		0	0 0
° ′	m	285° 19 h		270° 18 h		255° 17 h		240° 16 h		m	° ′